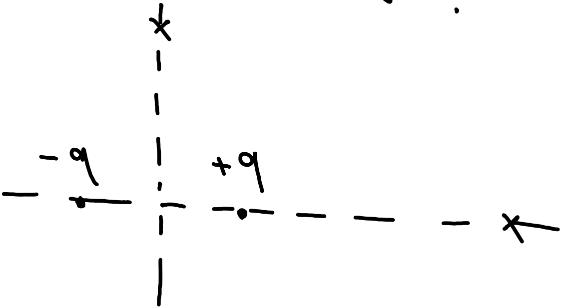
March 5, 2015

Hand in your HW!



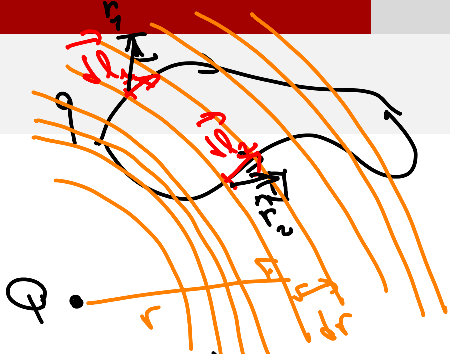
Potential - Potential Energy

$\vec F$

$$\vec F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}\hat r_{12}$$

If a force is conservative

$\int_{P_1}^{P_2} \vec F \cdot d\vec l$ is independent of path.



$$W_1 = \vec{F}_1 \cdot d\vec{l}_1$$

$$W_2 = \vec{F}_2 \cdot d\vec{l}_2$$

$$W_1 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,\hat{r}_1 \cdot d\vec{l}_1$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,(dl_1)_{\parallel}$$

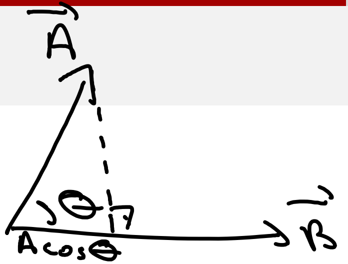
$$\boxed{W_1 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,dr}$$

$$W = \int \vec{F} \cdot d\vec{l}$$

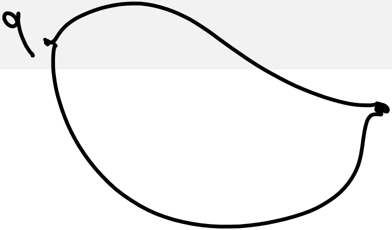
$$W_2 = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,\hat{r}_2 \cdot d\vec{l}_2$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,(dl_2)_{\parallel}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}\,dr = W_1$$



$$\vec{A}\cdot\vec{B} = A(B\cos\theta)$$
$$= (A\cos\theta)\,B$$
$$= B\,A_{\parallel}$$

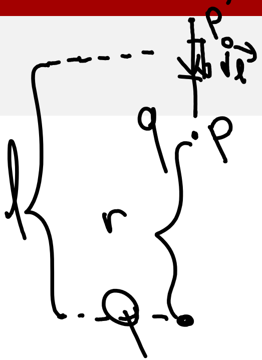


$$
\left.
\begin{aligned}
W &= \int \vec{F}_T \cdot d\vec{\ell} \\
&= \int \sum \vec{F}_i \cdot d\vec{\ell} \\
&= \sum \underbrace{\int \vec{F}_i \cdot d\vec{\ell}}_{\text{indep. of path}}
\end{aligned}
\right\} \text{sum is indep of path}
$$

$\Longrightarrow$ path

$$U(P) = U(P_0) - \int_{P_0}^{P} \vec{F} \cdot d\vec{\ell}$$

$$\boxed{U(P) = U(P_0) + \int_{P_0}^{P} (-\vec{F}) \cdot d\vec{\ell}}$$



$$U(P) = U(P_0) - \int_{P_0}^{P} \vec{F}\cdot\vec{dl}$$

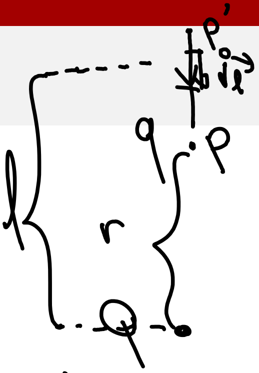
$$= U(P_0) - \cancelto{0}{\int_{P_0}^{P_0'} \vec{F}\cdot\vec{dl}} - \int_{P_0'}^{P} \vec{F}\cdot\vec{dl}$$

on a circle with charge Q at the center

$$\vec{dl} = dl\,\hat{r}$$

$$dl < 0$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{l^2}\hat{r}$$



$$\vec{F}\cdot\vec{dl} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{l^2}\,dl$$

$$U(P) = U(P_0) - \int_{r_0}^{r}\frac{1}{4\pi\varepsilon_0}\frac{qQ}{l^2}\,dl$$

$$\vec{dl} = dl\,\hat{r}$$

$$U(P) = U(P_0) + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{l}\Bigg|_{l=r_0}^{r}$$

$$dl < 0$$

$$U(P) = U(P_0) +$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{l^2}\,\hat{r}$$

r_0: distance of P_0 from Q

$$U(P) = U(P_0) + \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r} - \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r_0}$$

$$U(P) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} + \left(U(P_0) - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r_0}\right)$$

choose $\boxed{U(P_0) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r_0}}$

$$\boxed{U(P) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}}$$

$$U(P) = U(P_0) - \int_{P_0}^{P} \vec{F} \cdot d\vec{l}$$: Electric potential energy

$$\vec{F} = q\vec{E}$$

$$\lim_{q\to 0} \frac{U(P)}{q} = \lim_{q\to 0} \frac{U(P_0)}{q} - \int_{P_0}^{P} \vec{E} \cdot d\vec{l}$$

$$\equiv$$

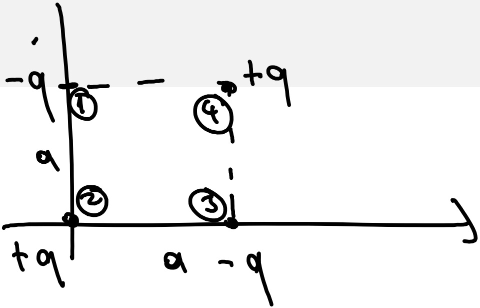
$$V(P) = V(P_0) - \int_{P_0}^{P} \vec{E} \cdot d\vec{l}$$: Electric Potential

$$[V] = 1 \text{ volt}$$

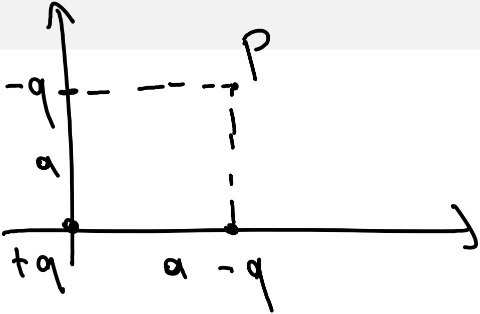
$$[\vec{E}] = 1 \text{ volt}/\text{m}$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



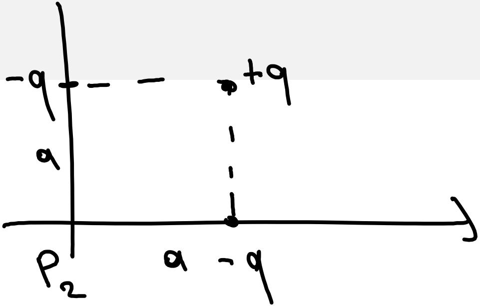
$$U_{1234} = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{a} + \frac{q^2}{a\sqrt{2}} - \frac{q^2}{a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{a\sqrt{2}}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-4 + \frac{2}{\sqrt{2}}\right]$$



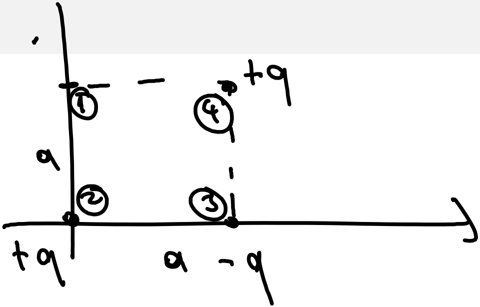
$$V(P) = ?$$

$$V_{123}(P_4) = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{a} + \frac{-q}{a} + \frac{q}{a\sqrt{2}}\right]$$

$$\boxed{V_{123}(P_4) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}\left[-2 + \frac{1}{\sqrt{2}}\right]}$$



$$V_{134}(P_2) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}\left(-2+\frac{1}{\sqrt{2}}\right)$$



$$V_{234}(P_1) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}\left[2-\frac{1}{\sqrt{2}}\right]$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{r_{14}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_4}{r_{24}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_4}{r_{34}} + \dots$$

$$U = \sum_{j>i} \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

$$U = \frac{1}{2}\sum_{i\neq j} \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

$$U = \frac{1}{2}\sum_{i \neq j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2}\sum_j q_j \left(\sum_i \frac{q_i}{r_{ij}} \frac{1}{4\pi\varepsilon_0} \right)$$

$$U = \frac{1}{2}\sum_j q_j V(q_j)$$

Electric Field from potential

$$V(P)-V(P_0) = -\int_{P_0}^{P} \vec{E}\cdot d\vec{l}$$

$\vec{\Delta l} \equiv d\vec{l}$

$P_0 \to P$

$$\Delta V \equiv V(P)-V(P_0)$$

$$\boxed{\Delta V \simeq -\vec{E}\cdot\vec{\Delta l}} \quad \forall\, \vec{\Delta l} \text{ (sufficiently small)}$$

$\vec{\Delta l} = (\Delta x)\hat{x}$

$\Rightarrow \vec{E}\cdot\vec{\Delta l} = E_x(\Delta x)$

$$E_x = -\lim_{\Delta x\to 0}\frac{\Delta V}{\Delta x} = -\lim_{x\to x_0}\frac{V(x,y,z)-V(x_0,y,z)}{x-x_0}$$

$$E_x(x,y,z) = -\frac{\partial V}{\partial x}(x,y,z)$$

$$E_y(x,y,z) = -\frac{\partial V}{\partial y}(x,y,z)$$

$$E_z(x,y,z) = -\frac{\partial V}{\partial z}(x,y,z)$$

$$\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$$

$$= -\left[\hat{x}\frac{\partial}{\partial x}V + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}\right]$$

$$= -\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)V$$

$$= -\vec{\nabla}V$$

$\hookrightarrow$ gradiant

Example

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0} Q\left(x^2+y^2+z^2\right)^{-1/2}$$

$$E_x = -\frac{\partial V}{\partial x} = \cancel{-}\frac{1}{4\pi\varepsilon_0} Q\left(\cancel{-\frac{1}{2}}\right)\left(x^2+y^2+z^2\right)^{-\frac{1}{2}-1}\left(\cancel{2}x\right)$$

$$E_x = \frac{1}{4\pi\varepsilon_0} Q \frac{x}{r^3}$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{1}{4\pi\varepsilon_0} Q \frac{y}{r^3}$$

$$E_z = \frac{1}{4\pi\varepsilon_0} Q \frac{z}{r^3}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^3} (x\hat{x} + y\hat{y} + z\hat{z})$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^3} \vec{r}$$

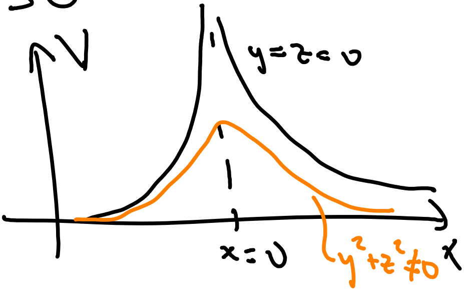
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}}$$

$$\boxed{\hat{r} = \frac{\vec{r}}{r}}$$

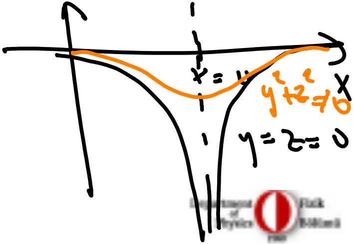
$Q.$ $$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

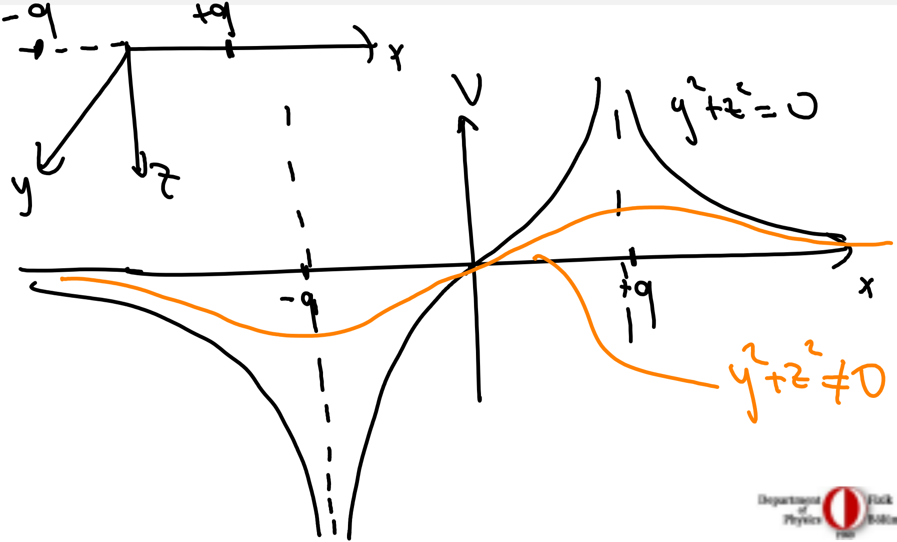
$$r = \sqrt{x^2+y^2+z^2}$$

$Q > 0$

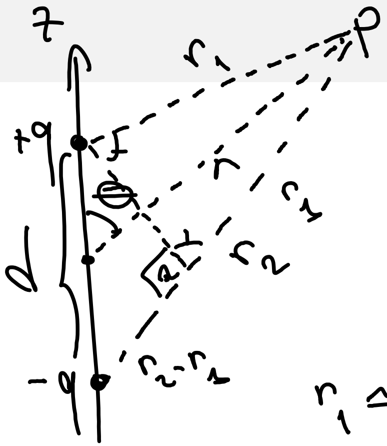


$Q < 0$




-q
+q
x
y
z
V
$y^2+z^2=0$
-q
+q
x
$y^2+z^2\neq 0$

$$V(P) = ?$$



$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\underset{(r_2)}{r_1}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\underset{(r_1)}{r_2}}$$

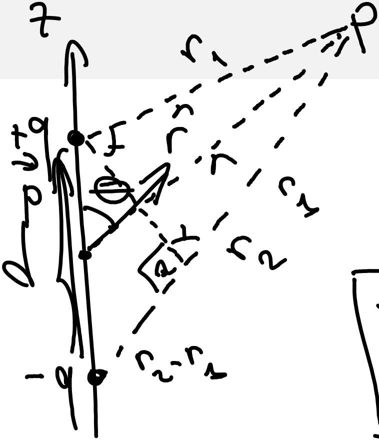
$$= \frac{1}{4\pi\varepsilon_0} q \left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$r_1 \simeq r_2 \simeq r$$

$$r_2 - r_1 \simeq d\cos\theta$$

$$r \gg d$$

$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{(qd)\cos\theta}{r^2}$$



$$V(P) = \frac{1}{4\pi\varepsilon_0}\frac{(qd)\cos\theta}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{|\vec{p}|\,|\hat{r}|\cos\theta}{r^2}$$

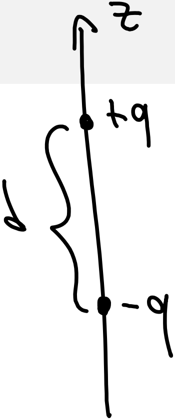
$$\boxed{V(P) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}}$$

HW!

$$V(x,y,z) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

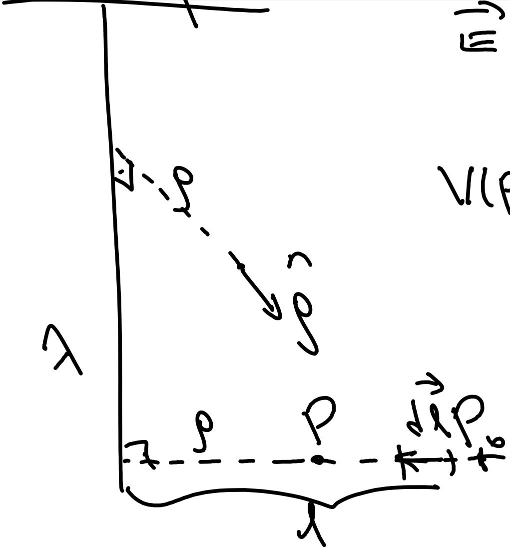
$$\hat{r} = \frac{\vec{r}}{r}, \quad r = (x^2+y^2+z^2)^{1/2},$$



$$\vec{p} \equiv (qd)\,\hat{z}$$


V
$\frac{1}{r^2}$ + corrections
r

Example

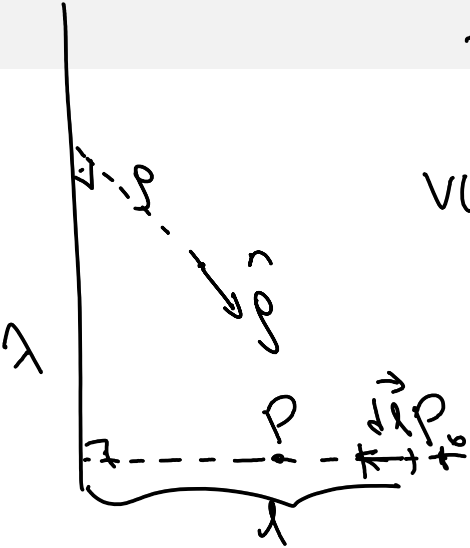


$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{\rho}\hat{\rho}$$

$$V(P) - V(P_0) = -\int_{P_0}^{P} \vec{E}\cdot\vec{dl}$$

$$\vec{dl} = dl\,\hat{\rho}$$

$$\vec{E}\cdot\vec{dl} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{l}\,dl$$



$$\vec{E}\cdot d\vec{l} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{l}\,dl$$

$$V(P) - V(P_0) = -\int_{\rho_0}^{\rho} \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{l}\,dl$$

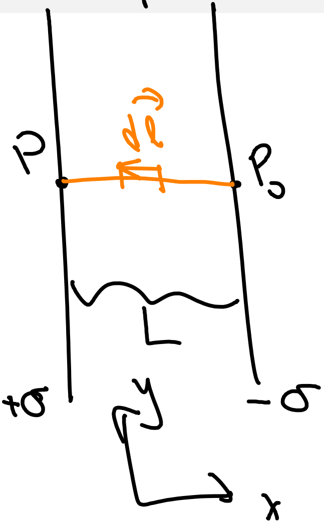
$$\boxed{= -\frac{1}{2\pi\varepsilon_0}\lambda \ln\frac{\rho}{\rho_0}}$$

$$\ln\left(\frac{\rho}{\rho_0}\right) \neq \ln\rho - \ln\rho_0$$

$$V(P) - V(P_0) = -\frac{1}{2\pi\varepsilon_0}\ln\frac{\rho}{\rho_0}$$

$$\Rightarrow V(P) \neq -\frac{1}{2\pi\varepsilon_0}\ln\rho$$

Example



$$\Delta V = ? \equiv V(P) - V(P_0)$$

$$= -\int_{P_0}^{P} \vec{E}\cdot\vec{dl}$$

$$\vec{dl} = dl\,\hat{x} \quad ; \quad dl<0$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

$$\Delta V = -\int_{P_0}^{P} \frac{\sigma}{\varepsilon_0} dl = \boxed{\frac{\sigma}{\varepsilon_0} L}$$

$$\frac{\Delta V}{L} = |\vec{E}|$$

$$V(x,y,z) = ?$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

$$E_y = -\frac{\partial V}{\partial y} = 0; \quad E_z = -\frac{\partial V}{\partial z} = 0$$

$$E_x = \frac{\sigma}{\varepsilon_0} = -\frac{\partial V}{\partial x} \Rightarrow \boxed{V = -\frac{\sigma}{\varepsilon_0}x}$$