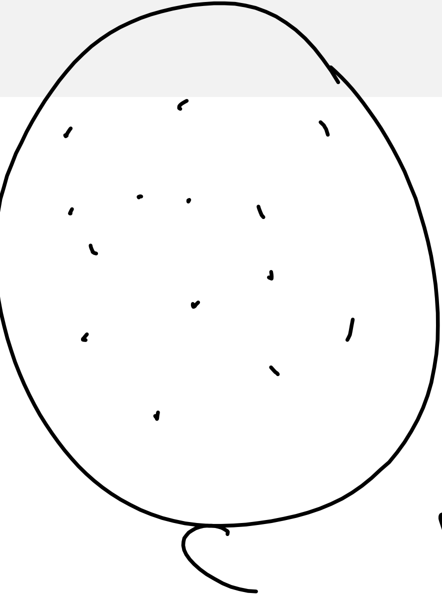
March 19, 2015

Hand in your HW!

Lecture videos are accesible from ODTUClass!



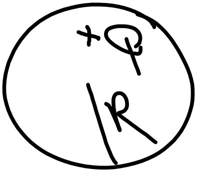
$$V = \sum \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_{ij}}$$

if charges are doubled, V is also doubled

$$V \propto Q$$

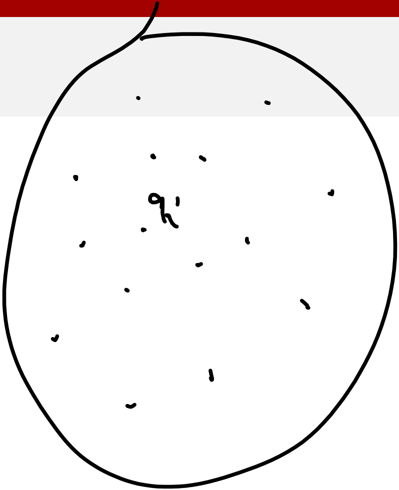
$$\boxed{Q = CV}$$

Example Spherical Capacitor

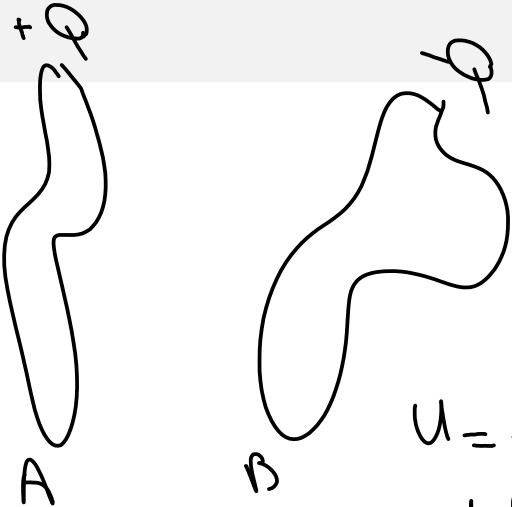


$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{Q}{C}$$

$$\boxed{C = 4\pi\varepsilon_0 R}$$



$$\boxed{U = \frac{1}{2}\sum q_i V(\vec{r}_i)}$$



$$U = \frac{1}{2}\sum q_i V(\vec{r}_i)$$

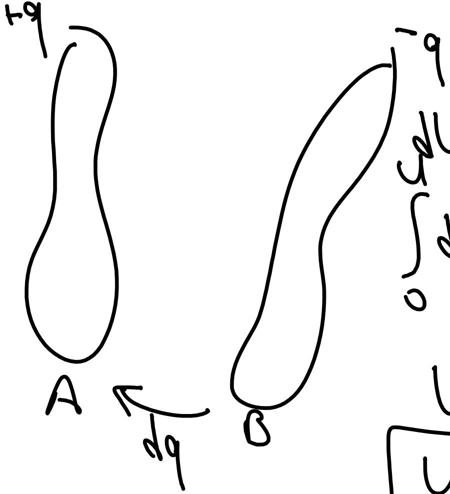
$$= \frac{1}{2}\sum_A q_i V_A + \frac{1}{2}\sum_B q_i V_B$$

$$U = \frac{1}{2}(+Q)V_A + \frac{1}{2}(-Q)V_B$$

$$= \frac{1}{2}Q(V_A - V_B)$$

$$\boxed{U = \frac{1}{2}QV}$$

2^{nd} derivation C is known!



$$V = \frac{q}{C}$$

$$dU = dW = dq\,V$$

$$\int_0^Q dU = \int_0^Q dq\,\frac{q}{C}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

$$\boxed{U = \frac{1}{2}QV}$$

Parallel Plate Capacitor

$$C = \varepsilon_0 \frac{A}{d}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$V = Ed = Q\frac{d}{A\varepsilon_0}$$

$$V = Ed$$

$$U = \frac{1}{2}V^2 C = \frac{1}{2}E^2 d^{\cancel{2}} \varepsilon_0 \frac{A}{\cancel{d}}$$

$$\boxed{U = \left(\frac{1}{2}\varepsilon_0 E^2\right)\underbrace{Ad}_{\text{volume}}}$$ when $\vec{E} \neq 0$

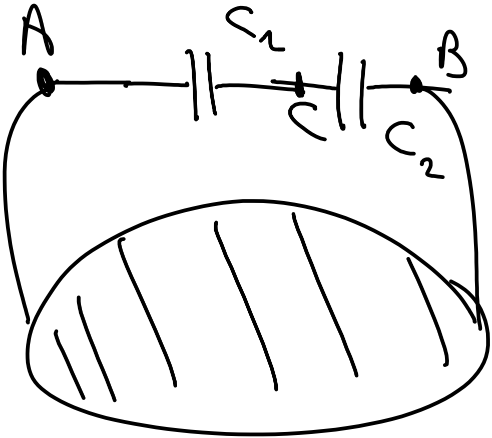
$$U = \int \left(\frac{1}{2}\varepsilon_0 E^2\right) d^3r$$

$$u_E = \frac{1}{2}\varepsilon_0 \vec{E}^2$$: energy density in the electric field.

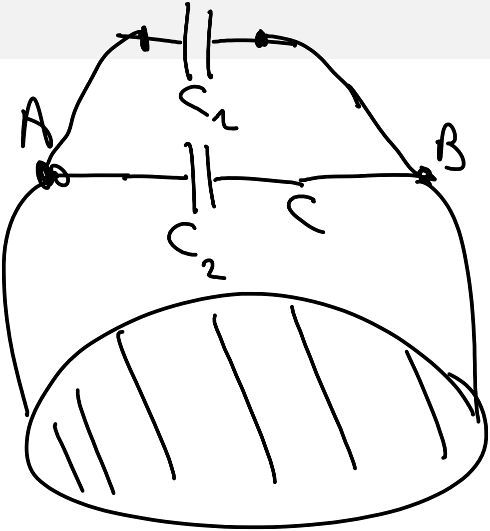
$$U = \frac{1}{2}\frac{Q^2}{C} \qquad (1)$$

$$U = \left(\frac{1}{2}\varepsilon_0 \vec{E}^2\right) A d \qquad (2)$$

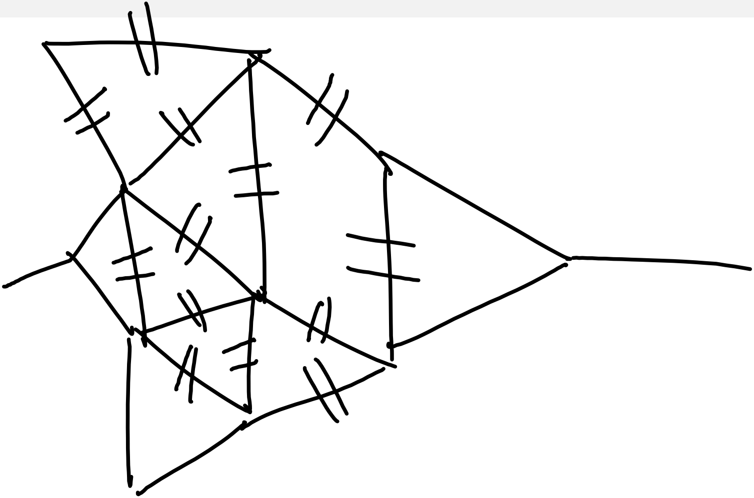
Connecting Capacitors



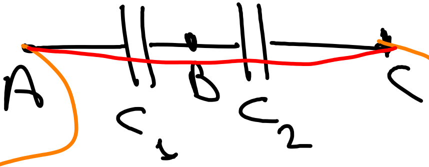
capacitors in series



capacitors in parallel



Capacitors in Series



$$V_{AC} = -\int_A^C \vec{E}\cdot\vec{dl}$$

$$V_{AC} = V_{AB} + V_{BC}$$

$\vec{E} = 0$ only if we neglect the fringing effects.

Example

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

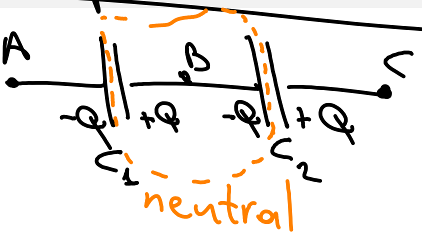
$\vec{r}$: position relative to the center.

$$V(A) - V(\infty) = -\int_{\infty}^{A} \vec{E}\cdot d\vec{l}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$V(A) = V(\infty) + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Capacitors in Series



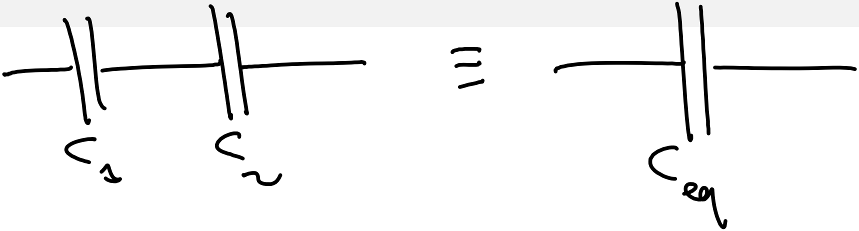
$$\underline{Q} = \underline{C}e\underline{V}at$$

$$V_{AB} = \frac{Q}{C_1}$$

$$V_{BC} = \frac{Q}{C_2}$$

$$V_{AC} = V_{AB} + V_{BC} = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

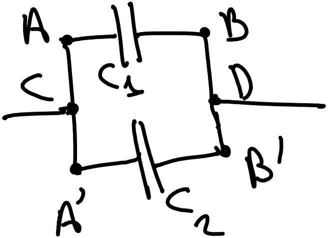
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \Leftarrow V_{AC} = \frac{Q}{C_{eq}}$$



$$U = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{Q^2}{2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

$$= \frac{1}{2}\frac{Q^2}{C_1} + \frac{1}{2}\frac{Q^2}{C_2}$$

Capacitors in Parallel



$$V_{AA'} = 0 \; ; \; V_{BB'} = 0$$

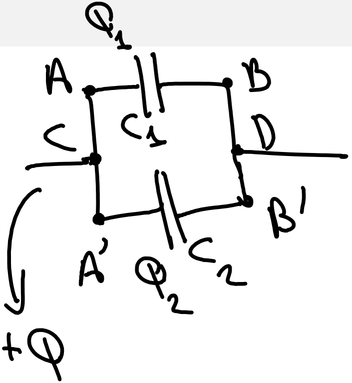
$$V_{AB} = V_{A'B'}$$

$$V_{AB} = V_A - V_B$$

$$V_{A'B'} = V_{A'} - V_{B'}$$

$$V_{AB} - V_{A'B'} = (V_A - V_{A'}) - (V_B - V_{B'})$$

$$= V_{AA'} - V_{BB'} = 0$$



$$V_{CD} = \frac{Q}{C_{eq}}$$

$$Q = C_{eq} V_{CD}$$

$+Q$

$$Q = Q_1 + Q_2$$

$$V_{AB} = V_{A'B'} = V_{CD} = V$$

$$C_{eq} V = C_1 V + C_2 V$$

$$\boxed{C_{eq} = C_1 + C_2}$$

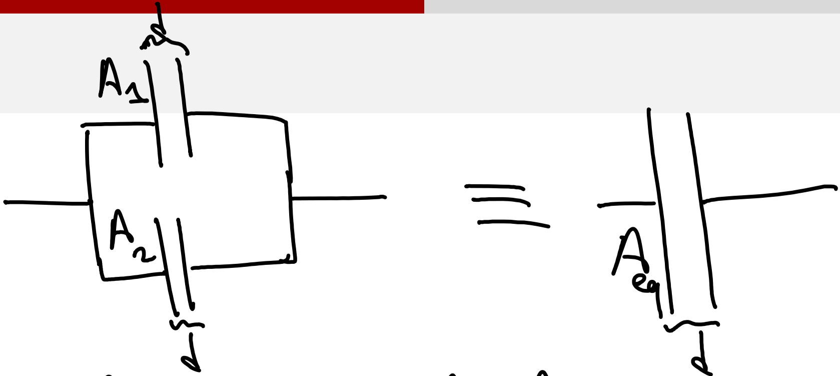
Parallel plate capacitor

$$C \sim \frac{A}{d}$$

$$\frac{1}{C_{eq}} \propto (d_1+d_2)$$

$$\frac{1}{C_1} \propto d_1$$

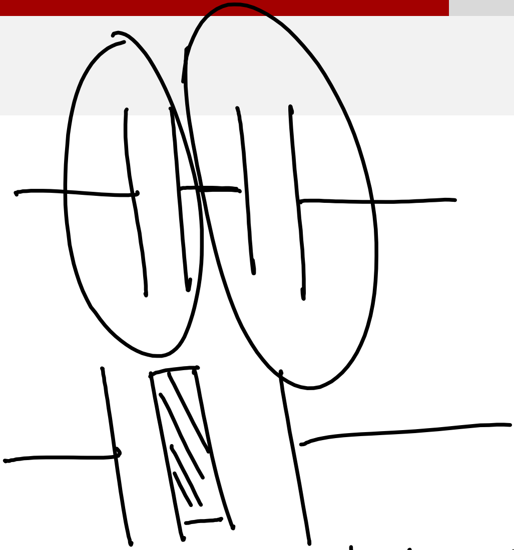
$$\frac{1}{C_2} \propto d_2$$



$C \propto A$

$$A_{eq} = A_1 + A_2$$

$$C_{eq} = C_1 + C_2$$



dielectric (insulator)

energy in parallel connection

$$U_{eq} = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$U_{eq} = U_1 + U_2$$

$$U = \frac{1}{2} \frac{Q^2}{C} \qquad U = \frac{1}{2} C V^2$$

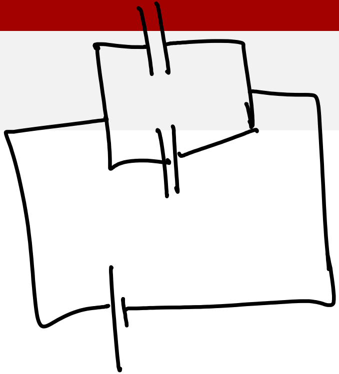
series:

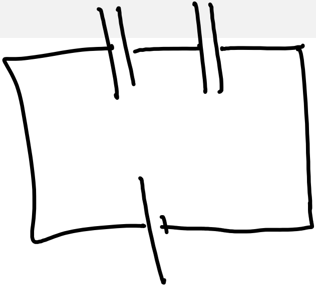
$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

$$= \frac{C_1 C_2}{C_1 + C_2} < C_{1,2}$$

parallel:

$$C_{eq} = C_1 + C_2 > C_{1,2}$$





U_1 U_2

$$U = \frac{1}{2} C V^2$$

$$U_1 > U_2$$

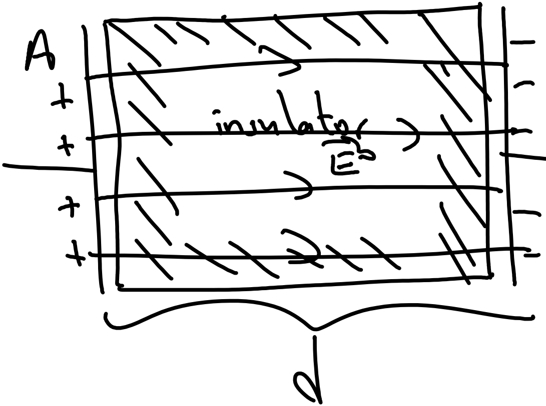
Q What happens to V if you decrease d in a capacitor that is NOT connected to a battery? ($Q_{capacitor} \neq 0$)

$$C = \varepsilon_0 \frac{A}{d} \qquad V = \frac{Q}{C}$$

if $d \Downarrow \Rightarrow C \Uparrow \Rightarrow V \Downarrow$

if $A \Downarrow \Rightarrow C \Downarrow \Rightarrow V \Uparrow$

Dielectrics



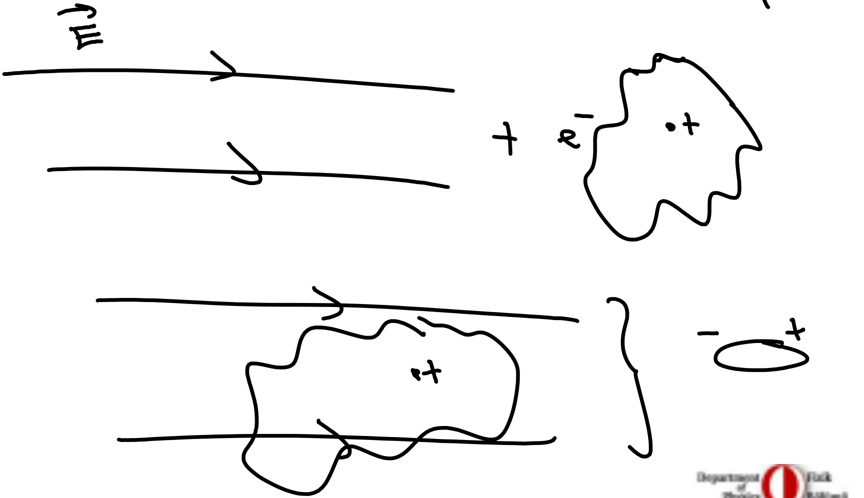
without insulator

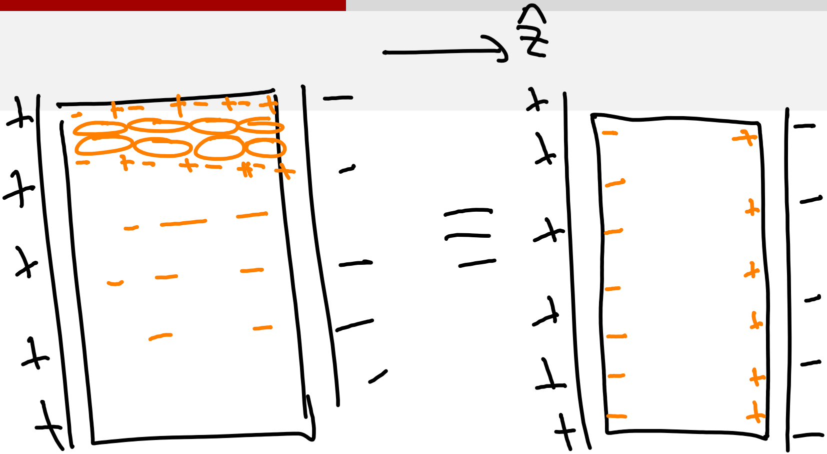
$$C = \varepsilon_0 \frac{A}{d}$$

$$|\vec{E}^0| = \frac{\sigma}{\varepsilon_0}$$

electric field in the absence of insulator

atom (molecule) in an electric field





$$\vec{E}^0 = \frac{\sigma_0}{\varepsilon_0}\hat{z}$$

$$\vec{E}_{ind} = \frac{\sigma_{ind}}{\varepsilon_0}(-\hat{z})$$

$$\vec{E} = \vec{E}^0 + \vec{E}_{ind} = \frac{(\sigma_0 - \sigma_{ind})}{\varepsilon_0}\hat{z}$$

$$\vec{E} = \frac{(\sigma^{\circ} - \sigma^{ind})}{\varepsilon}\hat{z}$$

$$|\vec{E}| < |\vec{E}^{\circ}| \Rightarrow V < V^{\circ}$$

$$V = \frac{Q}{C} \Rightarrow C > C^{\circ}$$

capacitance in the presence of the insulator

capacitance in the absence of insulator

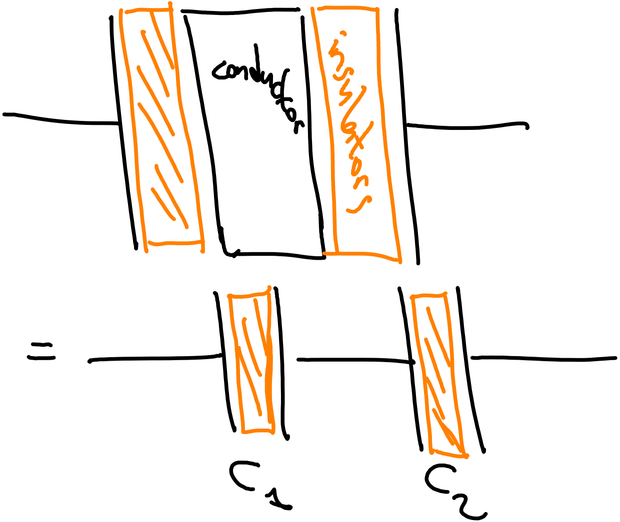
$$\vec{E}(\vec{E}^{\circ}) = K\vec{E}^{\circ} + \cdots$$ (linear dielectric)

$$= K\frac{\sigma}{\varepsilon_0}$$

$$\frac{\varepsilon_0}{K} \equiv \varepsilon$$: electric permeability of the dielectric.

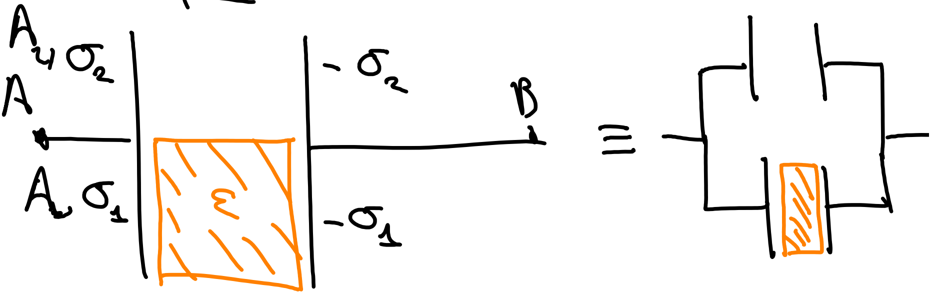
$$|\vec{E}| = \frac{\sigma}{\varepsilon} \quad ; \quad \boxed{C = \varepsilon\frac{A}{d}} = KC^{\circ}$$

Example



$C_{eq} < C_{1,2}$

Example



$$|\vec{E}_1| = \frac{\sigma_1}{\varepsilon}$$

$$|\vec{E}_2| = \frac{\sigma_2}{\varepsilon_0}$$

$$V_{AB} = E_1 d = E_2 d$$

$$\boxed{\frac{\sigma_1}{\varepsilon} = \frac{\sigma_2}{\varepsilon_0}}$$

$$Q = \sigma_1 A_1 + \sigma_2 A_2$$