IMPORTANT NOTES

- Acceleration is in the direction of change in velocity (either magnitude and/or direction)
- Force is in the direction of acceleration
- IF there is no force, then there is no acceleration, i.e. no change in velocity
- ALWAYS: a force is acted by something!

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Friction

- Electromagnetic in Origin
- Various type of friction:
  - Kinetic friction
  - Static friction
  - Rolling friction
- Friction always tries to oppose relative motion between surfaces in contact

Kinetic Friction

- Exists if two surfaces in contact are in relative motion
- Experimental Observation:

$$|\vec{F}_f| = \mu_k |\vec{N}| \tag{51}$$

  (Note: not a vectorial equation)
- The direction is parallel to the surface of contact
- μ_k:Coefficient of kinetic friction
- Independent of the contact area

Static Friction

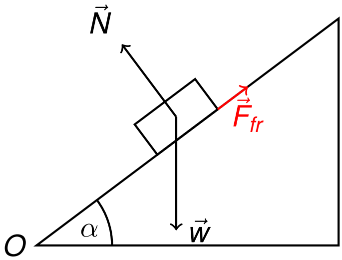
- The friction between two surfaces that are not in relative motion:

$$|\vec{F}_f| \leq \mu_s |\vec{N}| \tag{52}$$

- μ_s: Coefficient of static friction
- Usually $\mu_s > \mu_k$

Example 1

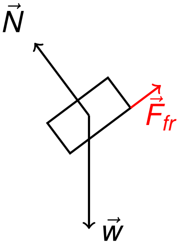
The object on the inclined surface



- Assume that initially the object is at rest on an inclined surface

Example 1

Free Body Diagram



- The forces acting on the mass:

$$\vec{N} = N\hat{y} \text{ (Unknown)} \tag{53}$$

$$\vec{w} = -mg\cos\alpha\hat{y} - mg\sin\alpha\hat{x} \tag{54}$$

$$\vec{F}_{fr} = F_{fr}\hat{x} \tag{55}$$

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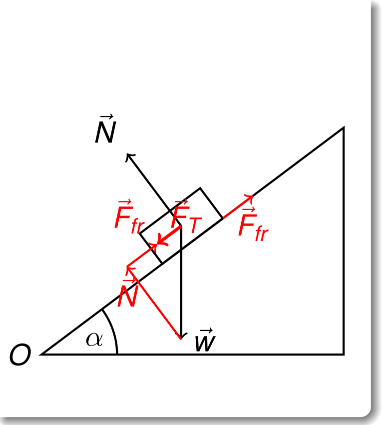
Example 1



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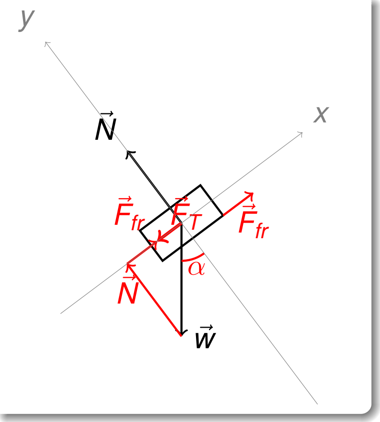
- The net force acting on the mass:

$$\vec{F}_T = (N - mg\cos\alpha)\hat{y} \tag{56}$$
$$+ (F_{fr} - mg\sin\alpha)\hat{x} \tag{57}$$

Example 1



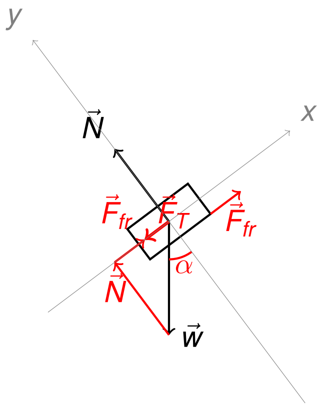
- The net force acting on the mass:

$$\vec{F}_T = (N - mg\cos\alpha)\hat{y} + (F_{fr} - mg\sin\alpha)\hat{x} \tag{53}$$

$$a_y = 0 \Longrightarrow N = mg\cos\alpha \tag{54}$$

$$a_x = \frac{F_{fr} - mg\sin\alpha}{m} \tag{55}$$

Example 1



$$a_y = 0 \Longrightarrow N = mg\cos\alpha \quad (53)$$

$$a_x = \frac{F_{fr} - mg\sin\alpha}{m} \quad (54)$$

If $F_{fr} = mg\sin\alpha$

- No acceleration, the object stays at rest
- $mg\sin\alpha = F_{fr} \leq \mu_s N = \mu_s mg\cos\alpha \Longrightarrow \tan\alpha < \mu_s$
- If α is such that $\tan\alpha > \mu_s$?

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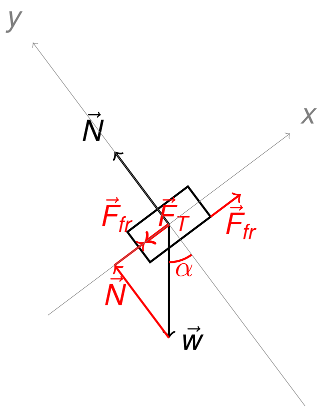
Example 1



$$a_y = 0 \Longrightarrow N = mg\cos\alpha \quad (53)$$

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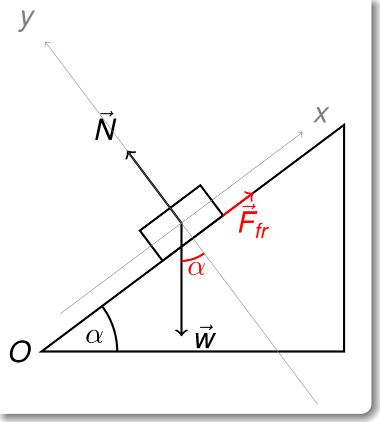
- The object will accelerate downwards: $\vec{a} = a_x\hat{x}$,

$$a_x = \frac{F_{fr} - mg\sin\alpha}{m} = \frac{\mu_k N - mg\sin\alpha}{m} = \frac{\mu_k mg\cos\alpha - mg\sin\alpha}{m} \quad (55)$$

Example 1



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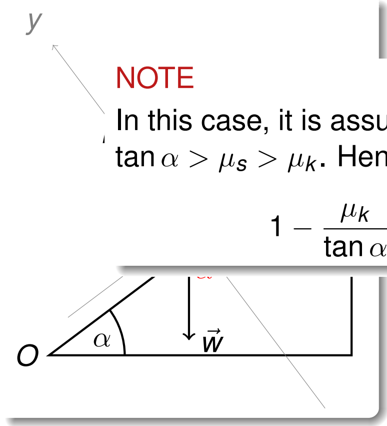
$$\begin{aligned} a_x &= \frac{F_{fr} - mg\sin\alpha}{m} = \frac{\mu_k N - mg\sin\alpha}{m} \\ &= \frac{\mu_k \not{m}g\cos\alpha - \not{m}g\sin\alpha}{\not{m}} \\ &= -g\sin\alpha\left(1 - \frac{\mu_k}{\tan\alpha}\right) \end{aligned} \quad (55)$$

Example 1



$$a_y = 0 \implies N = mg\cos\alpha \quad (53)$$

$$\frac{F_{f} - mg\sin\alpha}{m} \quad (54)$$

NOTE

In this case, it is assumed that $\tan\alpha > \mu_s > \mu_k$. Hence

$$1 - \frac{\mu_k}{\tan\alpha} > 0$$

ll accelerate

$= a_x\hat{x}$,

$$a_x = \frac{F_{fr} - mg\sin\alpha}{m} = \frac{\mu_k N - mg\sin\alpha}{m}$$

$$= \frac{\mu_k \not{m}g\cos\alpha - \not{m}g\sin\alpha}{\not{m}}$$

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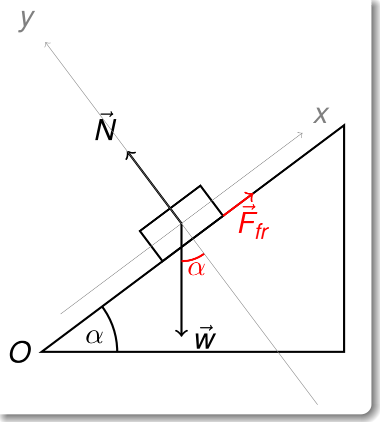
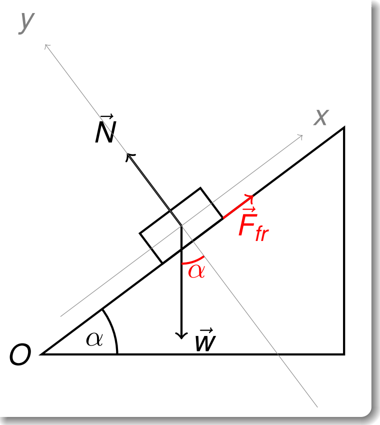
Example 1



$$a_x = -g\sin\alpha\left(1 - \frac{\mu_k}{\tan\alpha}\right) \quad (53)$$

- If $\mu_s > \tan\alpha > \mu_k$, the object will not slide if initially at rest, but will accelerate along the $-\hat{x}$ direction if initially moving along the $-\hat{x}$ direction.
- $\mu_s > \mu_k > \tan\alpha$, will accelerate along the $+\hat{x}$ direction, if initially the object is moving downwards along the $-\hat{x}$ direction

Example 1



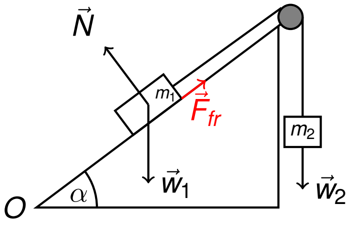
$$a_x = -g\sin\alpha\left(1 - \frac{\mu_k}{\tan\alpha}\right) \quad (53)$$

- If initially the object is moving along the $+\hat{x}$ direction, F_{fr} will point in the opposite direction: $\mu_k \rightarrow -\mu_k$

$$a_x = -g\sin\alpha\left(1 + \frac{\mu_k}{\tan\alpha}\right) \quad (54)$$

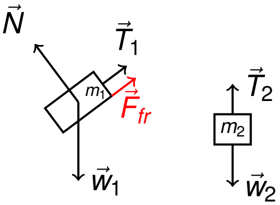
- The object will always have an acceleration along the $-\hat{x}$ direction

Example 2



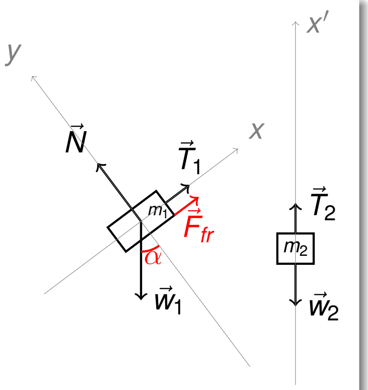
A mass m_1 is on an inclined plane. A second mass m_2 is attached to the first mass through a massless string and pulley system. The surface of the inclined plane has friction.

Example 2



Free body diagrams for the two masses

Example 2



- The forces acting on the masses:

$$\vec{w}_1 = -m_1 g \sin\alpha \hat{x} - m_1 g \cos\alpha \hat{y} \quad (55)$$

$$\vec{N} = N\hat{y}(\text{Unknown}) \quad (56)$$

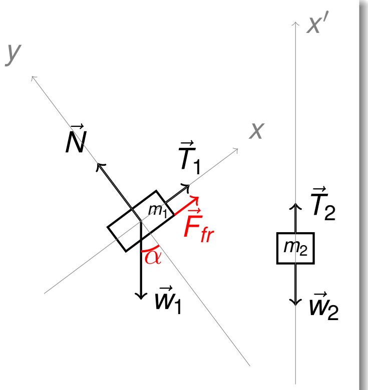
$$\vec{T}_1 = T\hat{x}(\text{Unknown}) \quad (57)$$

$$\vec{F}_{fr} = F_{fr}\hat{x}(\text{Unknown}) \quad (58)$$

$$\vec{w}_2 = -m_2 g\hat{x}' \quad (59)$$

$$\vec{T}_2 = T\hat{x}'(\text{Unknown}) \quad (60)$$

Example 2



- The forces acting on the masses:

$$\vec{w}_2 = -m_2 g \hat{x}' \quad (55)$$

$$\vec{T}_2 = T\hat{x}' (\text{Unknown}) \quad (56)$$

- Newton's Laws on the second mass:

$$\vec{a}_2 = a_2 \hat{x}' \quad (57)$$

$$\vec{F}_{T2} = (T - m_2 g)\hat{x}' \equiv m_2 a_2 \hat{x}' \quad (58)$$

$$\Longrightarrow T - m_2 g = m_2 a_2 \quad (59)$$

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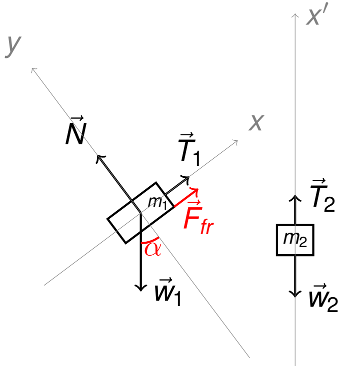
Example 2



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- Newton's Laws on the first mass:

$$\vec{a}_1 = a_1\hat{x} \quad (59)$$

(60)

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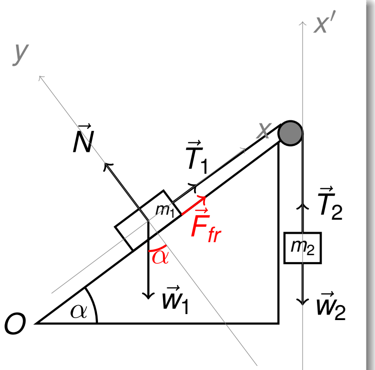
Example 2



- Newton's Laws on the first mass:

$$\vec{a}_1 = a_1 \hat{x} \quad (55)$$

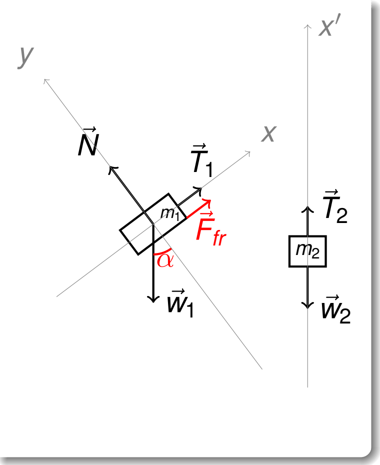
$$\vec{F}_{T1} = (T - m_1 g \sin\alpha + F_{fr})\hat{x} \quad (56)$$

$$+ (N - m_1 g \cos\alpha)\hat{y} \equiv m_1 a_1 \hat{x} \quad (57)$$

$$\Longrightarrow N - m_1 g \cos\alpha = 0 \quad (58)$$

$$T - m_1 g \sin\alpha + F_{fr} = m_1 a_1 \quad (59)$$

Example 2



- Results of Newton's Laws:

$$T - m_2 g = m_2 a_2 \quad (55)$$

$$N - m_1 g \cos\alpha = 0 \quad (56)$$

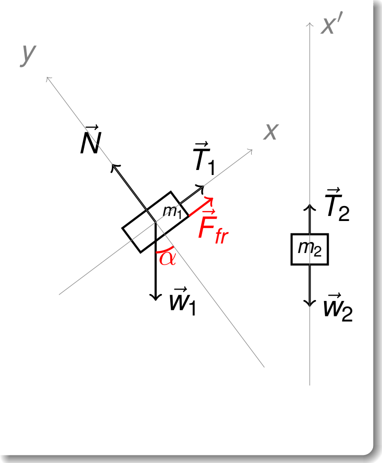
$$T - m_1 g \sin\alpha + F_{fr} = m_1 a_1 \quad (57)$$

- Unknowns: T, a_1, a_2, N, F_{fr}.
- Total 5 unknowns, need two more equations:

$$a_2 = -a_1 \quad (58)$$

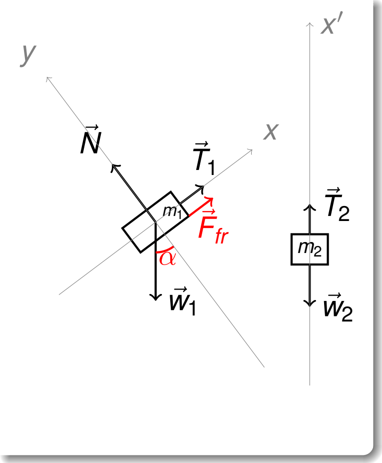
$$|F_{fr}| \leq \mu N \quad (59)$$

Example 2



- The direction of friction force should be determined.
- If the first mass is moving, $\vec{F}_{fr}$ will point in the opposite direction to its velocity
- If the first mass is at rest, $\vec{F}_{fr}$ will try to prevent it from starting to move
- First ignore friction to determine which direction the system will try to move

Example 2

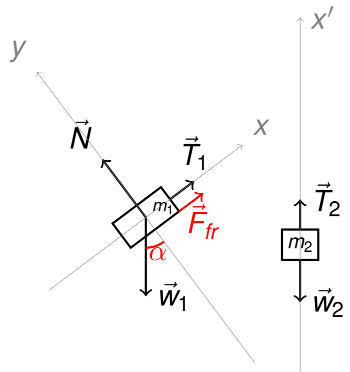


- Ignoring F_{fr}, the equations for the two unknowns T and a_1 are:

$$T - m_2 g = -m_2 a_1 \tag{55}$$

$$T - m_1 g \sin\alpha = m_1 a_1 \tag{56}$$

Example 2



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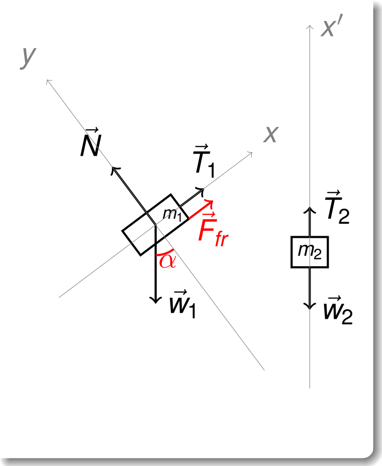
$$T - m_2 g = -m_2 a_1 \tag{55}$$

$$T - m_1 g \sin\alpha = m_1 a_1 \tag{56}$$

- Subtract the first equation from the second one to get:

$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \tag{57}$$

Example 2

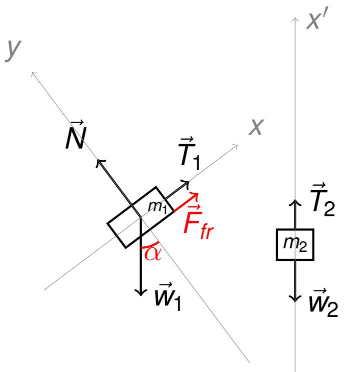


- Subtract the first equation from the second one to get:

$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \qquad (55)$$

- If the system is at rest and $m_2 g \geq m_1 g \sin\alpha$, the first mass will try to move in the $+\hat{x}$ direction, hence the friction force will be in the $-\hat{x}$ direction: $F_{fr} \leq 0$

Example 2



- Subtract the first equation from the second one to get:

$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \qquad (55)$$

- If the system is at rest and $m_2 g \le m_1 g \sin\alpha$, the first mass will try to move in the $-\hat{x}$ direction, hence the friction force will be in the $+\hat{x}$ direction: $F_{fr} \ge 0$.

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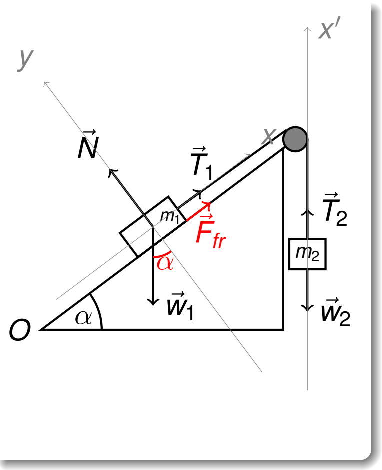
Example 2

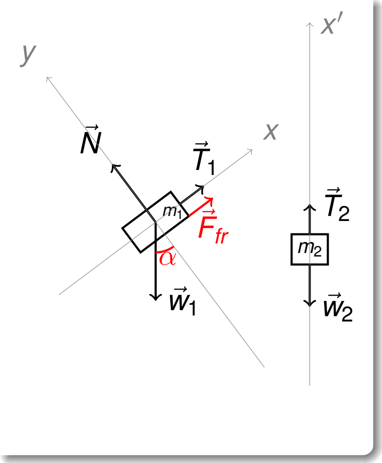


- Subtract the first equation from the second one to get:

$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \qquad (55)$$

- To proceed, assume the system is initially at rest and $m_2 > m_1 \sin\alpha$

Example 2

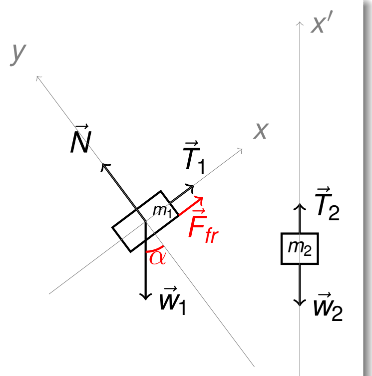


$$T - m_2 g = -m_2 a_1 \quad (55)$$

$$N = m_1 g \cos\alpha \quad (56)$$

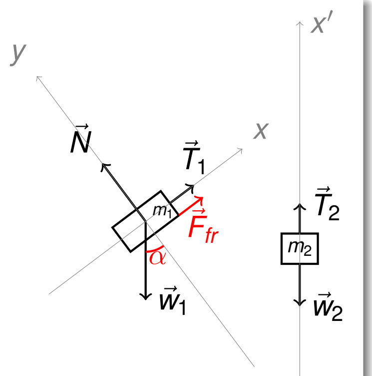
$$T - m_1 g \sin\alpha + F_{fr} = m_1 a_1 \quad (57)$$

Example 2



- To see if the system will stay at rest or not, set $a_1 = 0$.
- $T = m_2 g$
- $F_{fr} = (m_1 \sin\alpha - m_2)g$ (we had already assumed that $m_2 > m_1 \sin\alpha$)
- If $|F_{fr}| = (m_2 - m_1 \sin\alpha)g \leq \mu_s N$, then the system will stay at rest
- Otherwise if $(m_2 - m_1 \sin\alpha)g > \mu_s N$, then the system will start moving.

Example 2



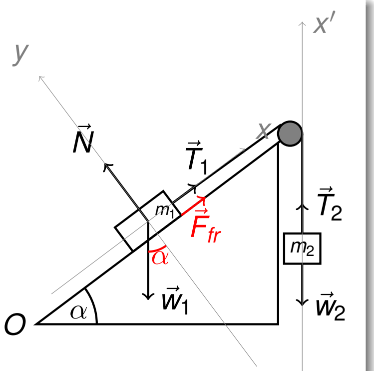
$$T - m_2 g = -m_2 a_1 \quad (55)$$

$$N = m_1 g \cos\alpha \quad (56)$$

$$T - m_1 g \sin\alpha + F_{fr} = m_1 a_1 \quad (57)$$

- After the system starts moving, $F_{fr} = -\mu_k N = -\mu_k m_1 g \cos\alpha$
- $m_2 g - m_1 g \sin\alpha - \mu_k m_1 g \cos\alpha = (m_1 + m_2) a_1$

Example 2

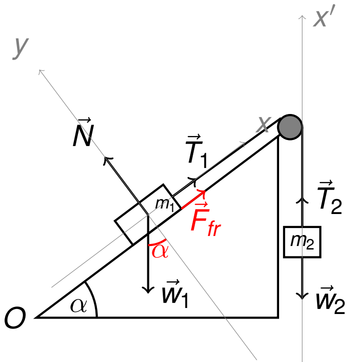


- Solving for a_1:

$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \tag{55}$$

$$- \frac{m_1}{m_1 + m_2}\mu_k g \cos\alpha \tag{56}$$

Example 2



- Solving for a_1:

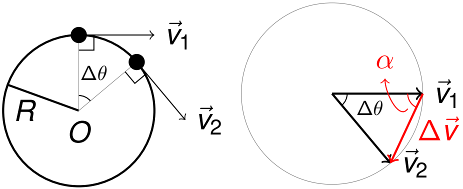
$$a_1 = g\frac{m_2 - m_1 \sin\alpha}{m_1 + m_2} \quad (55)$$

$$- \frac{m_1}{m_1 + m_2}\mu_k g \cos\alpha \quad (56)$$

- Compare with the previous result without the mass m_2 ($m_2 = 0$)

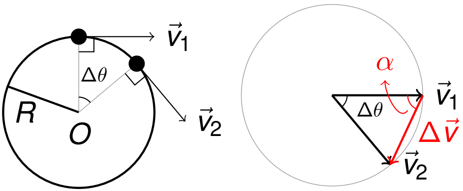
$$a_x = -g\sin\alpha - g\mu_k\cos\alpha \quad (57)$$

Uniform Circular Motion



- The object moves on a circle of constant radius with constant speed.
- The velocity is constantly changing $\Longrightarrow \vec{a} \neq 0$

Uniform Circular Motion



- The average acceleration is in the direction of $\Delta\vec{v}$
- For instantaneous acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\pi - \Delta\theta}{2} = \frac{\pi}{2} \tag{58}$$

$\vec{a}$ is perpendicular to $\vec{v}$

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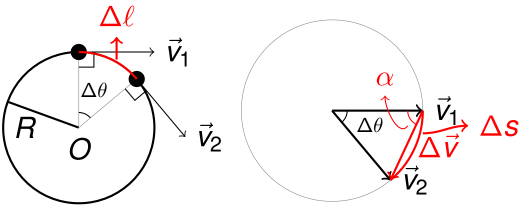
Uniform Circular Motion



$$\Delta s = v\Delta\theta$$
$$\Delta\ell = R\Delta\theta$$
$$\Delta\ell = v\Delta t$$

- Magnitude of $\vec{a}$:

$$|\vec{a}| \equiv a = \lim_{\Delta t\to 0}\frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\Delta\theta}{\Delta t} = v\frac{v}{R} = \frac{v^2}{R} \qquad (58)$$

- $\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t} \equiv \frac{v}{r}$ is called angular velocity

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- Let $\vec{r}$ be the position vector of the mass.
- $R^2 = \vec{r} \cdot \vec{r}$

$$0 = \frac{dR^2}{dt} = \vec{r} \cdot \frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt} \cdot \vec{r} \\ = 2\vec{r} \cdot \vec{v} \qquad (59)$$

Hence $\vec{v} \perp \vec{r}$.

- $v^2 = \vec{v} \cdot \vec{v}$

$$0 = \frac{dv^2}{dt} = \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v} \\ = 2\vec{v} \cdot \vec{a} \qquad (60)$$

Hence $\vec{a} \perp \vec{v}$

- Let $\vec{r}$ be the position vector of the mass.
- Let θ be the angle that $\vec{r}$ makes with the x axis:

$$\vec{r} = R(\cos\theta\hat{x} + \sin\theta\hat{y}) \equiv R\hat{r} \quad (59)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = R\frac{d\theta}{dt}(-\sin\theta\hat{x} + \cos\theta\hat{y}) \equiv R\frac{d\theta}{dt}\hat{\theta} \quad (60)$$

- $|\vec{v}| = R\frac{d\theta}{dt} \equiv v$ constant $\Longrightarrow \omega \equiv \frac{d\theta}{dt} = \frac{v}{R}$ is constant
- The acceleration:

$$\vec{v} = v(-\sin\theta\hat{x} + \cos\theta\hat{y}) \quad (61)$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt} = v\frac{d\theta}{dt}(-\cos\theta\hat{x} - \sin\theta\hat{y}) \equiv -v\omega\hat{r} \quad (62)$$

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