Circular Motion Dynamics

- Magnitude of acceleration of a uniform circulating body is

$$a = \frac{v^2}{R} \tag{63}$$

- Its direction points towards the center of circle
- A force has to be applied to the object to create this acceleration.
- By Newton's second Law, the magnitude of this force:

$$|\vec{F}| = m\frac{v^2}{R} \tag{64}$$

- This force has to be directed towards the center of circle

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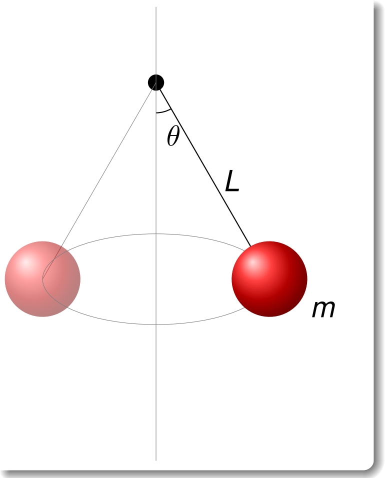
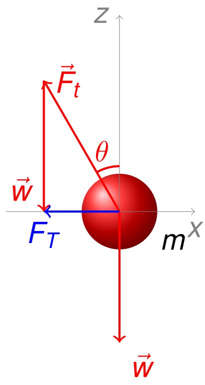
Conical Pendulum



- A mass is attached to a massless string. The mass makes uniform circular motion with speed v. Calculate v and the tension on the string

Conical Pendulum



- The forces acting on the mass:

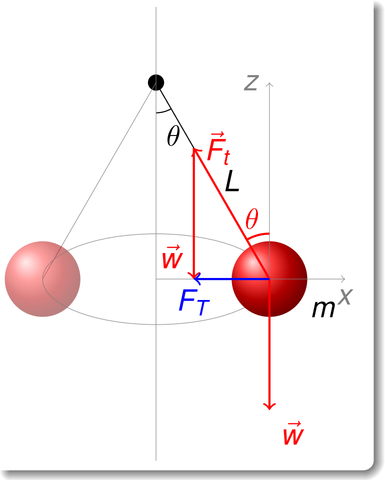
$$\vec{w} = -mg\hat{z} \tag{65}$$

$$\vec{F}_t = T\cos\theta\hat{z} - T\sin\theta\hat{x} \tag{66}$$

$$\vec{F}_T = (T\cos\theta - mg)\hat{z} - T\sin\theta\hat{x} \tag{67}$$

$$\equiv -m\frac{v^2}{R}\hat{x} \tag{68}$$

Conical Pendulum

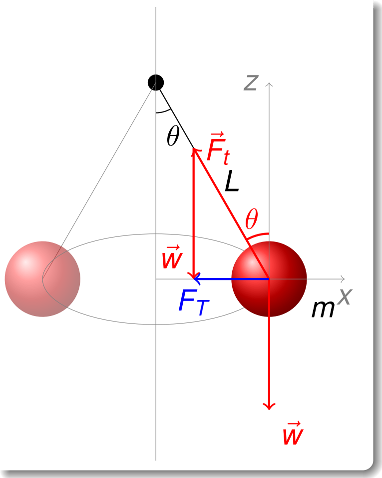


$$\vec{F}_T = (T\cos\theta - mg)\hat{z} - T\sin\theta\hat{x} \tag{65}$$

$$\equiv -m\frac{v^2}{R}\hat{x} = -m\frac{v^2}{L\sin\theta} \tag{66}$$

$$\Longrightarrow \begin{cases} T\cos\theta - mg & = & 0 \\ -T\sin\theta & = & -m\frac{v^2}{L\sin\theta} \end{cases} \tag{67}$$

Conical Pendulum



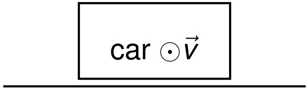
$$T\cos\theta - mg = 0 \implies T = \frac{mg}{\cos\theta} \quad (65)$$

$$T\sin\theta = m\frac{v^2}{L\sin\theta} \quad (66)$$

$$\implies gL\sin\theta\tan\theta = v^2 \quad (67)$$

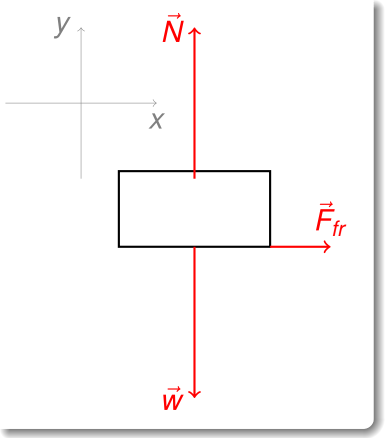
- Period of motion is $T = \frac{2\pi R}{v}$
- Check the units and the limits $v \to 0$ and $v \to \infty$

Skidding on a Curve



- A car of mass *m* takes a curve whose radius of curvature is *R*. What is its maximum velocity such that it will not skid?

Skidding on a Curve



- Forces acting on a car:

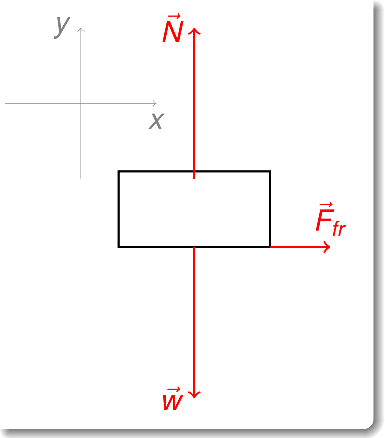
$$\vec{F}_{fr} = \mu_s N \quad (68)$$

$$\vec{N} = N\hat{y} \quad (69)$$

$$\vec{w} = -mg\hat{y} \quad (70)$$

$$\vec{F}_T = (N - mg)\hat{y} + \mu_s N\hat{x} \quad (71)$$

Skidding on a Curve



$$F_T = (N - mg)\hat{y} + \mu_s N\hat{x} \equiv m\frac{v_{max}^2}{R}\hat{x}$$

$$N - mg = 0 \quad (68)$$

$$\mu_s N = m\frac{v_{max}^2}{R} \quad (69)$$

$$\Longrightarrow v_{max} = \sqrt{\mu_s R g} \quad (70)$$

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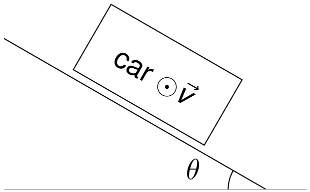
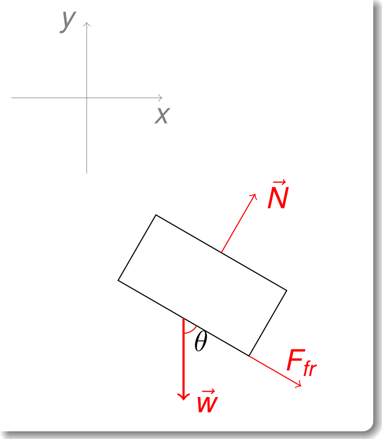
Banked Curves



- What should be the value of θ, such that a car moving at a speed v, can turn a curve with radius R without skidding? Ignore friction.

Banked Curves



- The forces acting on the car:

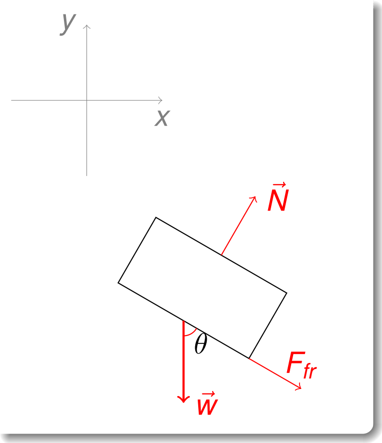
$$\vec{N} = N\sin\theta\hat{x} + N\cos\theta\hat{y} \tag{71}$$

$$\vec{F}_{fr} = 0 \tag{72}$$

$$\vec{w} = -mg\hat{y} \tag{73}$$

$$\vec{F}_T = N\sin\theta\hat{x} + (N\cos\theta - mg)\hat{y}$$
$$= m\frac{v^2}{R}\hat{x} \tag{74}$$

Banked Curves



$$\vec{F}_T = N\sin\theta\hat{x} + (N\cos\theta - mg)\hat{y}$$

$$= m\frac{v^2}{R}\hat{x}$$

$$\Longrightarrow \begin{cases} N\cos\theta - mg & = 0 \\ N\sin\theta & = m\frac{v^2}{R} \end{cases}$$

$$\Longrightarrow g\tan\theta = \frac{v^2}{R}$$