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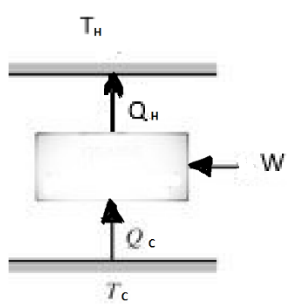
THE REFRIGERATION CYCLES SELF STUDY MODULE

Objective

The objective of this module is to guide you in analyzing the refrigeration cycles. You will be able to differentiate the refrigeration cycles relative to their energy demands while performing the cooling needed.

Compare

Now we start to learn about the refrigeration cycle. The figure shown demonstrates the general characteristics of a refrigeration cycle, summarized below:



1. The operation is cyclic, i.e. the process ends at the same state as it has started.
2. The cycle absorbs heat, Q_C, from the sink kept at constant T_C,
3. The cycle needs work, W
4. The cycle releases heat Q_H to the sink kept at constant temperature T_H. The cycle repeats itself.

Figure 1. The power cycle operating between a heat source at T_H and a heat sink operating at T_C

In this cycle the first law analysis reveals that

$\Delta U=Q_C-Q_H+W=0$ Because the operation is cyclic. Thus,

$W=Q_c-Q_H$ or, $Q_H= Q_c+W$

$\Delta S=0 =\frac{Q_C}{T_C}-\frac{Q_H}{T_H}+S_{generated}$ (Self Test: Why is $\Delta S=0$?)

Substituting Q_H in terms of Q_C and W in the entropy balance yields

$0= \frac{Q_C}{T_C}-\frac{Q_C+W}{T_H}+S_{generated}$

If the cycle is reversible, i.e., $S_{gen}=0$, we obtain a relationship between the amount of heat that we can remove from this cycle in terms of the amount of the work input and the difference between the source and the sink temperatures:

$$Coefficient\ of\ Performance\ COP = \frac{Q_C}{W} = \frac{T_C}{T_H - T_C}$$

This general characteristics of a refrigeration cycle is reflected in the hypothetical Carnot cycle, shown below. Notice that the cycle characteristics are the same. However, the cycles is operated in the opposite direction. This time, heat is absorbed at a lower temperature, and discarded at a higher temperature

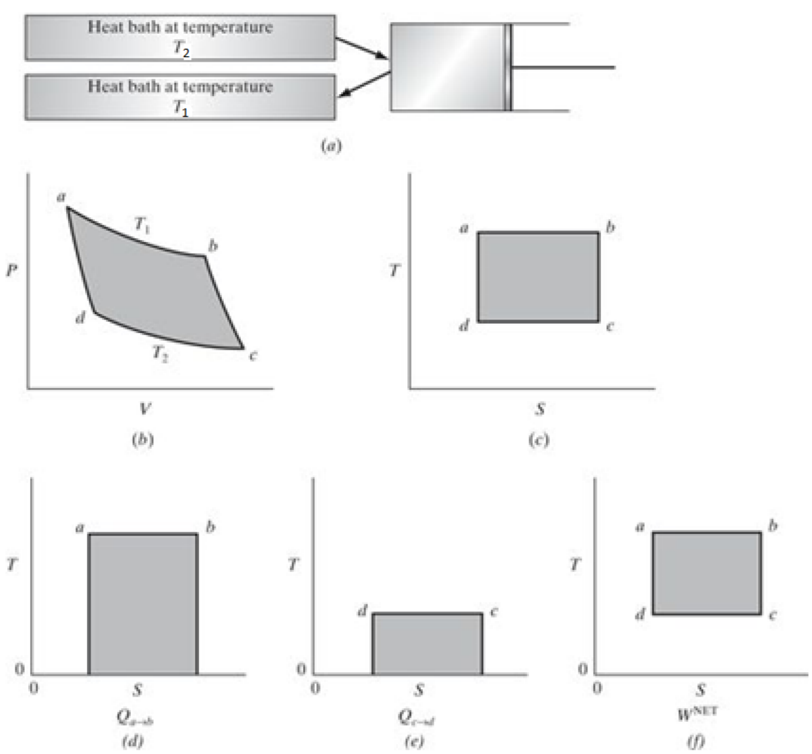


Figure 2. Sandler Figure 4.3-2 The Carnot Cycle and the heat and work interactions shown on a PV and TS diagrams. Notice that the same Carnot cycle operated in reverse direction becomes a refrigeration cycle. The only thing that changes is the temperatures at the source and at the sink.

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The tables below summarize the states, and the changes during the processes between these states.

State	T	P	V	S
a	T_1	P_a	V_a	S_a
b	T_1	P_b	V_b	S_b
c	T_2	P_c	V_c	S_b
d	T_2	P_d	V_d	S_a

Let us walk through this process and its energy and entropy balances

For the closed system, an ideal gas in the cylinder, the overall energy balance simplifies to

$$\frac{dU}{dt} = Q - P\frac{dV}{dt}$$

This governing equation is used in all steps of the refrigeration cycle to estimate its heat and work interactions. The cycle is reversible, and we will use our previously derived expression for the entropy change of an ideal gas given below.

$$\Delta S = n\,C_v \ln\left(\frac{T_2}{T_1}\right) + n\,R\ln\left(\frac{V_2}{V_1}\right)$$

Path	ΔU	Q	W	ΔS
d→c	0	$nRT_2\ln(V_c/V_d)$	$-nRT_2\ln(V_c/V_d)$	$nR\ln(V_c/V_d)$
c→b	$nC_v(T_1-T_2)$	0	$nC_v(T_1-T_2)$	0
b→a	0	$nRT_1\ln(V_a/V_b)$	$-nRT_1\ln(V_a/V_b)$	$nR\ln(V_a/V_b)$
a→d	$nC_v(T_2-T_1)$	0	$nC_v(T_2-T_1)$	0
Overall cycle	0	$-nR(T_2-T_1)\ln(V_a/V_b)$	$nR(T_2-T_1)\ln(V_a/V_b)$	0

For the overall cycle $\Delta S=0= nR\ln(V_c/V_d)+ nR\ln(V_a/V_b)=0$

$Q_{overall}= nRT_2 ln(V_c/V_d)+ nRT_1 ln(V_a/V_b)$

$ln(V_a/V_b)=- ln (V_c/V_d)$; $Q_{overall}=nR(T_2-T_1) ln (V_a/V_b)$;

$W_{overall}=-Q_{overall}$ such that the substitution yields

$COP=Q_c/W_{net}=\frac{T_C}{T_H-T_C}$

This expression is known as the Carnot refrigeration cycle coefficient of performance. Similar to the Carnot power cycle efficiency, the coefficient of performance is a very important identifier about the refrigeration cycle performances.

The Rankine Refrigeration Cycle

The general perspectives of the refrigeration cycle we discussed so far will prevail here also. We will have a low temperature source, a high temperature sink and while heat flows from cold to hot, we need to do work. Now let us remember the thermodynamic landscape, we have four different independent variables, as seen in the sketch on the right side, P,V, T,S. In the Carnot cycle we have navigated in the T, S domain of the processes. The Carnot cycle followed a route with constant T, constant S, constant T, constant S, in other words, TSTS sequence. Now we will analyze a PSPS cycle, the Rankine refrigeration cycle. The Rankine Refrigeration Cycle has the following features:

I. Reversible adiabatic, i.e. isentropic expansion, S
II. Constant pressure condensation, P
III. Reversible adiabatic, i.e. isentropic compression, S
IV. Constant pressure evaporation, P

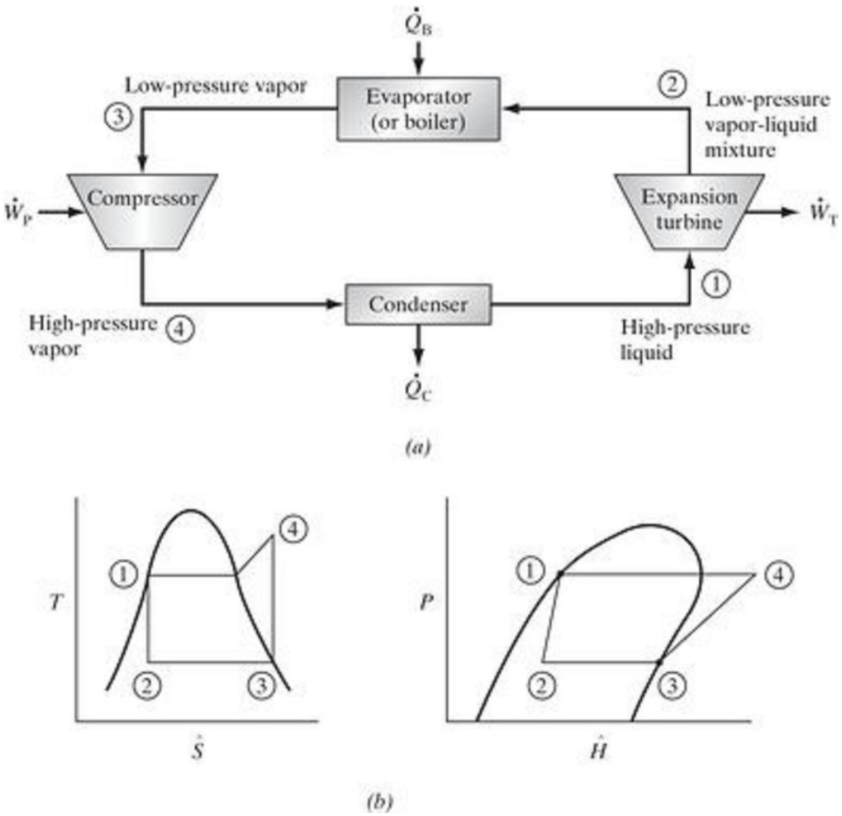

Figure 3. (this is FIGURE 5.2-3 in Sandler) Rankine refrigeration cycle

In the figure shown above the flow chart of a Rankine refrigeration cycle is summarized. Now we will analyze they cycle step by step.

I. System is selected around the expansion turbine, i.e. between points 1 and 2. The turbine operates adiabatically and reversibly (Self test: Why do we make such approximations?).The energy balance around this system simplifies to

$$0 = \dot{m}\left(\widehat{H}_{in} - \widehat{H}_{out}\right) + \dot{W}_{shaft}$$

Or,

$$\dot{W}_{turbine} = \dot{m}\left(\widehat{H}_2 - \widehat{H}_1\right)$$

II. System is selected around the evaporator (or boiler)

$$0 = \dot{m}(\widehat{H}_{in} - \widehat{H}_{out}) + \dot{Q}_{boiler}$$

Or,

$$\dot{Q}_{boiler} = \dot{m}(\widehat{H}_3 - \widehat{H}_2)$$

III. System is selected around the compressor, i.e. between points 3 and 4. The compressor operates adiabatically and reversibly (Self test: Why do we make such approximations?)

$$0 = \dot{m}(\widehat{H}_{in} - \widehat{H}_{out}) + \dot{W}_{shaft}$$

Or,

$$\dot{W}_{compressor} = \dot{m}(\widehat{H}_4 - \widehat{H}_3)$$

IV. The system is selected around the condenser.

$$0 = \dot{m}(\widehat{H}_{in} - \widehat{H}_{out}) + \dot{Q}_{boiler}$$

Or,

$$\dot{Q}_{condenser} = \dot{m}(\widehat{H}_1 - \widehat{H}_4)$$

If we can determine the states of the refrigeration fluid at the given process points of 1,2,3, and 4 we can determine the quantitative amounts of heat, net work and the coefficient of performance of the cycle.

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The refrigeration fluid

Those of you curious about the history of the refrigeration fluid, should read about the chlorofluorocarbons and their impact on the arctic ozone layers. These fluids were designed to be stable and their stability nearly destroyed Earth's protective ozone layer. Currently, HFC family of refrigerants are in use. We will base our design on HFC-134a, which is also the refrigeration fluid that is circulating in the refrigerator in your own kitchen. These fluids were designed to have certain physical properties: Their atmospheric boiling point should be close to -17 °C (are you curious about the reason?). Also, in order to be able to discard the heat to warm environments, the saturation pressure at around 50 °C should be moderate (again are you curious about the reason?).

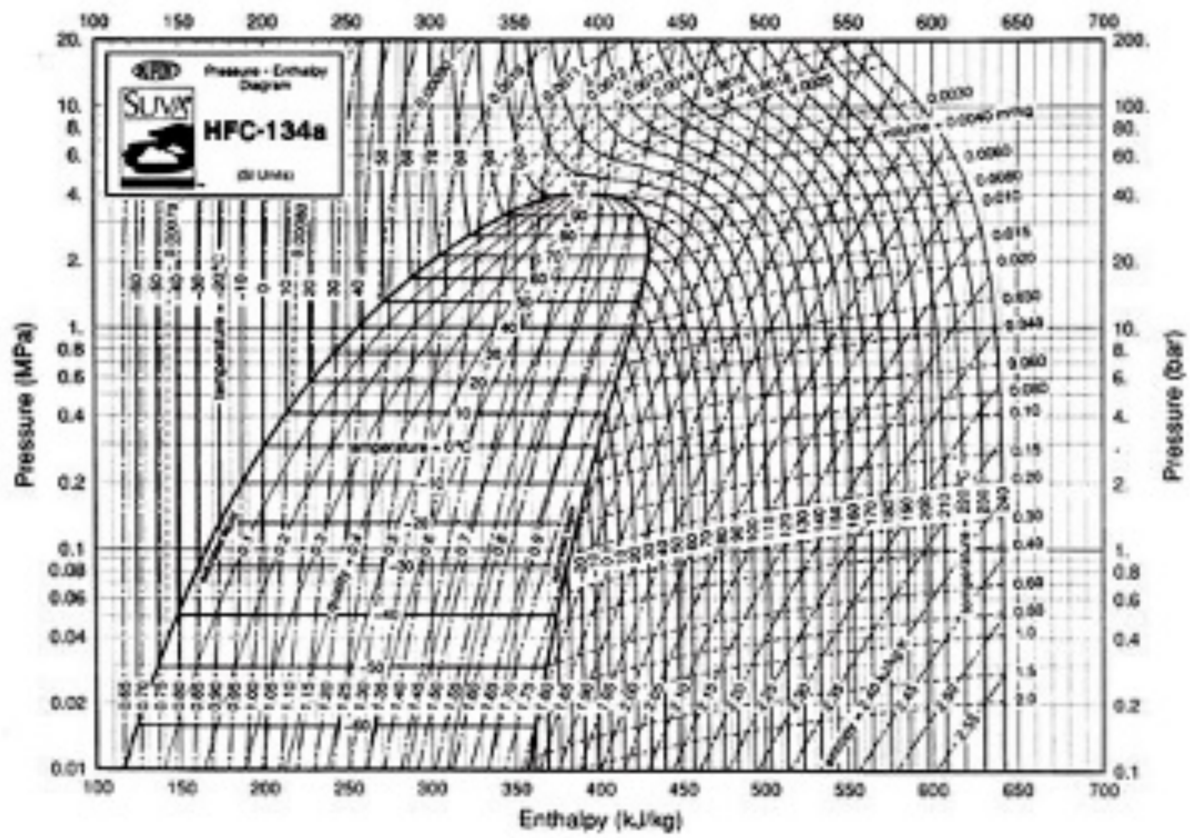

Figure 4. The PH diagram of HFC 134a

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Design a Refrigeration Cycle

Now we will design our own refrigeration cycle by selecting our own temperatures and pressures for this process. We will need the limits of refrigeration. Do we design a freezer, an air conditioner or a beverage cooler for food courts.

Design a refrigerator with a freezer

The coldest point of the refrigerator is its freezer. Our cycle will absorb heat from the freezer. The heat absorption is done by evaporating the refrigeration fluid at the needed temperature. Most refrigerators have freezer temperatures of around -17 °C (why?). The heat is discarded to the room, therefore the condenser should operate at around 50 °C in order to be able to discard heat to a space in which the temperatures can get as high as 45 °C. Given these pieces of information, and a knowledge of the refrigeration fluid, HFC 134a, it is possible to generate a navigation space in the thermodynamic landscape.

You are going to fill the table below, by reading from the relevant information from the PH diagram of HFC 134a shown above.

State	T(°C)	P	$\hat{H}$	$\hat{S}$
1				
2				
3				
4				

Now fill this table with the information you collected and using the energy and entropy balances.

Process unit	ΔH	Q	W	ΔS
The Turbine				
Boiler				
Compressor				
Condenser				
Overall cycle				

After filling the table with the relevant information and performing the necessary calculations, determine the cycle coefficient of performance.

Coefficient of performance $=Q_{cold}/W_{net}$

Compare this efficiency with that of the Carnot refrigeration cycle coefficient of performance. Make sure to use absolute temperature scale. Then we will discuss our results.

COP for Carnot$=\frac{T_H-T_C}{T_c}$

The vapor compression refrigeration cycle

This cycle is the simpler version of the Rankine refrigeration cycle. The turbine is replaced with a throttle valve at the expense of the recovered energy from the turbine. The flow chart and the cycle steps on a PH diagram is shown in Figure 5.

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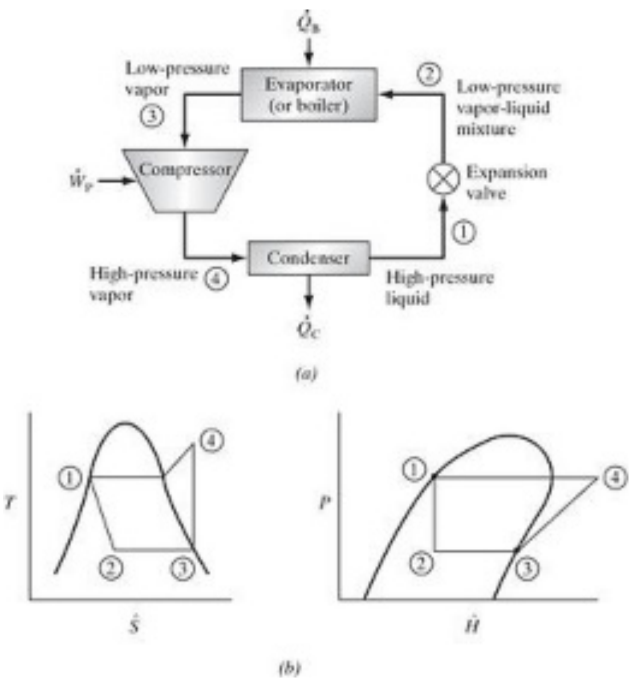

Figure 5. The vapor compression refrigeration cycle

This time, your responsibility is to repeat the same calculations, around the same temperatures, for the vapor compression refrigeration cycle.

You are going to fill the table below, by reading the relevant information from the PH diagram of HFC 134a shown above.

State	T(°C)	P	$\hat{H}$	$\hat{S}$
1				
2				
3				
4				

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Now fill this table with the information you collected and using the energy and entropy balances.

Process unit	ΔH	Q	W	ΔS
The Turbine				
Boiler				
Compressor				
Condenser				
Overall cycle				

After filling the table with the relevant information and performing the necessary calculations, determine the cycle coefficient of performance.

Coefficient of performance $=Q_{cold}/W_{net}$

Calculate

1. An air conditioner is designed to cool the rooms in Antalya. You are the engineer to select the cycle type, (Rankine or VCRC) select the air conditioner operating temperatures, ie Tc and Th, and estimate the capacity of the air conditioner that will provide a comfort zone in a room of 5mx5mx3 m.
2. This time your assignment is to choose the freezer for a meat packaging plant. Choose the temperature of the freezer, and determine the capacity of the cycle for a cold room that is about 30 m^2. How much electrical energy will be consumed as a result?

3. If you were to design a beverage cooler for a food court, what would be your choice of the evaporator temperature? What would be the cycle COP as a result?
4. The new generation of the refrigerators have their condensers inside the refrigerator itself. Hence, the heat discards from the freezer to the cooler. Given this information, design your refrigeration cycle and determine the coefficient of performance.

Evaluate yourself

What are the muddiest points, what are the points that you understood well? What should you do to clarify the parts that are not at all clear.

Why do we need -17 °C in the freezer?

What is the difference between an energy efficient refrigerator and a not so efficient refrigerator?

Reading assignment

Read Chapter 5 of Sandler's thermodynamics, 5th edition.

References

The content of this lecture notes is based on various editions of thermodynamics textbooks of Sandler (Wiley).