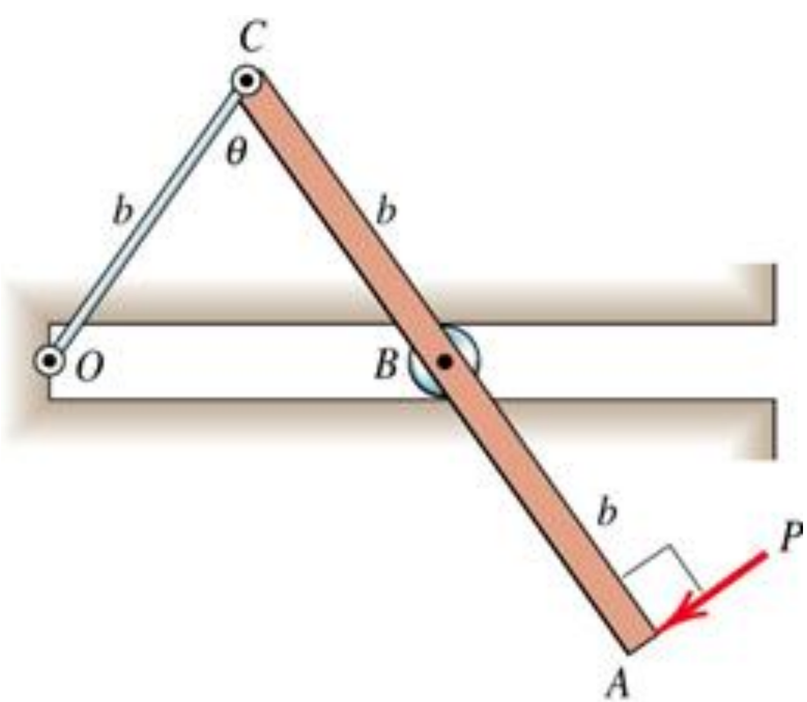
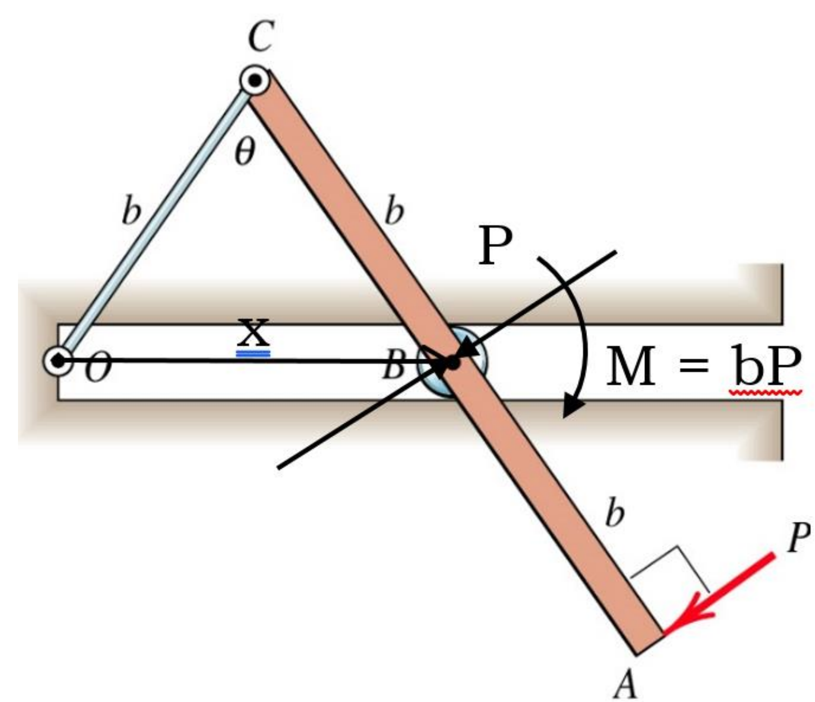
6/152 (4th), 6/151 (5th), None (6th), None (7th), None (8th)

The uniform bar ABC has a mass m and starts from rest with θ = 180° where A, B, C, and O are collinear. If the applied force P is constant in magnitude, determine the angular velocity ω of the bar as B reaches O with θ = 0. The mass of the roller at B and the mass of the strut OC are negligible.

Hint: Replace P by a force P at B and a couple moment.





This is a constrained system

$$U_{1\to 2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to 2} = \int_1^2 dU$$

$$x = 2b\sin\frac{\theta}{2}$$

$$dx = b\cos\frac{\theta}{2}d\theta$$

$$dU = -M\frac{d\theta}{2} - P\cos\frac{\theta}{2}dx = -Pb\left(\frac{1}{2} + \cos^2\frac{\theta}{2}\right)$$

$$U_{1\to 2} = \int_\pi^0 -Pb\left(\frac{1}{2} + \cos^2\frac{\theta}{2}\right)d\theta = Pb\pi$$

$$\Delta T = \frac{1}{2}m{v_G}^2 + \frac{1}{2}I_G\omega^2$$

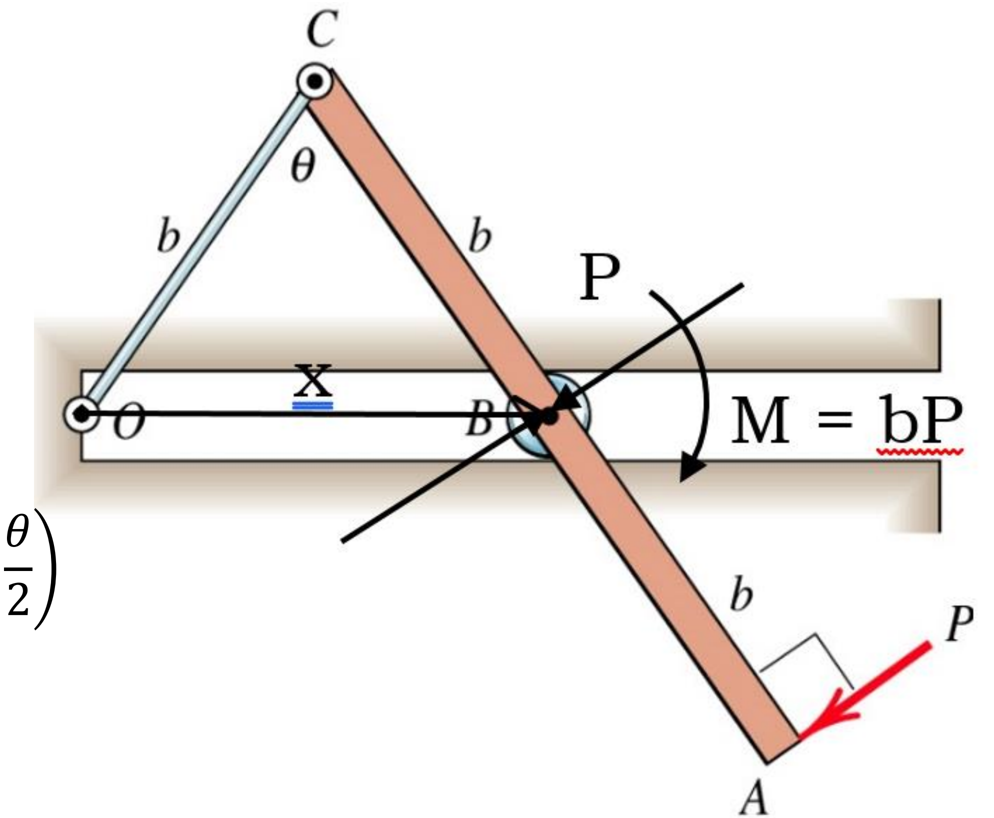
$$v_G = v_B = -\dot{x} = -b\cos\frac{\theta}{2}\dot{\theta}$$

$$\Delta T = \frac{1}{2}m\left(2b\cos\frac{0}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m(2b)^2\right)\omega^2 = \frac{13}{6}mb^2\omega^2$$

$$\Delta V_g = 0$$

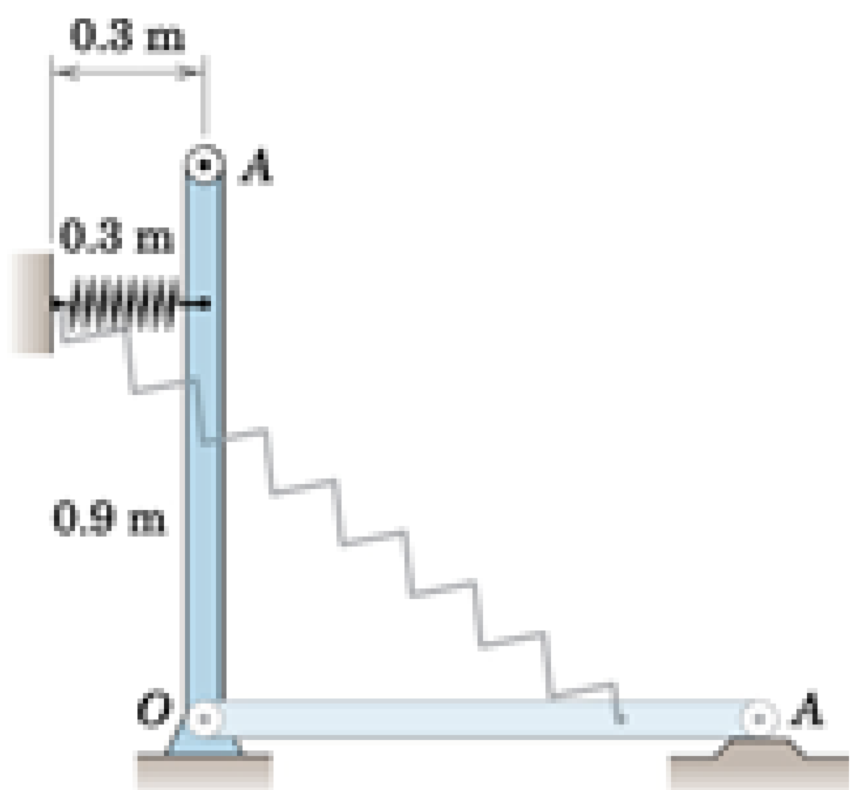
$$\Delta V_e = 0$$

$$\omega = \sqrt{\frac{6\pi P}{13mb}}$$



6/213 (4th), 6/219 (5th), 6/220 (6th), None (7th), 6/214 (8th)

The uniform slender bar has a mass of 30 kg and is released from rest in the near-vertical position shown, where the spring of stiffness 150 N/m is unstretched. Calculate the velocity with which end A strikes the horizontal surface.



$$U_{1\to2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to2} = 0$$

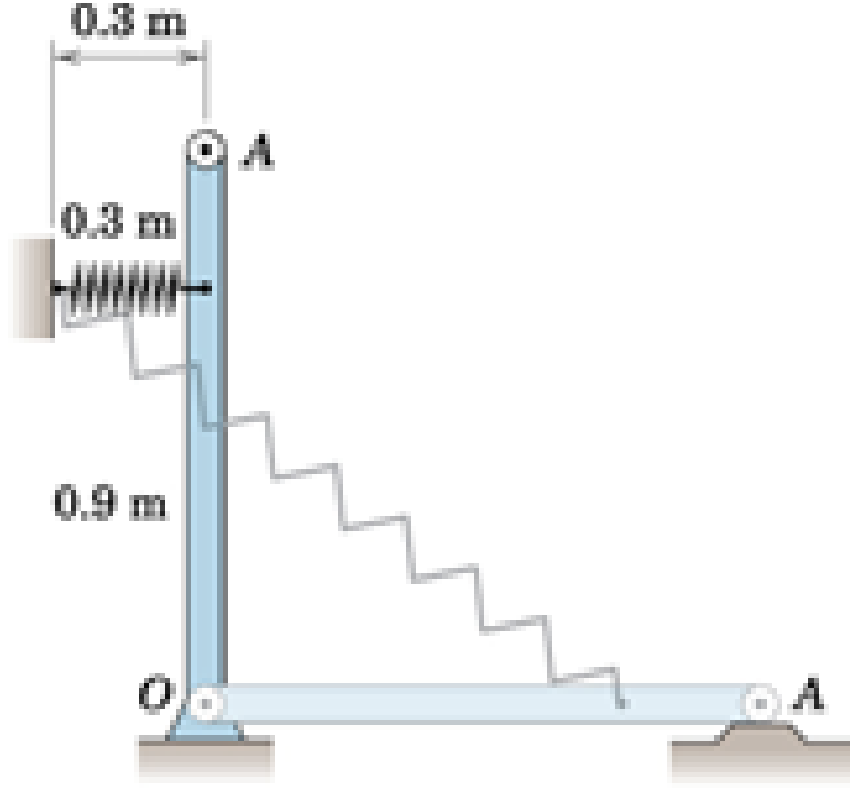
$$\Delta T = \frac{1}{2} I_O \omega^2 = \frac{1}{2}\frac{1}{3} m\ell^2 \left(\frac{v_A}{\ell}\right)^2 = 5{v_A}^2$$

$$\Delta V_g = mg\Delta h = 30 * 9.81 * (-0.6) = -176.6\ J$$

$$\Delta V_e = \frac{1}{2} kx^2, x = \sqrt{0.9^2 + 1.2^2} - 0.3 = 1.2\ m$$

$$\Delta V_e = \frac{1}{2} 150 * 1.2^2 = 108\ J$$

$$v_A = 3.70\ m/s$$



C. Impulse and Momentum

6/8 Impulse-Momentum Equations

- *Linear Momentum*

$$\vec{G} = \frac{d}{dt}\sum_{i=1}^{n} m_i\vec{v}_i = mv_G$$

$$\sum \vec{F} = \dot{\vec{G}}$$

$$\int_{t_1}^{t_2}\sum \vec{F}\,dt = \vec{G}(t_2) - \vec{G}(t_1) = \Delta\vec{G}\begin{cases} x - y \\ n - t \\ r - \theta \end{cases}$$

- *Angular Momentum*

$$\vec{H}_G = \sum_{i=1}^{n} \vec{\rho}_i \times m_i \vec{v}_i = \sum_{i=1}^{n} \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i \,,$$

For a rigid body

$$\dot{\vec{\rho}}_i = \vec{\omega} \times \vec{\rho}_i, \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i = m_i(\vec{\rho}_i \times \vec{\omega} \times \vec{\rho}_i) = m_i {\rho_i}^2 \omega \hat{k}$$

$$\sum_{i=1}^{n} m_i {\rho_i}^2 = I_G$$

$$\vec{H}_G = I_G \vec{\omega} = I_G \omega \hat{k}$$

$$\sum \vec{M}_G = \dot{\vec{H}}_G$$

$$\int_{t_1}^{t_2} \sum \vec{M}_G \, dt = \vec{H}_G(t_2) - \vec{H}_G(t_1) = \Delta \vec{H}_G$$

$$H_P = I_G \omega + moment\ of\ m\vec{v}_G\ about\ P$$

$$H_O = I_O \omega$$

- *Conservation of Momentum*

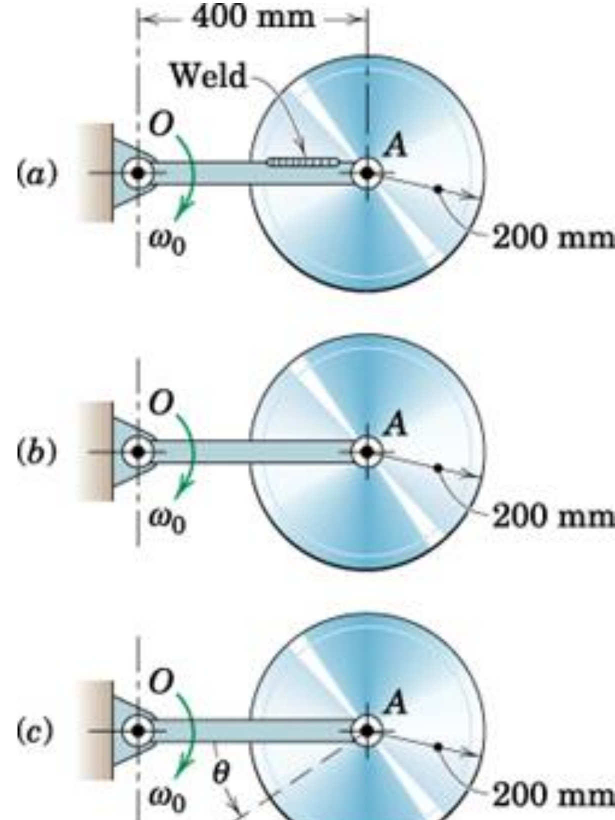
If $\sum \vec{F} = \vec{0}$ for a certain interval of time then $\Delta\vec{G} = \vec{0}$

If $\sum M_P = 0$ for a certain interval of time then $\Delta H_P = 0$

Example 6/180 (4th), 6/181 (5th), 6/187 (6th), None (7th), None (8th)

The uniform circular disk of 200 mm radius has a mass of 25 kg and is mounted on the rotating bar OA in three different ways. In each case the bar rotates about its vertical shaft at O with a clockwise angular velocity $\omega_0 = 4$ rad/s. In case (a) the disk is welded to the bar. In case (b) the disk which is pinned freely at A moves with curvilinear translation therefore has no rigid body rotation. In case (c) the relative angle between the disk and bar is increasing at the rate 8 rad/s.

Calculate the angular momentum of the disk about point O for each case.



Mass of the rod is negligible (since not stated) therefore its mass moment of inertia and angular momentum are also negligible.

For a uniform disk:

$$I_G = I_A = \frac{1}{2}mr^2 = \frac{1}{2}25 * 0.2^2 = 0.5\,kg.m^2$$

a. $\omega_{disk} = \omega_0$

$$H_O = \begin{cases} I_G\omega_0 + mv_G|AO| = 0.5 * 4 + 25 * 0.4 * 4 * 0.4 = 18\,kg.m^2/s\ (CW) \\ I_O\omega_0 = [I_G + m|OA|^2]\omega = [0.5 + 25 * 0.4^2] * 4 = 18\,kg.m^2/s\ (CW) \end{cases}$$

b. $\omega_{disk} = 0$

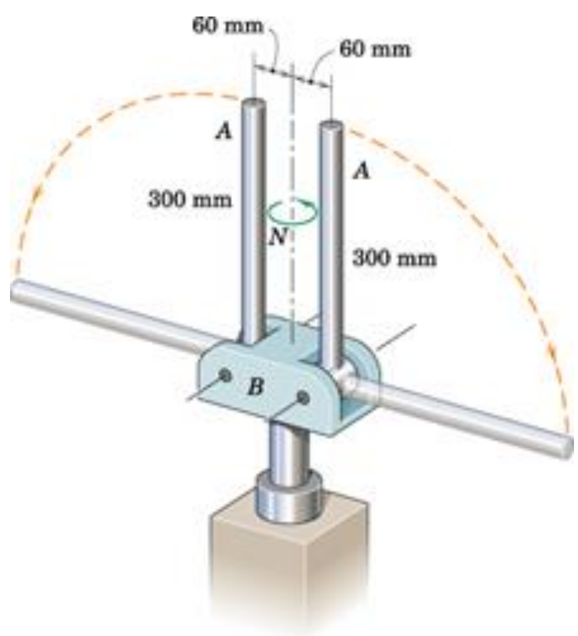
$$H_O = I_G\omega_{disk} + mv_G|AO| = 0.5 * 0 + 25 * 0.4 * 4 * 0.4$$
$$= 16\,kg.m^2/s\ (CW)$$

c. $\omega_{disk} = \omega_0 + \dot{\theta} = 4 + (-8) = -4\,rad/s\ CCW\ (i.e.\ 4\,rad/s\ CW)$

$$H_O = I_G\omega_{disk} + mv_G|AO| = 0.5 * (-4) + 25 * 0.4 * 4 * 0.4$$
$$= 14\,kg.m^2/s\ (CW)$$

6/187 (4th), 6/193 (5th), 6/201 (6th), None (7th), 6/186 (8th)

Each of the two 300 mm uniform rods A has a mass of 1.5 kg and is hinged at its end to the rotating base B. The 4 kg base has a radius of gyration of 40 mm and is initially rotating freely about its vertical axis with a speed of 300 rev/min and with the rods latched in the vertical positions. If the latches are released and the rods assume the horizontal positions, calculate the new rotational speed N of the assembly.



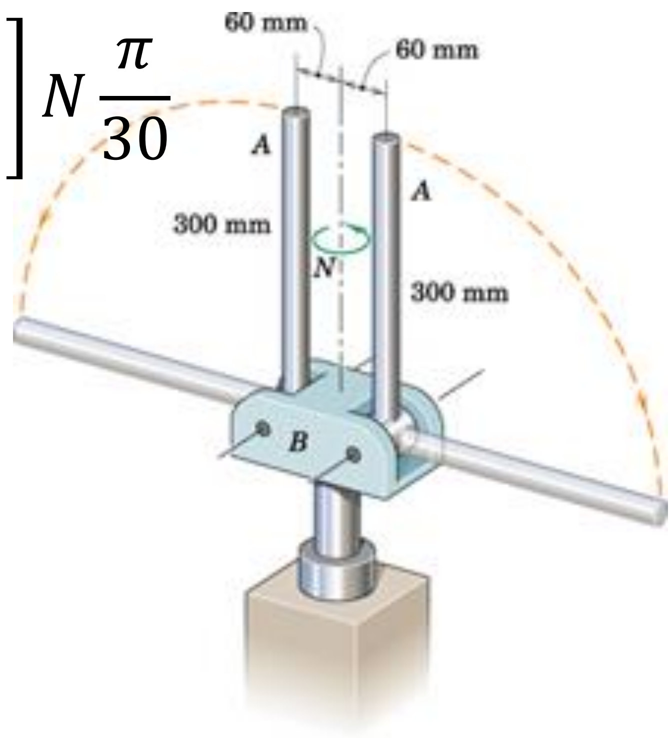
Since there is no moment about the rotation axis angular momentum about the rotation axis is conserved.

$$\Delta H = 0$$

$$H_1 = (2I_{rod} + mk^2)\omega_1 = [2(0 + 1.5 * 0.06^2) + 4 * 0.04^2]300\frac{\pi}{30}$$
$$= 0.172\pi\ kg.m^2/s$$

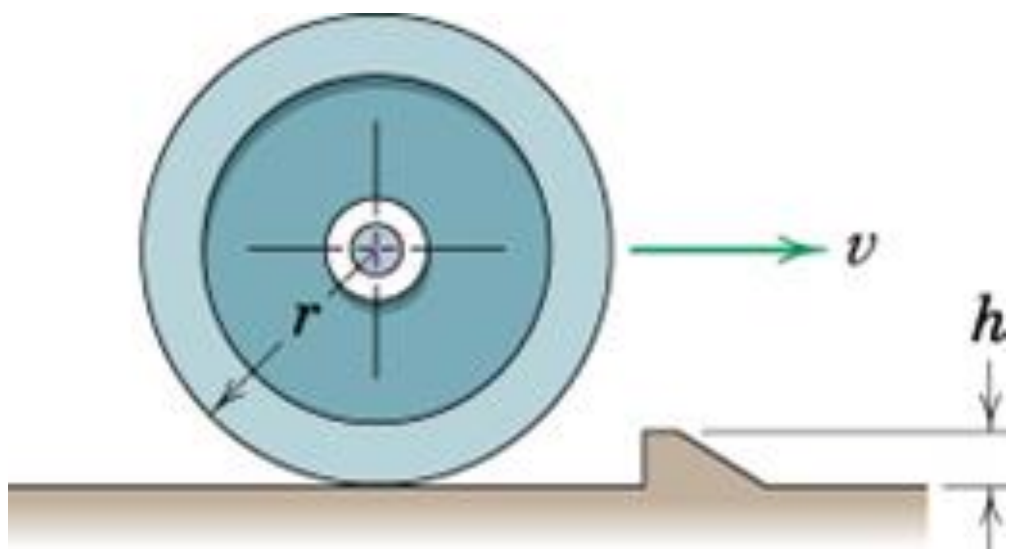
$$H_2 = (2I_{rod} + mk^2)\omega_2$$
$$= \left[2\left(\frac{1}{12}1.5 * 0.3^2 + 1.5 * 0.21^2\right) + 4 * 0.04^2\right]N\frac{\pi}{30}$$

$$N = 32.0\ rpm$$

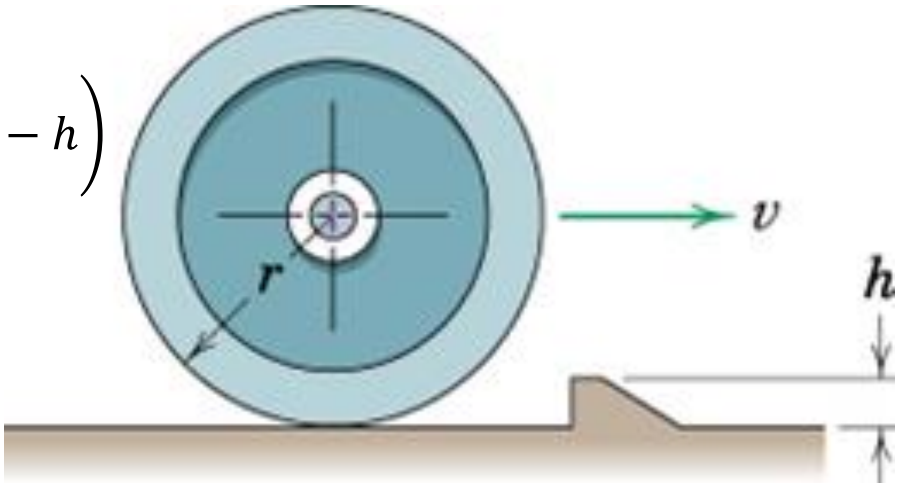


6/200 (4th), 6/204 (5th), 6/206 (6th), 6/199 (7th), None (8th)

Determine the minimum velocity which the wheel must have to just roll over the obstruction. The centroidal radius of gyration of the wheel is k, and it is assumed that the wheel does not slip.



The wheel hits the obstruction the reaction forces from the obstruction has an impulsive character therefore impulse of weight of the wheel, compared to these forces are negligible. During hit (1→2) conservation of angular momentum about the point of hit (say A) may assumed to be conserved. After impact, while climbing over the obstruction (2→3) energy is conserved.



$$\Delta H_A = 0$$

$$H_{A_1} = I_G\omega_{min} + mv_{min}(r-h) = mv_{min}\left(\frac{k^2}{r} + r - h\right)$$

$$H_{A_2} = I_A\frac{v_2}{r} = m(k^2+r^2)\frac{v_2}{r}$$

$$v_2 = v_{min}\left(1 - \frac{rh}{k^2+h^2}\right)$$

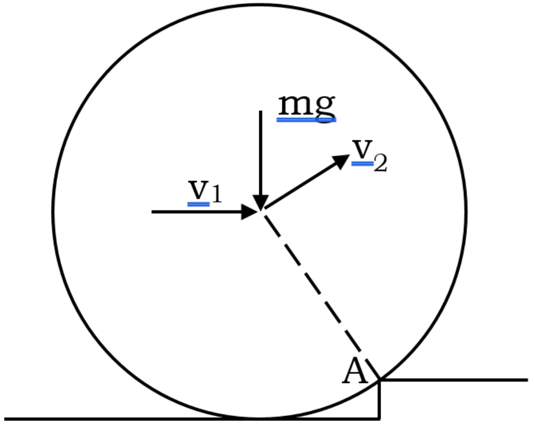
$$U_{2\to3} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{2\to3} = \Delta V_e = 0$$

$$\Delta T = 0 - \frac{1}{2}m(k^2+r^2)\left(\frac{v_2}{r}\right)^2$$

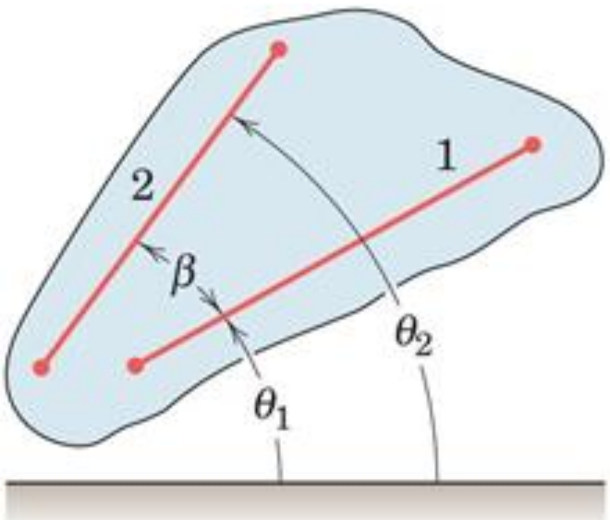
$$\Delta V_g = mgh$$

$$v_{min} = \frac{r\sqrt{2gh(k^2+r^2)}}{k^2+r^2-rh}$$



Differences between 2-D & 3-D Dynamics

Angular displacement
$\Delta\theta$ is unique!



Orientation of a rigid body may be obtained using a *sequence* of rotations which is not **unique**.

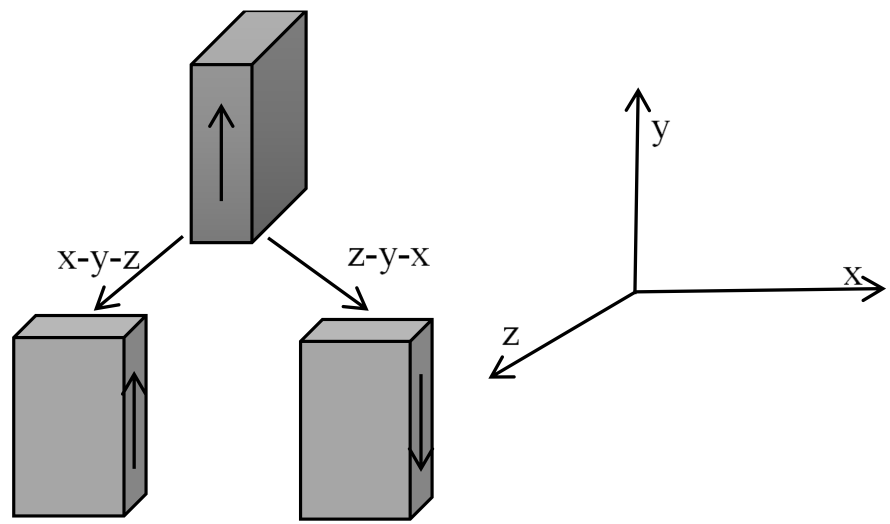


Figure from: "*Effect of Rotation Sequence on Evaluation of Joint Angles about Anatomical Axes in Gait Analyses*" by Metin Biçer, Sebahat Aydil, Ergin Tönük, Güneş Yavuzer

Differences between 2-D & 3-D Dynamics

Angular velocity:

$\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}$

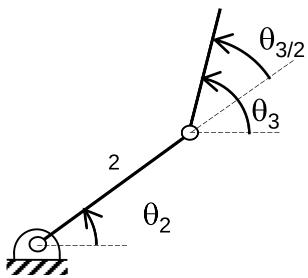
Angular velocity:

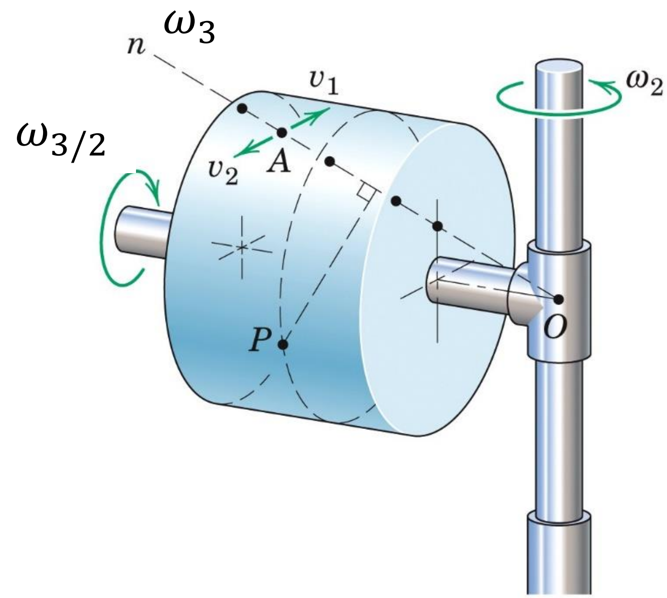
$\vec{\omega} = \omega_x\hat{\imath} + \omega_y\hat{\jmath} + \omega_z\hat{k}$

$\neq \dot{\theta}_x\hat{\imath} + \dot{\theta}_y\hat{\jmath} + \dot{\theta}_z\hat{k}$

Differences between 2-D & 3-D Dynamics

Addition Theorem for Angular Velocity





$$\omega_3 = \omega_2 + \omega_{3/2}$$

$$\vec{\omega}_3 = \vec{\omega}_2 + \vec{\omega}_{3/2}$$

Differences between 2-D & 3-D Dynamics

Transformation of a Time Derivative (Transport or Coriolis Theorem)

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \left(\frac{d\vec{V}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{V}$$

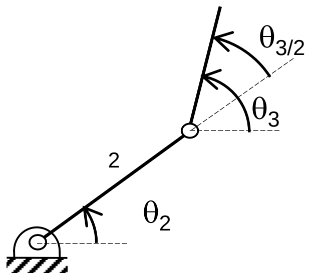
Let $\vec{V} = \vec{\omega}$

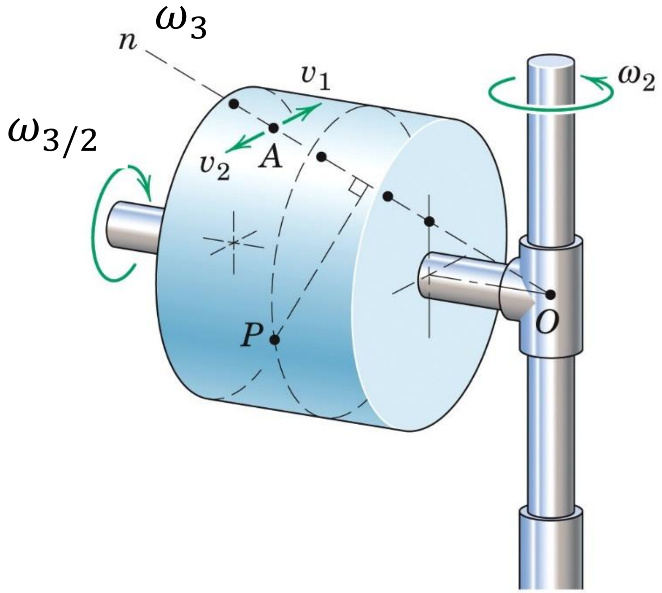
$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{\omega}$$

$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{\omega}$$

Differences between 2-D & 3-D Dynamics

Angular Acceleration





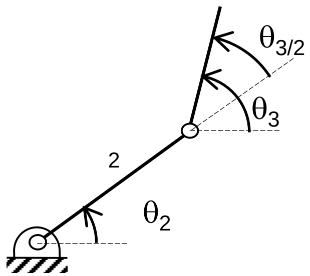
$$\left(\frac{d\vec{\omega}_3}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}_2}{dt}\right)_{X-Y} + \left(\frac{d\vec{\omega}_{3/2}}{dt}\right)_{x-y} + \vec{\omega}_2 \times \vec{\omega}_{3/2}$$

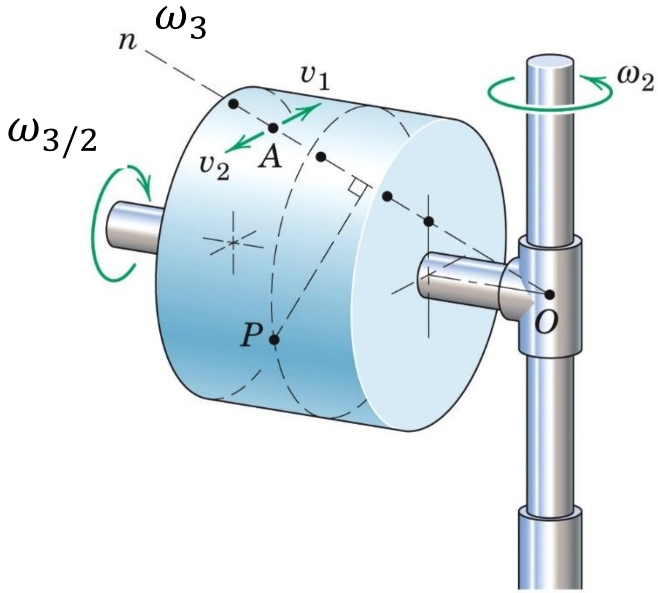
$$\vec{\omega}_i = \omega_i \hat{k}$$

$$\left(\frac{d\vec{\omega}_3}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}_2}{dt}\right)_{X-Y} + \left(\frac{d\vec{\omega}_{3/2}}{dt}\right)_{x-y} + \vec{\omega}_2 \times \vec{\omega}_{3/2}$$

Differences between 2-D & 3-D Dynamics

Angular Acceleration





$\vec{\alpha}_3 = \vec{\alpha}_2 + \vec{\alpha}_{3/2}$

$\vec{\alpha}_3 = \vec{\alpha}_2 + \vec{\alpha}_{3/2} + \vec{\omega}_2 \times \vec{\omega}_{3/2}$

The gyroscopic acceleration!

Differences between 2-D & 3-D Dynamics

Angular Momentum Change

$$\dot{\vec{H}}_G = \frac{d}{dt}\left|\vec{H}_g\right|\hat{k}$$

Change in angular momentum is due to change in its magnitude only. The direction is fixed!

$$\dot{\vec{H}}_G = \frac{d}{dt}\left|\vec{H}_g\right|\frac{\vec{H}_g}{\left|\vec{H}_g\right|} + \left|\vec{H}_g\right|\frac{d}{dt}\left(\frac{\vec{H}_g}{\left|\vec{H}_g\right|}\right)$$

Change in angular momentum is due to change in its magnitude and due to change in its direction which is **not** fixed anymore!