Review of Integration

- $\int_{t_1}^{t_2} f(t)dt$ is just a symbol
  - Meaning of the symbol: What does it stand for?
  - Value of the symbol: What does that symbol equal to?

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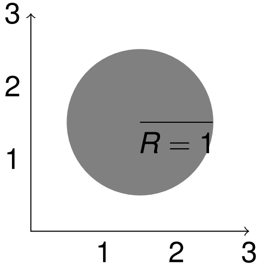
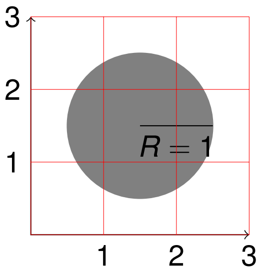
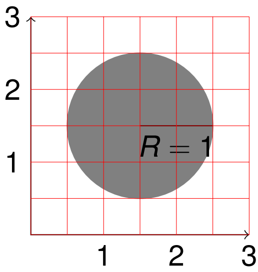


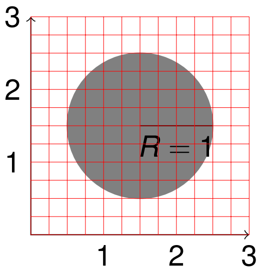
- The area is $A = \pi R^2 \simeq 3.21$



- The area is $A = \pi R^2 \simeq 3.21$
- The number of squares lying entirely inside the circle: $A_< = 1$
- The number of squares that have a part inside the circle: $A_> = 9$
- Area of each square: $a_0 = 1$
- $a_0 A_< < A < a_0 A_>$ $\Longrightarrow 1 < A < 9$



- The area is $A = \pi R^2 \simeq 3.21$
- The number of squares lying entirely inside the circle: $A_< = 4$
- The number of squares that have a part inside the circle: $A_> = 16$
- Area of each square: $a_0 = 1/4$
- $a_0 A_< < A < a_0 A_>$ $\Longrightarrow 1 < A < 4$



- The area is $A = \pi R^2 \simeq 3.21$
- The number of squares lying entirely inside the circle: $A_< = 32$
- The number of squares that have a part inside the circle: $A_> = 60$
- Area of each square: $a_0 = 1/16$
- $a_0 A_< < A < a_0 A_>$ $\Longrightarrow 2 < A < 3.75$

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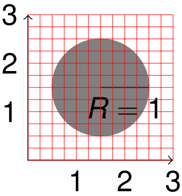
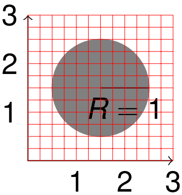


- $a_0 A_< = \sum_{\substack{\text{squares completely}\\\text{in circle}}} 1 \times a_0$
- $a_0 A_> = \sum_{\substack{\text{squares containing at least}\\\text{a fraction inside the circle}}} 1 \times a_0$

- Always $a_0 A_< < A < a_0 A_>$
- As the grid size get smaller $a_0 A_<$ increases as $a_0 A_>$ decreases, and A is always in between
- Eventually $a_0 A_< \simeq a_0 A_> \simeq A$: Mathematical expression:

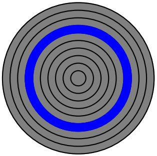
$$\lim_{a_0 \to 0} a_0 A_< = \lim_{a_0 \to 0} a_0 A_> = A \tag{23}$$



- $a_0 A_< = \sum_{\substack{\text{squares completely}\\ \text{in circle}}} 1 \times a_0$
- $a_0 A_> = \sum_{\substack{\text{squares containing at least}\\ \text{a fraction inside the circle}}} 1 \times a_0$

- In the limit $a_0 \to 0$, each sum contains infinitely many terms
- The contribution of each term, i.e. $1 \times a_0$, becomes zero.
- In the limit $a_0 \to 0$, we are summing infinitely many zeroes: the result is finite.
- rather than writing "limit as $a_0 \to 0$, sum the areas of all the squares that lie completely inside the circle," we write

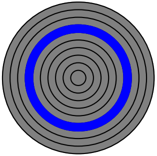
$$\int_{\text{disc}} 1 \times da \tag{23}$$



Area of a circle

- Let r be the inner radius of the chosen ring
- $r + \delta r$ be the area of the outer radius of the chosen ring
- The ring can be straightened out and it will fit inside a rectangle whose width is δr are height $2\pi(r + \delta r)$
- The ring will contain a rectangle whose width is δr and height $2\pi r$.
- $2\pi r\delta r < \delta A < 2\pi(r + \delta r)\delta r$ where δA is the area of the ring

Area of a circle



- $2\pi r\delta r < \delta A < 2\pi(r+\delta r)\delta r$ where δA is the area of the ring
- Total area of the disk is the sum of all such rings from $r=0$ upto $r=R$:

$$\sum_r 2\pi r\delta r < A = \sum_{r=0}^{R} \delta A < \sum_d 2\pi(r+\delta r)\delta r \tag{24}$$

- In the limit $\delta r \to 0$

$$\tag{25}$$

$$A = \int_0^R dr 2\pi r = \pi R^2 \tag{26}$$

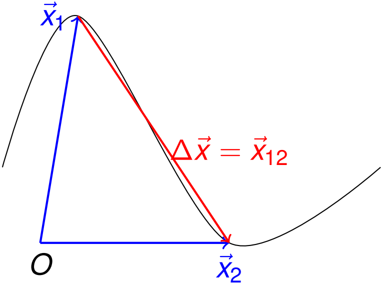
POP QUIZ

$$\vec{A} = \hat{x} + \hat{y} + \hat{z} \quad (27)$$

$$\vec{B} = \hat{x} + \hat{y} - 2\hat{z} \quad (28)$$

1. Show that $\vec{A}$ and $\vec{B}$ are perpendicular.
2. Calculate $\vec{A} + \vec{B}$, $\vec{A} - \vec{B}$, $\vec{A} \cdot \vec{B}$
3. Calculate $|\vec{A}|$, $|\vec{B}|$

Displacement Vector



The displacement vector is $\Delta\vec{x} = \vec{x}_{12} = \vec{x}_2 - \vec{x}_1$

Motion in 3D

The discussions on motion in 1D can be generalized to 3D by just representing positions, velocities and acceleration with 3D vectors:

- $\vec{x}_i$ and $\vec{x}_f$ are initial and final positions of the particle.
- Displacement vector is $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$
- Average velocity is $\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t}$. ($\bar{\vec{v}}$ is a vector times a number, hence it is also a vector)
- Componentwise $\bar{v}_x = \frac{\Delta x}{\Delta t}$, $\bar{v}_y = \frac{\Delta y}{\Delta t}$, $\bar{v}_z = \frac{\Delta z}{\Delta t}$.
- Instantaneous velocity $\vec{v}(t) = \lim_{\Delta t\to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$
- Componentwise $v_x(t) = \frac{dx}{dt}$, $v_y(t) = \frac{dy}{dt}$, $v_z(t) = \frac{dz}{dt}$

Motion in 3D

The discussions on motion in 1D can be generalized to 3D by just representing positions, velocities and acceleration with 3D vectors:

- $\vec{v}_i$ and $\vec{v}_f$ are initial and final velocities of the particle.
- Average acceleration is $\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t}$. ($\bar{\vec{a}}$ is a vector times a number, hence it is also a vector)
- Componentwise $\bar{a}_x = \frac{\Delta v_x}{\Delta t}$, $\bar{a}_y = \frac{\Delta v_y}{\Delta t}$, $\bar{a}_z = \frac{\Delta v_z}{\Delta t}$.
- Instantaneous acceleration $\vec{a}(t) = \lim_{\Delta t\to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$
- Componentwise $a_i(t) = \frac{dv_i}{dt}$

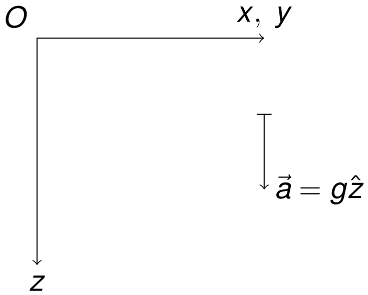
Motion in 3D

The discussions on motion in 1D can be generalized to 3D by just representing positions, velocities and acceleration with 3D vectors:

- $\vec{v}(t) = \frac{d\vec{x}}{dt} \Longleftrightarrow \vec{x}(t) = \vec{x}(t_i) + \int_{t_i}^{t} \vec{v}(t')dt'$
- $\vec{a}(t) = \frac{d\vec{v}}{dt} \Longleftrightarrow \vec{v}(t) = \vec{v}(t_i) + \int_{t_i}^{t} \vec{a}(t')dt'$

Motion In Earth's Gravity

- In the absence of friction, all objects have the same acceleration under the gravitational attraction of Earth.
- Close to the surface of Earth, this acceleration is uniform, and is denoted by $g \simeq 9.8\ m/s^2$ and points toward the center of Earth.
- Choose a coordinate axis: one possible choice is z pointing downwards and x and y axis horizontal. Choose $z = 0$ plan to lie on the surface of earth.



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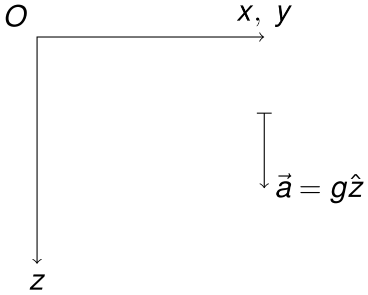

- $\vec{a} = g\hat{z}$.
- $\vec{v}(t) = \vec{v}(t_i) + \int_{t_i}^{t} \vec{a}(t')dt' = \vec{v}_i + g\hat{z}(t - t_i)$

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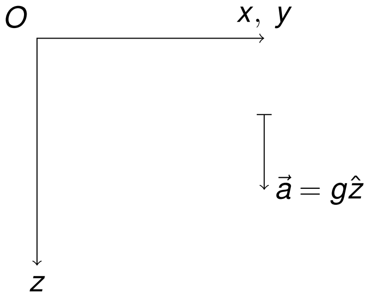

- $\vec{v}(t) = \vec{v}_i + g\hat{z}(t - t_i)$
- $\vec{x}(t) = \vec{x}(t_i) + \int_{t_i}^{t} \vec{v}(t')dt' = \vec{x}_i + \vec{v}_i(t - t_i) + \frac{1}{2}g\hat{z}(t - t_i)^2$