MIDTERM

- **Date:** November 16, 2013 (this saturday)
- **Time:** 13:30
- **Duration:** 3 Hours
- **Place:** U1, U2 and U3
- **Topics:** Everything we have covered until the midterm
- Formulas will be provided in the exam
- You will use two points for any misplaced vector sign, units, etc., e.g. You solve a problem of 10 points “correctly”. You misplace 5 vector sign and one unit. You will get “-2” points.

Department of Physics Fizik Bölümü 1960

Definition

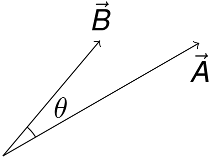
Consider a mass m and a *constant* force $\vec{F}$ acting ***on*** it. Under the influence of this force, the mass is displaced by $\Delta\vec{r}$. The work done ***by*** the force is

$$\Delta W = \vec{F} \cdot \Delta\vec{r}$$

/ The unit of work is $[W] = Nm \equiv J(oule)$.

Scalar Product-Review

- The scalar product of $\vec{A}$ and $\vec{B}$ is defined as $\vec{A} \cdot \vec{B} = AB\cos\theta$ where $A = |\vec{A}|$, $B = |\vec{B}|$ and θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.



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Scalar Product-Review

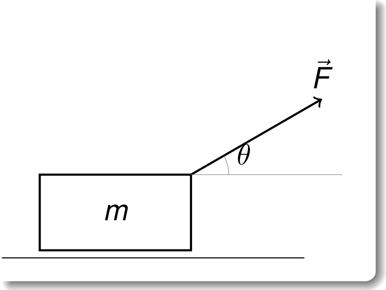
- In terms of their components, if

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \tag{90}$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} \tag{91}$$

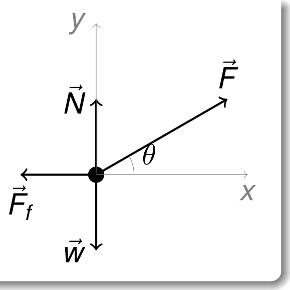
then $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z(\equiv AB\cos\theta)$

Work Done on a Block



- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

Work Done on a Block

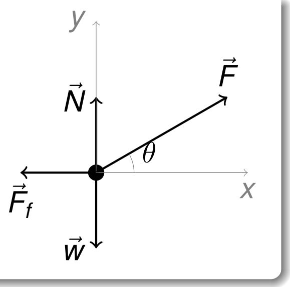


- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

- The work done by $\vec{F}$:

$$W_F = \vec{F} \cdot \Delta\vec{r} = F \Delta x \cos\theta$$

Work Done on a Block

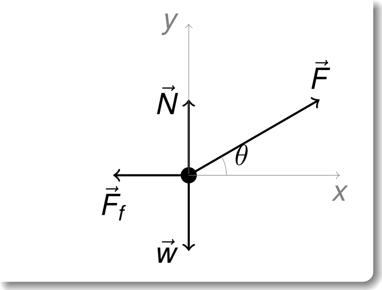


- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

- The work done by gravity:

$$W_w = \vec{w} \cdot \vec{\Delta} r = mg\Delta x \cos\frac{\pi}{2} = 0$$

Work Done on a Block

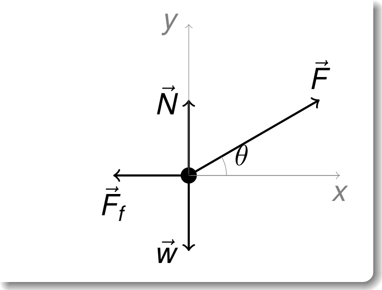


- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

- The work done by the Normal force

$$W_N = \vec{N} \cdot \vec{\Delta} r = N \Delta x \cos \frac{\pi}{2} = 0$$

Work Done on a Block

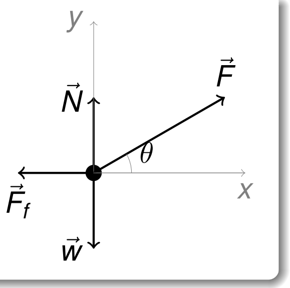


- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

- The work done by the friction force is:

$$W_f = \vec{F}_f \cdot \Delta\vec{r} = F_f \Delta x \cos\pi = -F_f \Delta x = -\mu_k mg\Delta x$$

Work Done on a Block



- The surface has friction, and the coefficient of kinetic friction is μ_k
- If the block moves horizontally by Δx then the work done on the mass by various forces are:

- The total work done on the mass is:

$$W_T = F\Delta x \cos\theta - \mu_k mg\Delta x = (F\cos\theta - \mu_k mg)\Delta x$$

Some Examples

Question

Suppose you hold your physics book, and walk on a flat surface with the book for 5 *secs*. If your speed is always constant during this time, what is the work done on the book by you? (Take the mass of the book to be 1 *kg*)

Answer

Some Examples

Question

Suppose you hold your physics book, and walk on a flat surface with the book for 5 *secs*. If your speed is always constant during this time, what is the work done on the book by you? (Take the mass of the book to be 1 *kg*)

Answer

None. The force you apply to the book, $\vec{F}$, is vertical, the displacement of the book, $\Delta\vec{r}$ is horizontal. Hence $W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos\frac{\pi}{2} = 0$

Some Examples

Question

Suppose you hold your physics book, and walk on a flat surface with the book for 5 *secs*. If during this time, your acceleration is $\vec{a} = a\hat{x}$ with $a = 0.1\ m/s^2$, what is the work done on the book by you?(Take the mass of the book to be 1 *kg*)

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Some Examples

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Answer

Since the book is accelerating, You should be applying an additional horizontal force with magnitude $F = (1\ kg)(0.1\ m/sec^2) = 0.1\ N$. The horizontal distance covered in the x direction during 5 *sec* is $\Delta x = \frac{1}{2}at^2 = \frac{1}{2}(0.1\ m/s^2)(5\ s)^2 = 2.5\ m$. The force and the displacement are in the same direction, hence the work done on the book by you is $W = (0.1\ N)(2.5\ m) = 0.25\ J$

Some Examples

Question

Suppose you take the elevator with the book from the ground floor to the first floor which is 3 m above. What is the work done on the book by you? by its weight? Assume that the the elevator is always moving very slowly so that neglect any acceleration.

Answer

Some Examples

Question

Suppose you take the elevator with the book from the ground floor to the first floor which is 3 m above. What is the work done on the book by you? by its weight? Assume that the the elevator is always moving very slowly so that neglect any acceleration.

Answer

If the upward direction is taken as the positive z direction, then the forces acting on the book are: The force of the hand: $\vec{F}_h = mg\hat{z}$; the weight of the book: $\vec{w} = -mg\hat{z}$; the displacement of the book: $\Delta\vec{r} = \Delta x\hat{z}$ where $\Delta x = 3\ m$. The work done by these forces are:

$$W_h = \vec{F}_h \cdot \Delta\vec{r} = mh\Delta x \qquad (92)$$

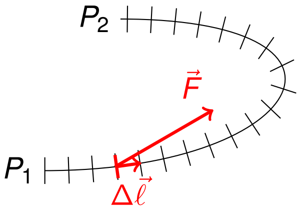
$$W_w = \vec{w} \cdot \Delta\vec{r} = -mg\Delta x \qquad (93)$$

Some Examples

Question

If you are just holding your book, but not doing anything else, you are not doing any work. Why do you get tired if you are not doing any work?

Work Done By a Variable Force



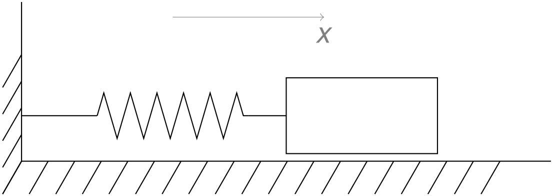
- Divide the path of the object into infinitesimal segments $\Delta\vec{\ell}$
- Within each segment, the force is (almost) constant

$$\Delta W = \vec{F}(\vec{r}) \cdot \Delta\vec{\ell}$$

- The total work is the sum of all ΔW in the limit $|\Delta\vec{\ell}| \to 0$

$$W = \int_{P_i}^{P_f} \vec{F} \cdot d\vec{\ell}$$

Work Done By a Spring

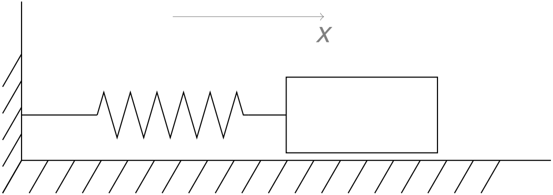


- If the mass is displaced by x from its equilibrium position, the string exerts a force of magnitude, $F = kx$. Its direction is towards the equilibrium position (Hooke' Law) :

$$\vec{F} = -kx\hat{x}.$$

k is called the spring constant.

Work Done By a Spring



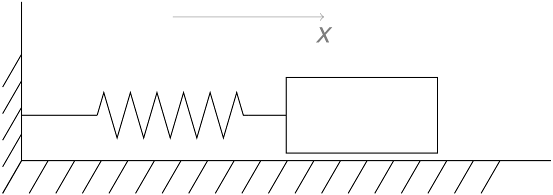
- If the mass is displaced by x from its equilibrium position, the string exerts a force of magnitude, $F = kx$. Its direction is towards the equilibrium position (Hooke' Law) :

  $$\vec{F} = -kx\hat{x}.$$

  k is called the spring constant.
- If the mass is initially at the position x and is displaced by dx, then $\Delta\vec{r} = dx\hat{x}$, hence,

  $$\Delta W = \vec{F} \cdot \Delta\vec{r} = (-kx\hat{x}) \cdot (dx\hat{x}) = -kxdx$$

Work Done By a Spring



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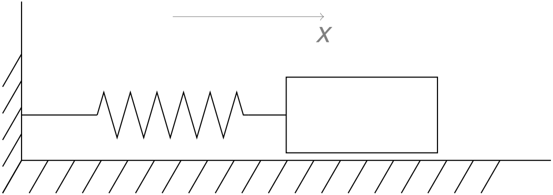
$$\Delta W = \vec{F}\cdot\Delta\vec{r} = (-kx\hat{x})\cdot(dx\hat{x}) = -kxdx$$

- Summing over x from $x=0$ upto $x=L$, the total work done by the string is

$$W_s = \int_0^L (-kxdx) = -k\left.\frac{x^2}{2}\right|_{x=0}^{x=L} = -\frac{1}{2}kL^2$$

Work Done By a Spring



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- If a force $\vec{F}_{ext}$ is applied to displace the mass with zero acceleration, then $\vec{F}_{ext} = -\vec{F}$ and hence

$$W_{ext} = -W_s = \frac{1}{2}kL^2$$

Kinetic Energy and Work-Energy Principle

- In the previous exercise, assume that at time t, the mass is at position x, with velocity $\vec{v}_i = v_i \hat{x}$. Its acceleration is $-kx/m$.
- In time dt, it will be displaced by $\delta x = \frac{v_i + v_f}{2} dt$. Its new speed will be $v_f = v_i - kx/m dt$
- The work done on the mass is

$$dW = -kxdx = -kx\frac{v_i + v_f}{2}dt = kx\frac{v_i + v_f}{2}\frac{m}{kx}(v_f - v_i) \quad (92)$$

$$= \frac{m}{2}v_f^2 - \frac{m}{2}v_i^2 = d\left(\frac{1}{2}mv^2\right) \Longrightarrow W = \Delta\left(\frac{1}{2}mv^2\right) \quad (93)$$

- $\frac{1}{2}mv^2$ is defined as the kinetic energy of the particle
- The work done on a mass is equal to the change in its kinetic energy.

General Derivation of Work-Energy Principle

- Definition of Work, and Newton's second Law (note that $\vec{F}$ is the total force):

$$W = \int \vec{F} \cdot d\vec{\ell} = \int m\vec{a} \cdot d\vec{\ell} \tag{94}$$

- $d\vec{\ell}$ is the displacement of object (in a time dt), hence $d\vec{\ell} = \vec{v}dt$:

$$= \int m\frac{d\vec{v}}{dt} \cdot \frac{d\vec{\ell}}{dt}dt = \int m\frac{d\vec{v}}{dt} \cdot \vec{v}dt \tag{95}$$

- Simplifying:

$$= \int \frac{d}{dt}\left(\frac{1}{2}m\vec{v}^2\right) dt = \int d\left(\frac{1}{2}m\vec{v}^2\right) = \Delta\left(\frac{1}{2}m\vec{v}^2\right) \tag{96}$$