March 10, 2015

$$U = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_4}{r_{14}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_4}{r_{24}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_4}{r_{34}} + \cdots$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}$$

$$U = \sum_{j=1}^{N} q_j \sum_{i=1}^{j-1} \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_{ij}} = \sum_{i<j=1}^{N} \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}$$

$$U = \sum_{i<j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2} \sum_{i \neq j} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{r_{ij}}$$

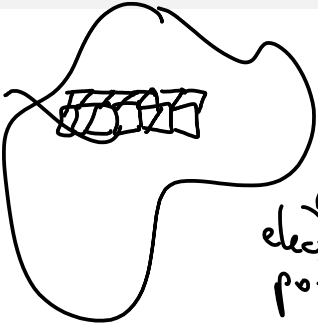
$$\boxed{U = \frac{1}{2} \sum_i q_i V(\vec{r}_i)}$$

$\vec{r}\,'$

volume

$$V(\vec{r}\,') = \sum_{\vec{r}} \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{r})\, dV}{|\vec{r} - \vec{r}\,'|}$$

ΔV



electric pot.

$$V(\vec{r}\,') = \int d^3r \, \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{r})}{|\vec{r} - \vec{r}\,'|}$$

$$\Delta q = \rho(\vec{r})\, \Delta V$$

volume

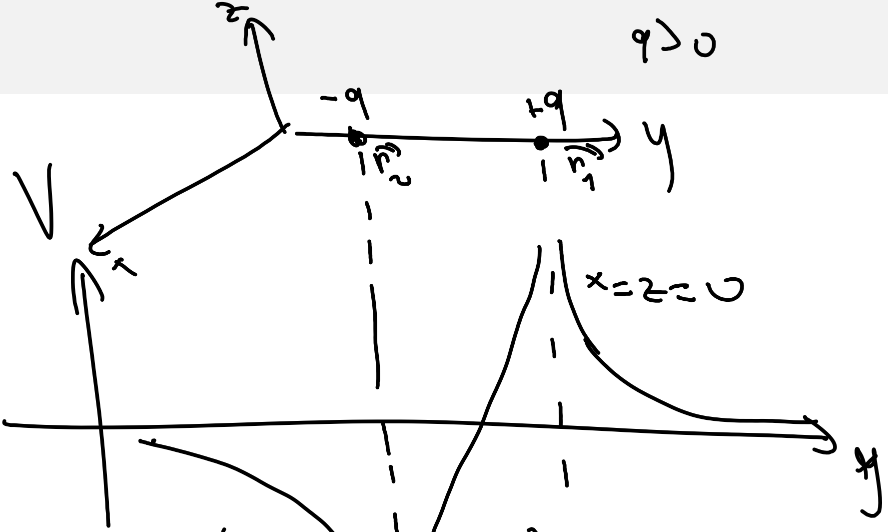
charge density

$$dV = d^3r$$

$$U = \frac{1}{2} \sum q_i V(\vec{r}_i)$$

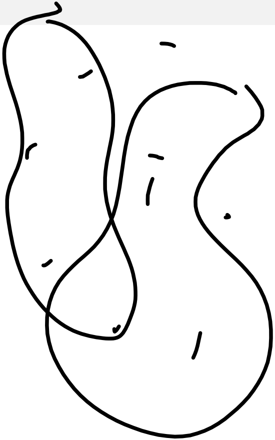
$$= \frac{1}{2} \sum_{n} \rho(\vec{r})\, d^3r\, V(\vec{r})$$

$$\boxed{U = \frac{1}{2} \int d^3r\, \rho(\vec{r})\, V(\vec{r})}$$



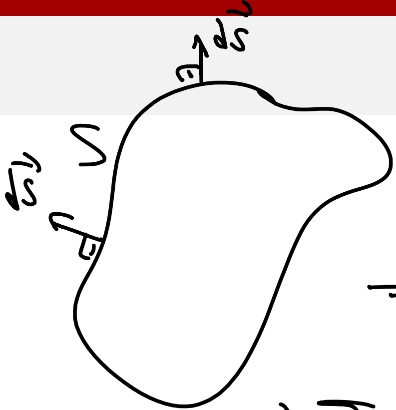
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{|\vec{r}-\vec{r}_1|} - \frac{q}{|\vec{r}-\vec{r}_2|}\right)$$

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r}}$$



$$E = \frac{1}{4\pi\varepsilon}\,\frac{q_1+q_2+q_3}{r^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}\,\hat{r}$$



$$\oint \vec{E}\cdot d\vec{S} = 0$$

$\Longrightarrow \vec{E} = 0$ on the surface

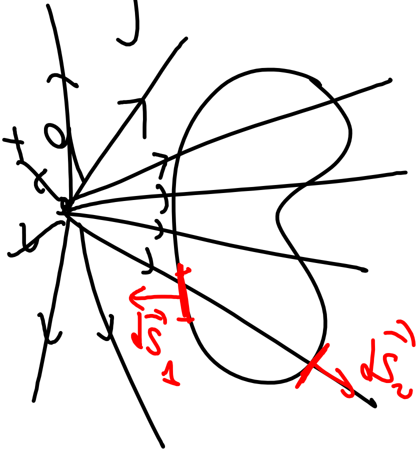
a) True

b) False

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}$$

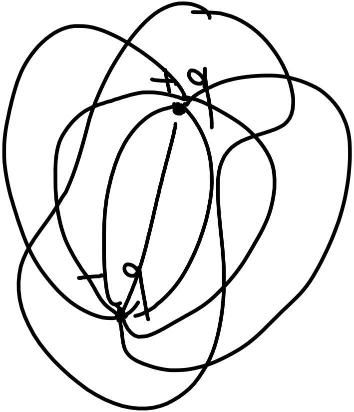
Gauss' Law

$$\oint \vec{E}\cdot d\vec{S} = 0 \Rightarrow Q_{enc} = 0$$



$$\vec{E}\cdot d\vec{S}_1 < 0$$

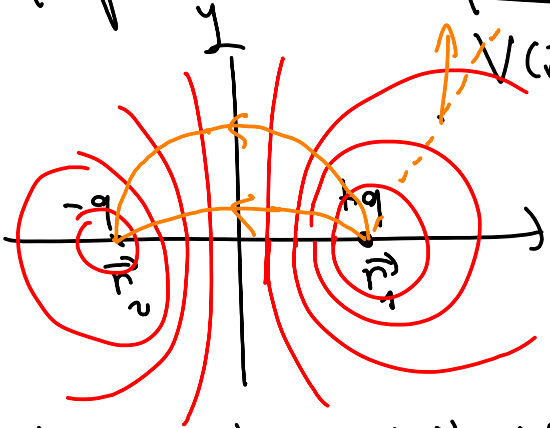
$$\vec{E}\cdot d\vec{S}_2 > 0$$



$$Q_{enc} = (+q) + (-q) = 0$$

$$\Rightarrow \oint \vec{E} \cdot d\vec{S} = 0$$

Equipotential Surfaces



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{r}_1|} - \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}-\vec{r}_2|}$$

all $\vec{r}$ for which $V(\vec{r}) = V_0$ defines a surface. $\Leftarrow$ equipotential surface.

$$\frac{\Delta V}{\Delta d} \sim |\vec{E}|$$

$$V(P_2) - V(P_1) = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$$

if P_1 & P_2 very close to each other

$$\boxed{dV = -\vec{E} \cdot d\vec{l}}$$

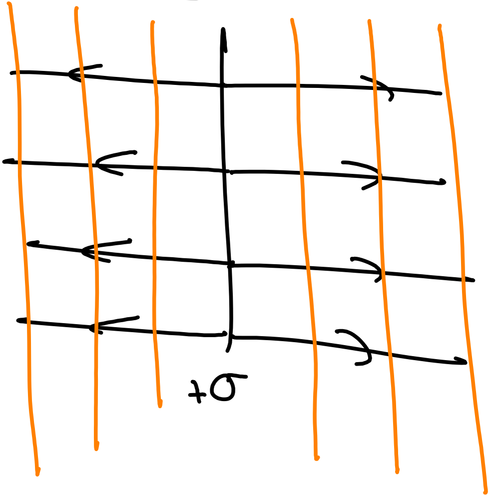
If P_1 & P_2 are on the same equipotential $\Rightarrow dV = 0$

$\Rightarrow \vec{E} \cdot d\vec{l} = 0$

$\Rightarrow \vec{E}$ is perpendicular to the surface!

Electric Field: a function that assigns a vector (Electric vector) at every point.

Example



Example A positive charge, initially at rest, moves towards

a) smaller potential smaller pot. En.

b) " " larger " "

c) larger " smaller pot. En.

d) " " larger " "

$$\Delta V = \lim_{q \to 0} \frac{\Delta U}{q}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$

Example A ~~positive~~ negative charge initially at rest moves towards

a) smaller potential smaller pot. En.

b) " " larger " "

c) larger " smaller pot. En.

d) " " larger " "