# EE-464 STATIC POWER CONVERSION-II 

Three Phase Inverters

Ozan Keysan<br>keysan.me

Office: C-113 • Tel: 2107586

## Three Phase Inverters

## ABB



Different Sized Variable Frequency Drives (VFD)

## Three Phase Inverters



Three Phase Voltage-Source Inverters

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Three inverter legs are connected in parallel

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. Current can flow through the switch or anti-parallel diodes.

## PWM Techniques

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There are many different PWM techniques that will be covered:
. Square-wave (Six-step) PWM
. Sinusoidal PWM (SPWM)

- Hysteresis (Bang-Bang) Control
. Space-Vector PWM (SVPWM)
. Third harmonic injection


## Six-Step Inverter

## Commonly used in BLDC motor Drives



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## Six-Step Inverter

- Each switch has $50 \%$ duty ratio.
- Each leg has a phase difference of 120 degrees
- One switching action takes place at every 60 degrees


## Six-Step Inverter



## Six-Step Inverter



Six-Step Inverter
Line-to-line voltage: $V_{A B}=V_{A 0}-V_{B 0}$


## Line-to-line voltages:





## Square Wave Operation



BLDC Drive with square wave

Switching Sequence


## Line-to-Line Voltages




(c)

## Equivalent Phase Voltages



## Line-to-Line Harmonics

Fourier Coefficients
$\hat{V}_{n, l-l}=\frac{1}{n} \frac{4}{\pi} V_{d c} \cos \left(n \frac{\pi}{6}\right)$
For: $n=6 k \pm 1=1,5,7,11,13 \ldots$

- No even harmonics
- No third order harmonics

Line-to-Line Harmonics
RMS of the fundamental component?

$$
V_{1, l-l, r m s}=\frac{1}{\sqrt{2}} \frac{4}{\pi} V_{d c} \frac{\sqrt{3}}{2}=0.78 V_{d c}
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$V_{1, l-l, r m s}=\frac{1}{\sqrt{2}} \frac{4}{\pi} V_{d c} \frac{\sqrt{3}}{2}=0.78 V_{d c}$
Harmonics RMS:
$V_{n, l-l, r m s}=\frac{1}{n} 0.78 V_{d c}$
For: $n=6 k \pm 1=1,5,7,11,13 \ldots$

Line-to-Line Harmonics


## Line-to-Neutral voltages:

Neutral point is floating


Voltage level changes every 60 degrees (that's why it's a six-step inverter!)

## Line-to-Neutral Harmonics

Fourier Coefficients
$\hat{V}_{n, l-N}=\frac{1}{n} \frac{2}{3 \pi} V_{d c}\left(2+\cos \left(\frac{\pi n}{3}\right)-\cos \left(\frac{2 \pi n}{3}\right)\right.$.
For: $n=6 k \pm 1=1,5,7,11,13 \ldots$
Simpler Form
$\hat{V}_{n, l-N}=\frac{1}{n} \frac{2}{\pi} V_{d c}$

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## Example: (D. Hart. 8-12)

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For the six-step three phase inverter shown below, Vin=100V, $f_{\text {out }}=60 \mathrm{~Hz}$. The load is Y -connected to load with a phase load of $R=10 \Omega, L=20 \mathrm{mH}$.


Calculate the total harmonic distortion (THD) of the load current and voltage.

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Amplitude for load current at each frequency:
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Table 8.7 Fourier Components for the Six-Step Inverter of
Example 8-12

| $\boldsymbol{n}$ | $\boldsymbol{V}_{\boldsymbol{n}, \boldsymbol{L}-\boldsymbol{N}}(\mathbf{V})$ | $\boldsymbol{Z}_{\boldsymbol{n}}(\Omega)$ | $\boldsymbol{I}_{\boldsymbol{n}} \mathbf{( A )}$ | $\boldsymbol{I}_{\boldsymbol{n}, \text { rms }}(\mathbf{A})$ |
| ---: | :---: | :---: | :---: | :---: |
| 1 | 63.6 | 12.5 | 5.08 | 3.59 |
| 5 | 12.73 | 39.0 | 0.33 | 0.23 |
| 7 | 9.09 | 53.7 | 0.17 | 0.12 |
| 11 | 5.79 | 83.5 | 0.07 | 0.05 |
| 13 | 4.90 | 98.5 | 0.05 | 0.04 |

## Voltage THD=

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## Voltage THD=

$\frac{\sqrt{\sum_{n=2}^{\infty} V_{n}^{2}}}{V_{1, r m s}} \approx \frac{\sqrt{12.73^{2}+9.09^{2}+5.79^{2}+4.90^{2}}}{63.6}=0.31=31 \%$

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## Current THD=

$\frac{\sqrt{\sum_{n=2}^{\infty} I_{n}{ }^{2}}}{I_{1, r m s}} \approx \frac{\sqrt{0.23^{2}+0.12^{2}+0.05^{2}+0.04^{2}}}{3.59}=0.07=7 \%$

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Voltage Plot, Current Plot

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## Sinusoidal PWM (SPWM)

Harmonics in the line voltage

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## Harmonics in the line voltage



Harmonics at the side bands,
Like the unipolar but starts at mf.

## Sinusoidal PWM (SPWM)

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If mf is small, it is better to use synchronized PWM, and mf should be an odd interger, preferably multiple of 3 to reduce harmonics.

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Harmonics in the line voltage

## Sinusoidal PWM (SPWM)

## Harmonics in the line voltage

Table 8-2 Generalized Harmonics of $v_{L L}$ for a Large and Odd $m_{f}$ That Is a Multiple of 3 .

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| $m_{f} \pm 2$ | 0.010 | 0.037 | 0.080 | 0.135 | 0.195 |
| $m_{f} \pm 4$ |  | 0.245 | 0.367 | 0.490 | 0.612 |
| $2 m_{f} \pm 1$ | 0.116 | 0.200 | 0.227 | 0.192 | 0.111 |
| $2 m_{f} \pm 5$ |  |  |  | 0.008 | 0.020 |
| $3 m_{f} \pm 2$ | 0.027 | 0.085 | 0.124 | 0.108 | 0.038 |
| $3 m_{f} \pm 4$ |  | 0.007 | 0.029 | 0.064 | 0.096 |
| $4 m_{f} \pm 1$ | 0.100 | 0.096 | 0.005 | 0.064 | 0.042 |
| $4 m_{f} \pm 5$ |  |  | 0.021 | 0.051 | 0.073 |
| $4 m_{f} \pm 7$ |  |  |  | 0.010 | 0.030 |

Voltage Levels?

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Linear Region $\left(m_{a}<1\right)$

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$$
V_{l-l, r m s}=0.612 V_{d}(\text { max in linear region })
$$

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V_{l-l, r m s} & =\frac{\sqrt{3}}{\sqrt{2}} \frac{4}{\pi} m_{a} \frac{V_{d}}{2} \\
V_{l-l, r m s} & =0.78 V_{d}
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& V_{l-l, r m s}=0.78 V_{d} \\
& V_{l-l, r m s, h}=\frac{0.78}{h} V_{d} \text { for } h=6 n \pm 1
\end{aligned}
$$

Voltage Levels?

## Voltage Levels?



## You can download this presentation from: keysan.me/ee464

Harici Slaytlar

## Push-Pull Inverter

## Push-Pull Inverter

Similar to Push-Pull Converter


## Push-Pull Inverter

Similar to Push-Pull Converter


But without the rectifying diodes

## Push-Pull Inverter

## Push-Pull Inverter



## Push-Pull Inverter



T1, T2 operates in sequence

## Push-Pull Inverter

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Voltage output can be adjusted by the turns-ratio

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\hat{V}_{o 1}=m_{a} \frac{V_{d}}{n}
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- Transistors have common ground (no isolation required for gate drives)


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- A good transformer, with high coupling is required (to reduce energy in the leakage inductance)


## Switch Utilization in Single Phase Inverters

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$=\frac{V_{o 1} I_{o, \text { max }}}{q V_{T} I_{T}}$

## Switch Utilization in Single Phase Inverters

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Assume highly inductive load, no current harmonics (just the fundamental)
$=\frac{V_{o 1} I_{o, \text { max }}}{q V_{T} I_{T}}$
Maximum utilization occurs at square wave

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Half Bridge Inverter

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Output Voltage: $V_{o 1, \max }=\frac{4}{\pi \sqrt{2}} \frac{V_{d, \max }}{2}$

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Output Voltage: $V_{o 1, \max }=\frac{4}{\pi \sqrt{2}} \frac{V_{d, \max }}{2}$
Max. Switch Utilization
$q=2$
$=\frac{1}{2 \pi} \approx 0.16$

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Voltage, Current Ratings?

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$=\frac{1}{2 \pi} \frac{\pi}{4} m_{a}=\frac{1}{8} m_{a}$
Linear Region
$=0.125$ when $m_{a}=1$

