Lecture 1

Basic Features of the Classical Triangle

"The Four Remarkable Points"

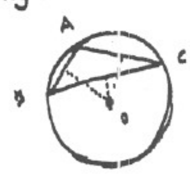
Cl~~ass~~ Basic Features of the Classical Triangle

§ 1

("The Four Remarkable Points")

~~1A~~

ABC : the "generic" triangle.



(1) Perpendicular Bisectors, Circum centre, Circum circle, Circum radius } O, (O), R.

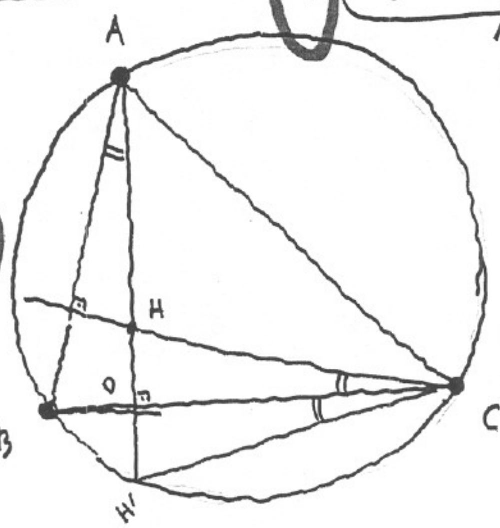
show "unusually proportioned" configurations...

(2) Altitudes, Orthocentre, Orthic Group } H, h_a, h_b, h_c

$$2\Delta = a.h_a = b.h_b = ch_c.$$

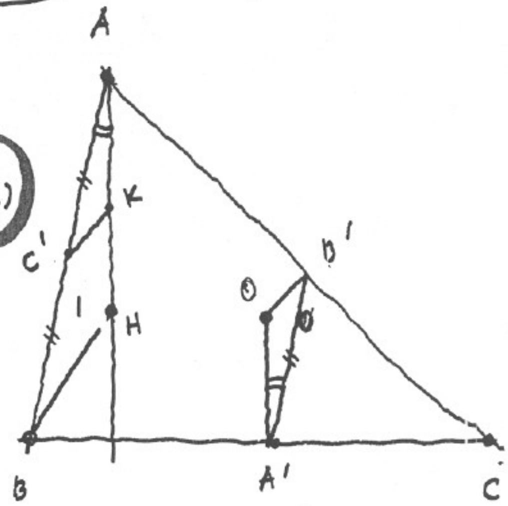
Interesting facts : a) $\cancel{h_b < h_c \Leftrightarrow b > c}$ better: $h_b = h_c \Longrightarrow b = c$

b)



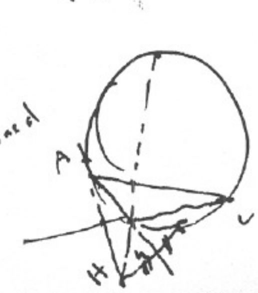
The reflection of H in each side of ABC lies on (O).

c)



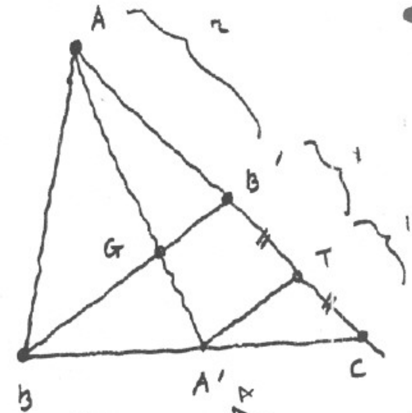
$2|OA'| = |AH|$

Unusually proportioned



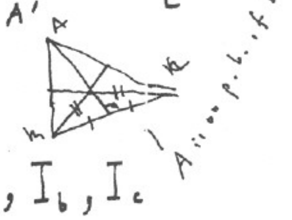
(3)

Medians
Centroid $\Big\}$ G, m_a, m_b, m_c



Elegant proof for $\boxed{\cancel{m_b < m_c \iff b > c}}$?

$\exists$ an elegant proof for $\boxed{m_b = m_c \Longrightarrow b = c}$

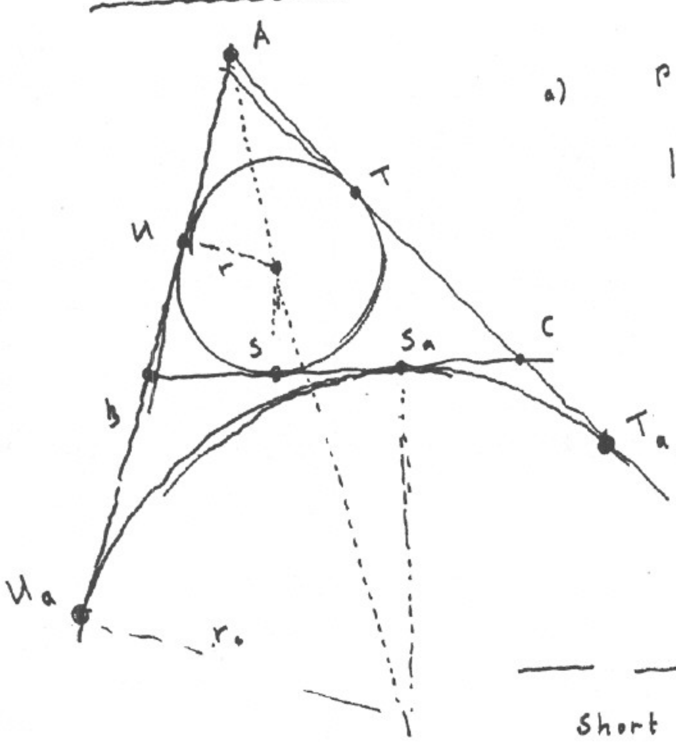


(4)

Angle bisectors (Internal, External)
Incircle (Excircles)
Inradius (Exradii)
Incenter (Excenter)

$\Bigg\}$ I, I_a, I_b, I_c
r, r_a, r_b, r_c
(I), (I_a) etc.

Interesting facts :



a) Putting $2s = a + b + c$

$|AT| = |AU| = s - a$ [redacted]

$|AT_a| = |AU_a| = s$

$|BS| = |CS_a| = s - b.$

etc.

b) $\Delta = s \cdot r = (s-a) r_a.$

Short interlude on the classical history of

O, H, G, I

$I_a, I_b, I_c.$

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Before Euler

1) (b)(c) in page 1
2) ABCH "orthic"

(5) **Enter Euler!**

Karl Wilhelm Feuerbach (1800–1834)
"Eigenschaften einiger merkwürdigen Punkte des geradlinigen Dreiecks und mehrerer durch sie bestimmten Linien und Figuren" (Nürnberg, 1822). → Johnson. p. 190.

a) Theorem: O, H, G collinear and $GO : GH = -1 : 2$ } "Euler line".

b) Theorem: The midpoints of [BC], [CA], [AB], [HA], [HB], [HC] and the feet of the altitudes lie on a circle whereof the centre is the midpoint of [OH].

Problem involving inversion + 9 p. circle: E 2419 AMM p.560 (1973).

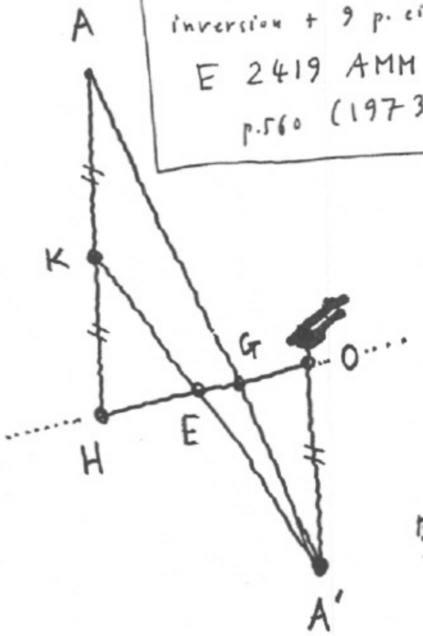
Notation: E

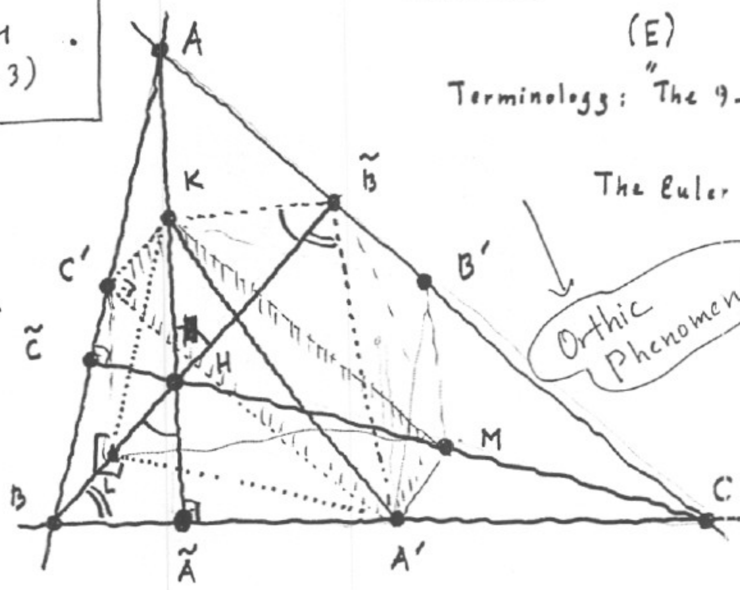
(E)

Terminology: "The 9-point circle"

The Euler circle.

Orthic Phenomena





QA113 E82

Lalescu's proof of the Feuerbach's theorem...

L. Euler

"Solutio facilis problematum quorundam geometricorum difficilorum"

"Novi Comment" Acad. Imp. Sci. Petropolitanae

11 (1765 published 1767)

pp. 103–123.