ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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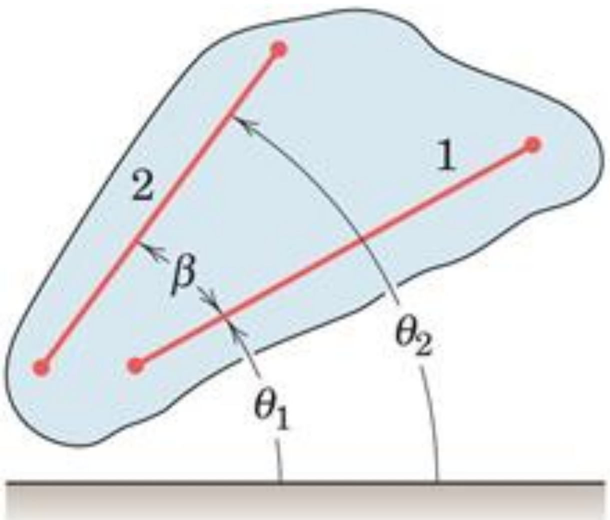
Differences between 2-D & 3-D Dynamics

Angular displacement (rotation) Δθ is unique!



Orientation of a rigid body may be obtained using a *sequence* of rotations which is not **unique**.

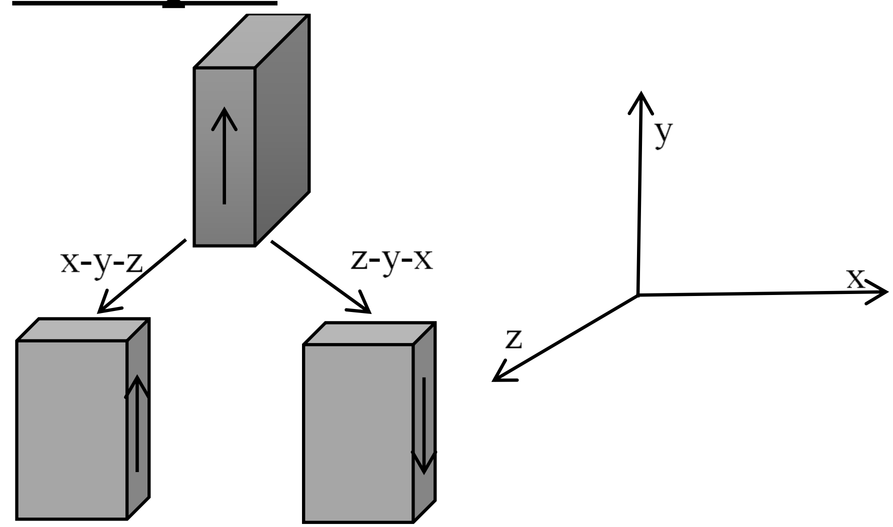


Figure from: "*Effect of Rotation Sequence on Evaluation of Joint Angles about Anatomical Axes in Gait Analyses*" by Metin Biçer, Sebahat Aydil, Ergin Tönük, Güneş Yavuzer

Differences between 2-D & 3-D Dynamics

Angular velocity:

$$\vec{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}$$

Only magnitude may change, direction is fixed.

$$\frac{d\vec{\omega}}{dt} = \dot{\omega}\hat{k} = \alpha\hat{k}$$

Angular velocity:

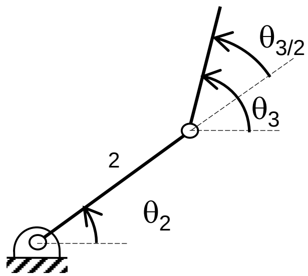
$$\vec{\omega} = \omega_x\hat{\imath} + \omega_y\hat{\jmath} + \omega_z\hat{k}$$

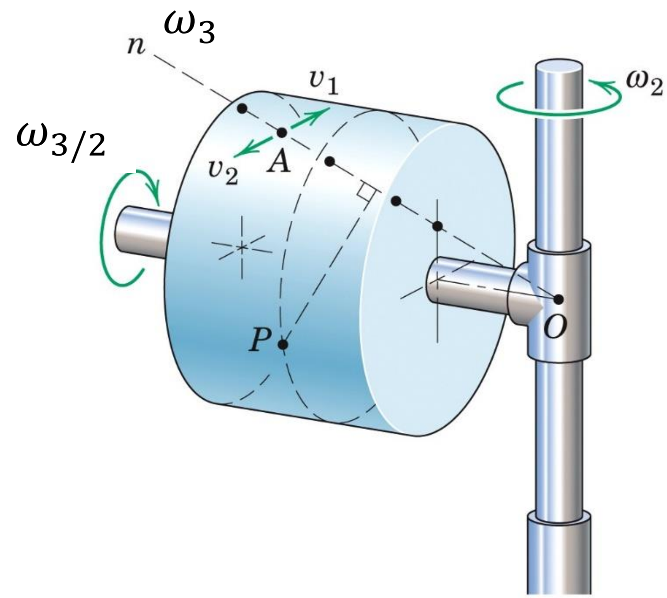
Not only magnitude but direction may change as well.

$$\frac{d\vec{\omega}}{dt} = \dot{\omega}\left(\frac{\vec{\omega}}{\omega}\right) + \omega\frac{d}{dt}\left(\frac{\vec{\omega}}{\omega}\right)$$

Differences between 2-D & 3-D Dynamics

Addition Theorem for Angular Velocity





$$\omega_3 = \omega_2 + \omega_{3/2}$$

$$\vec{\omega}_3 = \vec{\omega}_2 + \vec{\omega}_{3/2}$$

Differences between 2-D & 3-D Dynamics

Transformation of a Time Derivative (Transport or Coriolis Theorem)

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \left(\frac{d\vec{V}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{V}$$

Let $\vec{V} = \vec{\omega}$

$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{\omega}$$

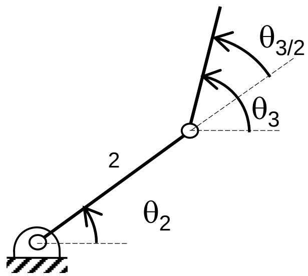
$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y} + \omega\hat{k} \times \omega\hat{k}$$

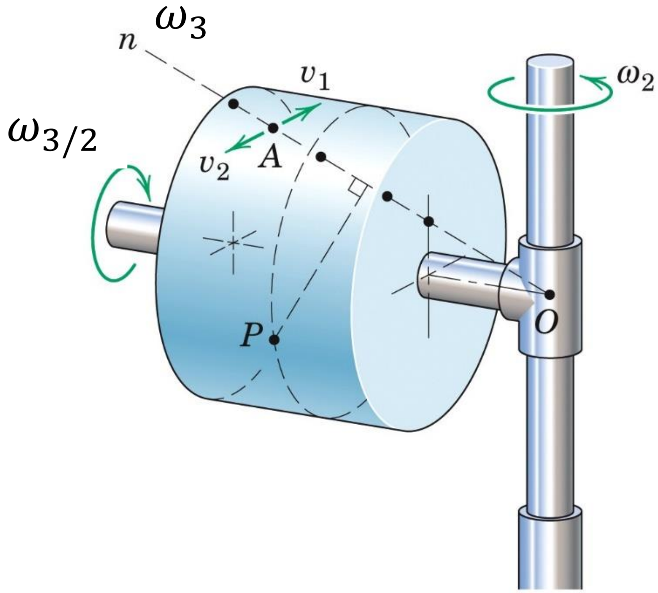
$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y}$$

$$\left(\frac{d\vec{\omega}}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{\omega}$$

Differences between 2-D & 3-D Dynamics

Angular Acceleration





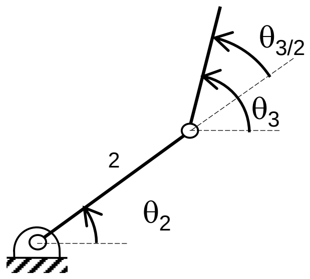
$$\left(\frac{d\vec{\omega}_3}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}_2}{dt}\right)_{X-Y} + \left(\frac{d\vec{\omega}_{3/2}}{dt}\right)_{x-y} + \vec{\omega}_2 \times \vec{\omega}_{3/2} \rightarrow \vec{0}$$

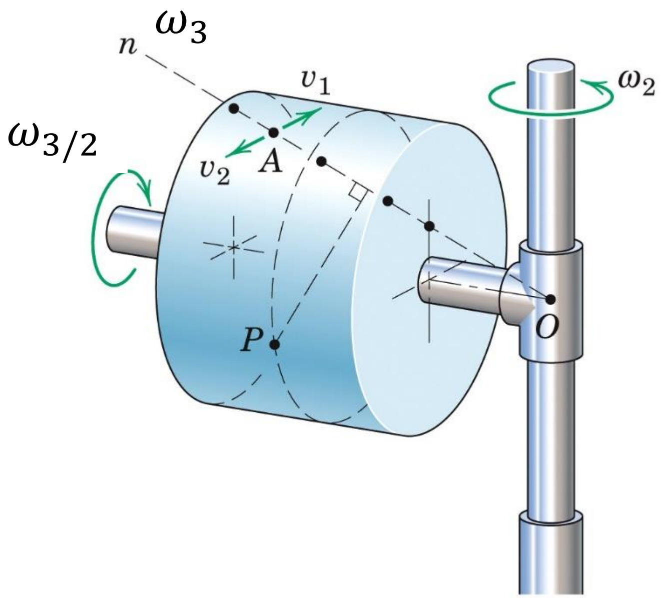
$\vec{\omega}_i = \omega_i \hat{k}$

$$\left(\frac{d\vec{\omega}_3}{dt}\right)_{X-Y} = \left(\frac{d\vec{\omega}_2}{dt}\right)_{X-Y} + \left(\frac{d\vec{\omega}_{3/2}}{dt}\right)_{x-y} + \vec{\omega}_2 \times \vec{\omega}_{3/2}$$

Differences between 2-D & 3-D Dynamics

Angular Acceleration





$$\vec{\alpha}_3 = \vec{\alpha}_2 + \vec{\alpha}_{3/2}$$

$$\vec{\alpha}_3 = \vec{\alpha}_2 + \vec{\alpha}_{3/2} + \vec{\omega}_2 \times \vec{\omega}_{3/2}$$

The gyroscopic acceleration!

Differences between 2-D & 3-D Dynamics

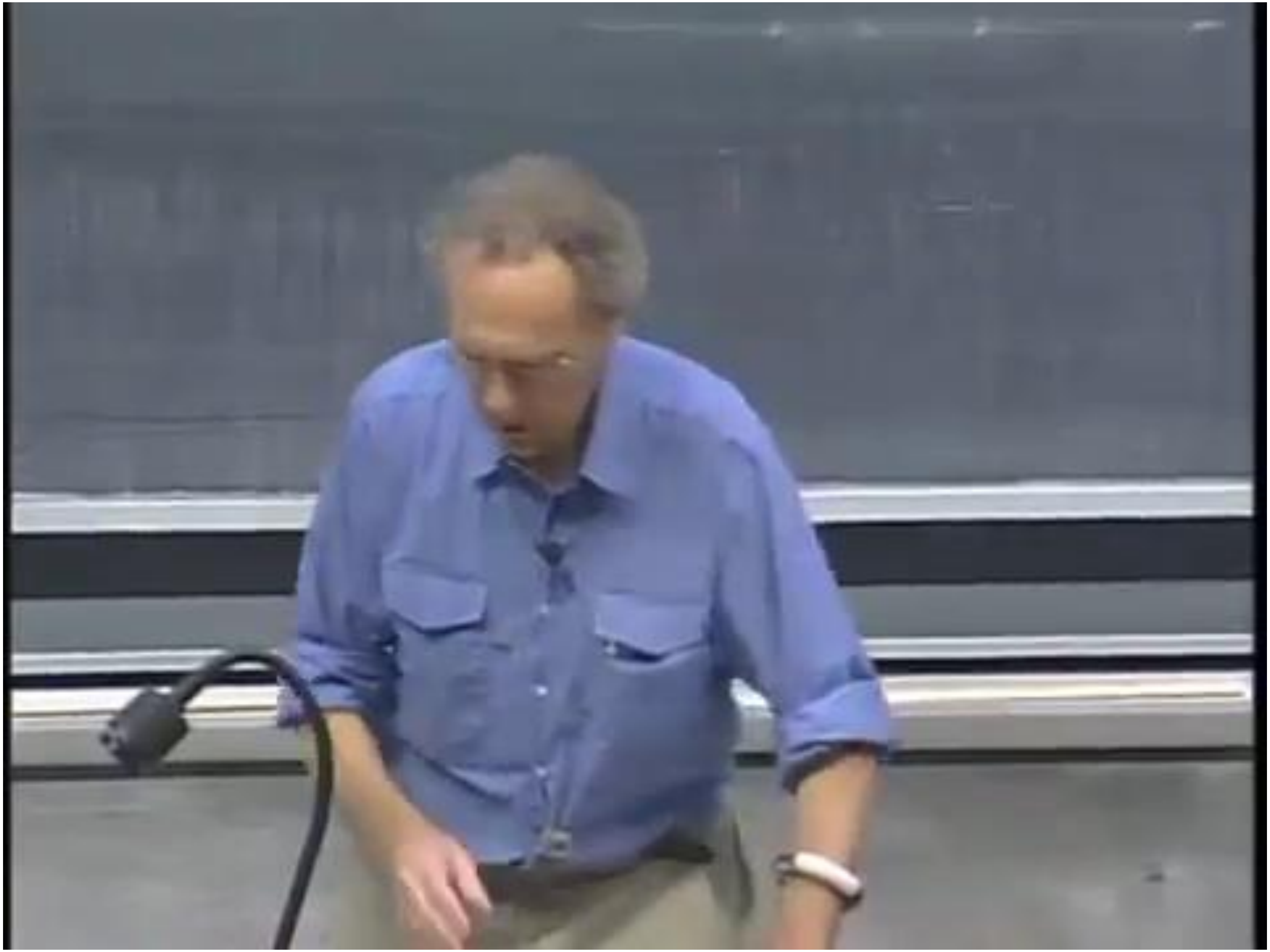
Angular Momentum Change

$$\dot{\vec{H}}_G = \frac{d}{dt}\left|\vec{H}_g\right|\hat{k}$$

Change in angular momentum is due to change in its magnitude only. The direction is fixed!

$$\dot{\vec{H}}_G = \frac{d}{dt}\left|\vec{H}_g\right|\frac{\vec{H}_g}{\left|\vec{H}_g\right|} + \left|\vec{H}_g\right|\frac{d}{dt}\left(\frac{\vec{H}_g}{\left|\vec{H}_g\right|}\right)$$

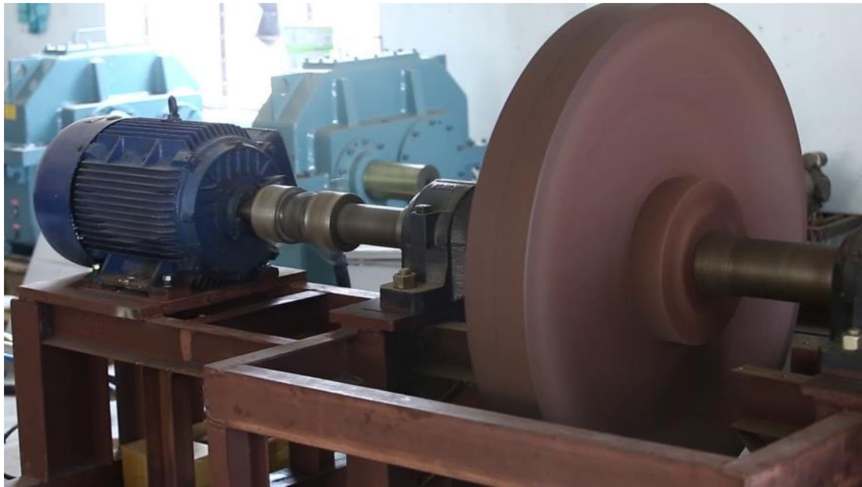
Change in angular momentum is due to change in its magnitude and due to change in its direction which is **not** fixed anymore!



Lectures by Walter Lewin
https://www.youtube.com/watch?v=XPUuF_dECVI

Problems on Kinematics of Rigid Bodies

The frictional resistance to the rotation of a flywheel consists of a retardation due to air friction which varies as the square of the angular velocity and a constant frictional retardation in the bearing. As a result the angular acceleration of the flywheel while it is allowed to coast is given by $\alpha = -K - k\omega^2$, where K and k are two positive constants. Determine an expression for the time required for the flywheel to come to rest from an initial angular velocity ω_0.



Since

$$\alpha = \alpha(\omega)$$

we can utilize

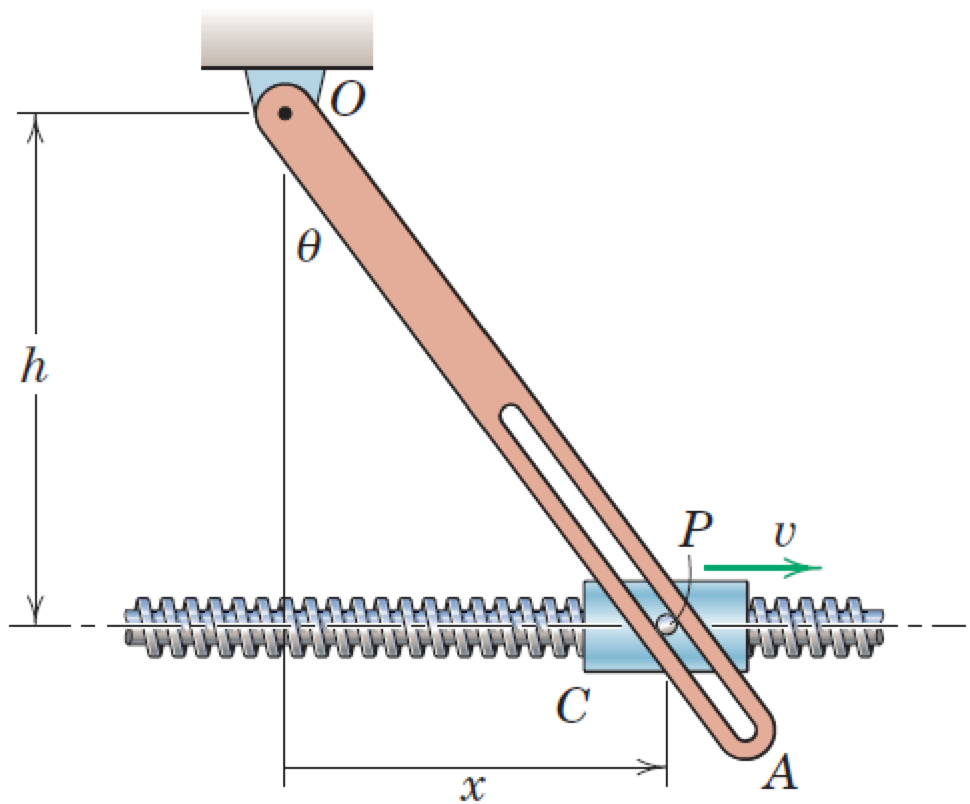
$$\alpha = \frac{d\omega}{dt}$$

$$dt = \frac{d\omega}{-K - k\omega^2}$$

$$\int_0^t dt = \int_{\omega_0}^0 \frac{d\omega}{-K - k\omega^2}$$

$$t = \frac{1}{\sqrt{kK}} arctan\left(\omega_0\sqrt{\frac{k}{K}}\right)$$

Rotation of the slotted bar OA is controlled by the lead screw that imparts a horizontal velocity v to collar C. Pin P is attached to the collar. Determine the angular velocity ω_{OA} of bar OA in terms of v and the displacement x.



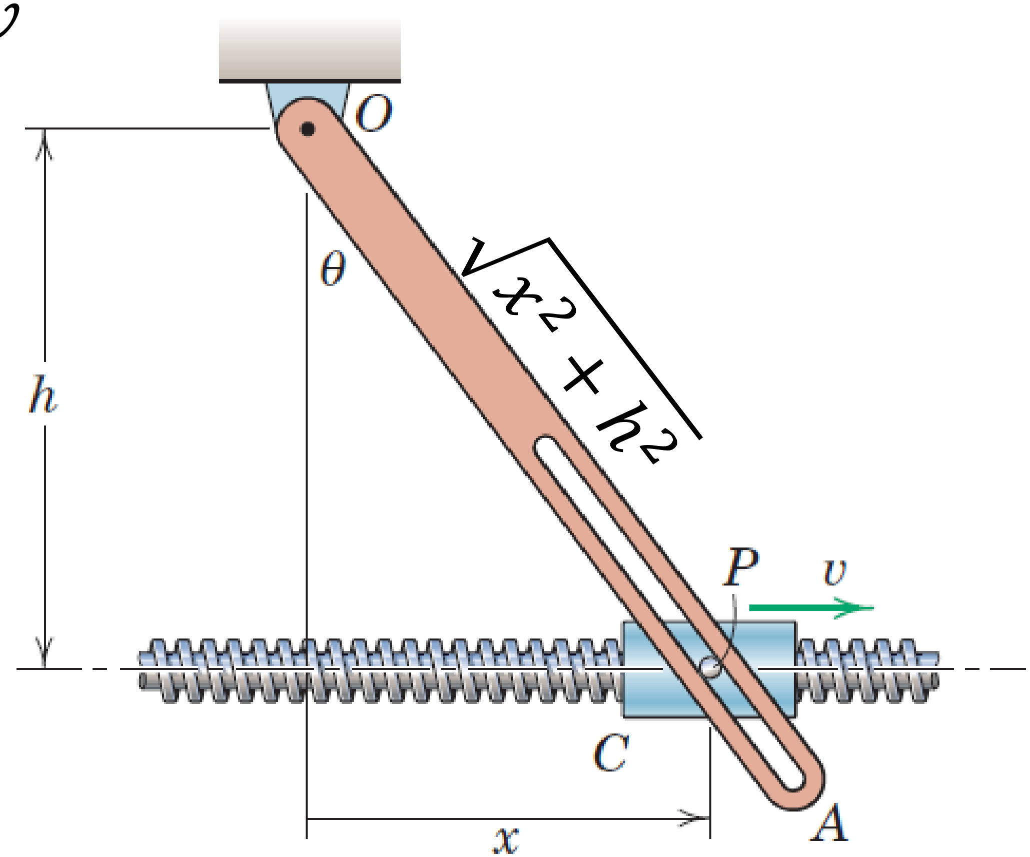
$$\omega_{OA} = \dot{\theta}$$

$$tan\theta = \frac{sin\theta}{cos\theta} = \frac{x}{h}$$

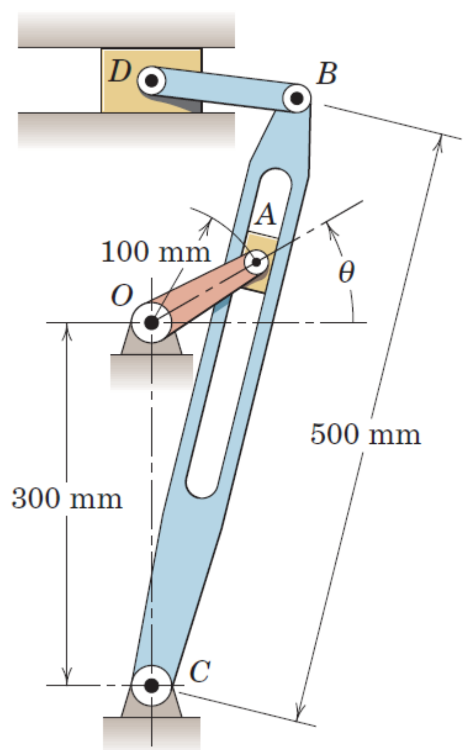
$$\frac{cos^2\theta + sin^2\theta}{cos^2\theta}\dot{\theta} = \frac{\dot{x}}{h}, \dot{x} = v$$

$$cos\theta = \frac{h}{\sqrt{x^2 + h^2}}$$

$$\dot{\theta} = \frac{x^2 + h^2}{h^3}v$$



The figure illustrates a commonly used quick-return mechanism which produces a slow cutting stroke of the tool (attached to D) and a rapid return stroke. If the driving crank OA is turning at the constant rate $\dot{\theta} = 3$ rad/s, determine the magnitude of the velocity of point B for the instant when $\theta = 30°$. Also propose how you would find $\vec{v}_D$ by ICZV.



$$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$$

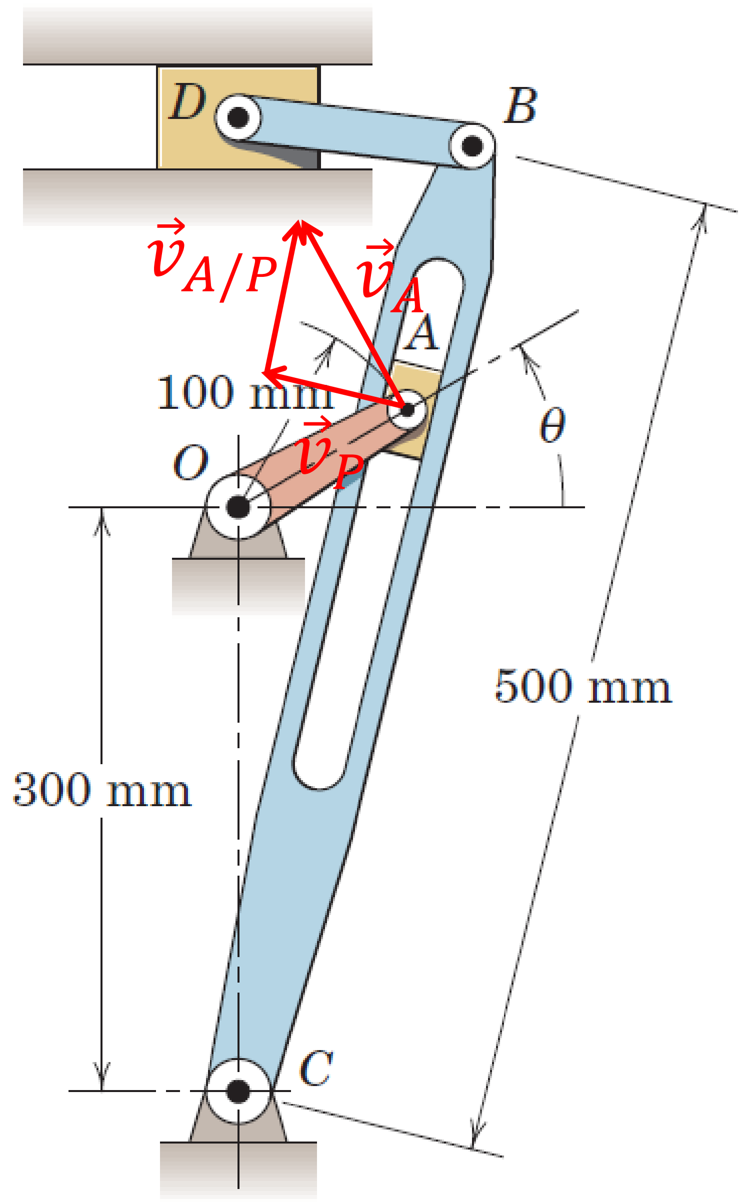
$$\vec{v}_A \perp |OA|$$

$$\vec{v}_P \perp |CB|$$

$\vec{v}_{P/A}$ is along the slot

$$\vec{v}_A = 0.1 * 3(sin\theta\hat{\imath} - cos\theta\hat{\jmath})$$

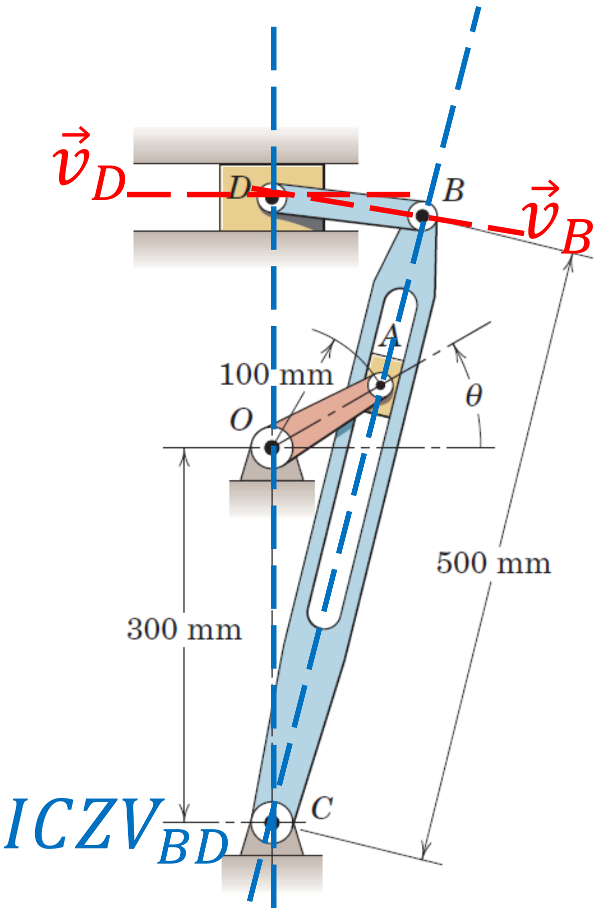
$$\omega_{CB} = \frac{v_P}{|CP|}$$



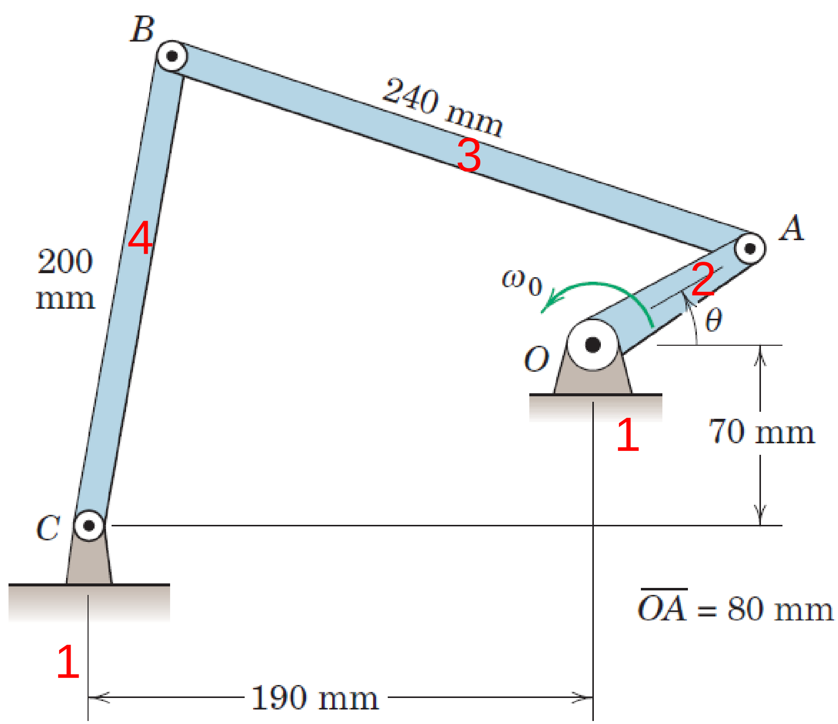
$$v_B = |CB|\omega_{CB}$$

$$\omega_{BD} = \frac{v_B}{|BC|}$$

$$v_D = |D\ ICZV_{BD}|\omega_{BD}$$



The crank OA of the four-bar linkage is driven at a constant counterclockwise angular velocity $\omega_0 = 10$ rad/s. Determine the expressions for angular velocities and accelerations of rods 3 and 4 for any θ.



$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

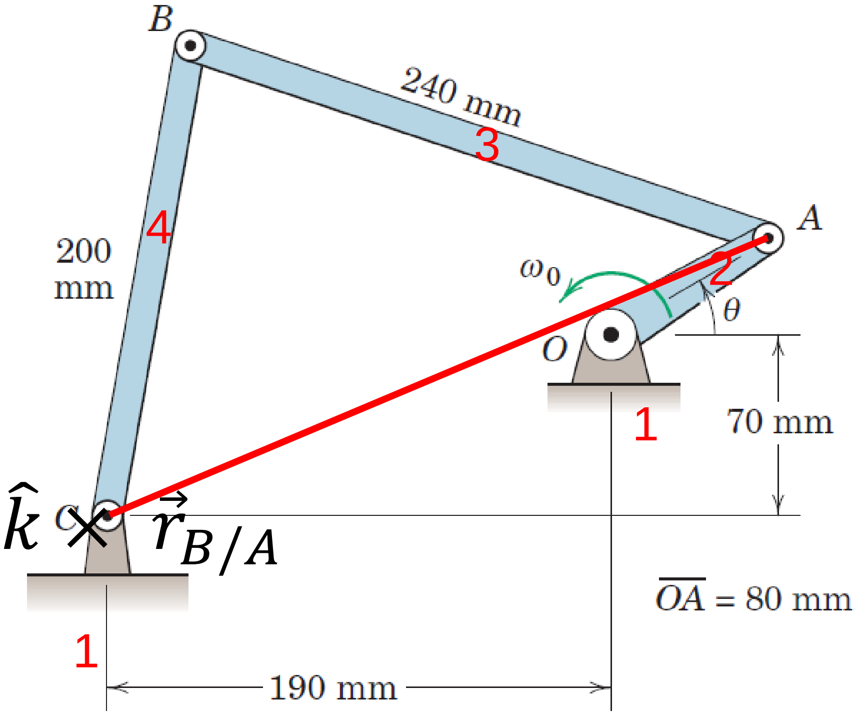
$$\vec{v}_A = \omega_0 \hat{k} \times \vec{r}_{A/O}$$

$$\vec{r}_{A/O} = |OA|(cos\theta\hat{\imath} + sin\theta\hat{\jmath})$$

$$\vec{v}_B = \omega_4 \hat{k} \times \vec{r}_{B/C}$$

$$\vec{v}_{B/A} = \omega_3 \hat{k} \times \vec{r}_{B/A}$$

$$\omega_4 \hat{k} \times \vec{r}_{B/C} = \omega_0 \hat{k} \times \vec{r}_{A/O} + \omega_3 \hat{k} \times \vec{r}_{B/A}$$



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_A = \alpha_0 \hat{k} \times \vec{r}_{A/O} - \omega_0{}^2 \vec{r}_{A/O}$$

$$\vec{a}_B = \alpha_4 \hat{k} \times \vec{r}_{B/C} - \omega_4{}^2 \vec{r}_{B/C}$$

$$\vec{a}_{B/A} = \alpha_3 \hat{k} \times \vec{r}_{B/A} - \omega_3{}^2 \vec{r}_{B/A}$$

$$\alpha_4 \hat{k} \times \vec{r}_{B/C} - \omega_4{}^2 \vec{r}_{B/C} = \alpha_0 \hat{k} \times \vec{r}_{A/O} - \omega_0{}^2 \vec{r}_{A/O} + \alpha_3 \hat{k} \times \vec{r}_{B/A} - \omega_3{}^2 \vec{r}_{B/A}$$