Problems in Geometry (4)

1. Consider a quadrangle $ABCD$ with points P, Q, R, S, T on BD, AB, AD, CB, CD respectively. Prove that if P, Q, R are collinear and P, S, T are collinear, then the lines QS, RT, AC are concurrent. [1]

2. The following is a problem by the late Prof. Demir who liked to present it with a quaintly arithmetic notation which is very suggestive and delightful. A numeral m written decimally will stand for a point and given such numerals m, n the line through them is denoted by $m \cdot n$:

Given distinct non-collinear points $1, 2, 3, 4$ consider points $12 \in 1 \cdot 2 - \{1, 2\}$, $23 \in 2 \cdot 3 - \{2, 3\}$, $34 \in 3 \cdot 4 - \{3, 4\}$. Let $1 \cdot 23$ and $12 \cdot 3$, $2 \cdot 34$ and $23 \cdot 4$ meet in 123, 234, respectively. Finally, let $1 \cdot 234$ intersect $123 \cdot 4$ in 1234. Prove that 12, 1234, 34 are collinear. [2]

3. Given triangle ABC, let D be a point on BC,and P, Q points on AB. Let PD meet AC in H, QD meet AC in K and CP meet AD in M and CQ meet AD in N. Prove that KM and HN meet on AB. [3]

4. Let ABC, DEF be triangles with AD, BE, CF concurrent. Prove that if AE, BF, CD are concurrent so are AF, BD and CE. [4]

---

[1] The Desargues Theorem

[2] The Desargues Theorem.

[3] The dual of the Pappus Theorem.

[4] The dual of the Pappus Theorem.