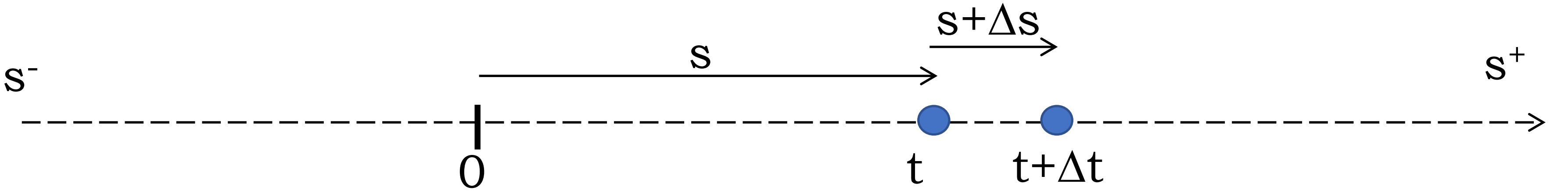
ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

Dr. Ergin TÖNÜK
Department of Mechanical Engineering
Graduate Program of Biomedical Engineering
tonuk@metu.edu.tr
http://tonuk.me.metu.edu.tr

2.2 Rectilinear Motion

Motion along a straight line.



$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} = \dot{s}$$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \dot{v}$$

$$v = \frac{ds}{dt} = \dot{s}$$

$$a = \frac{dv}{dt} = \dot{v}$$

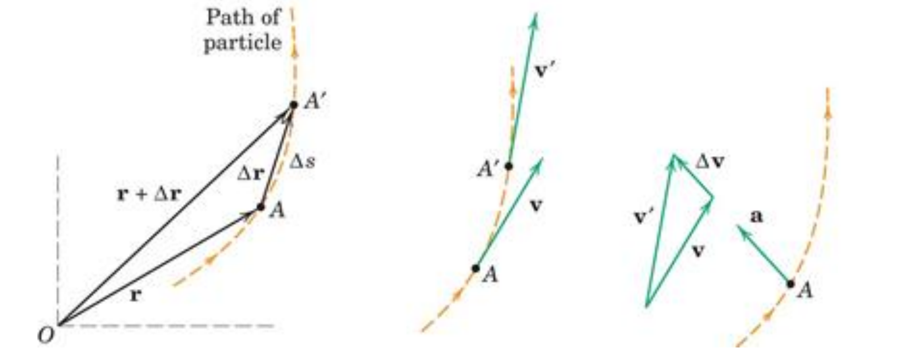
$$vdv = ads$$

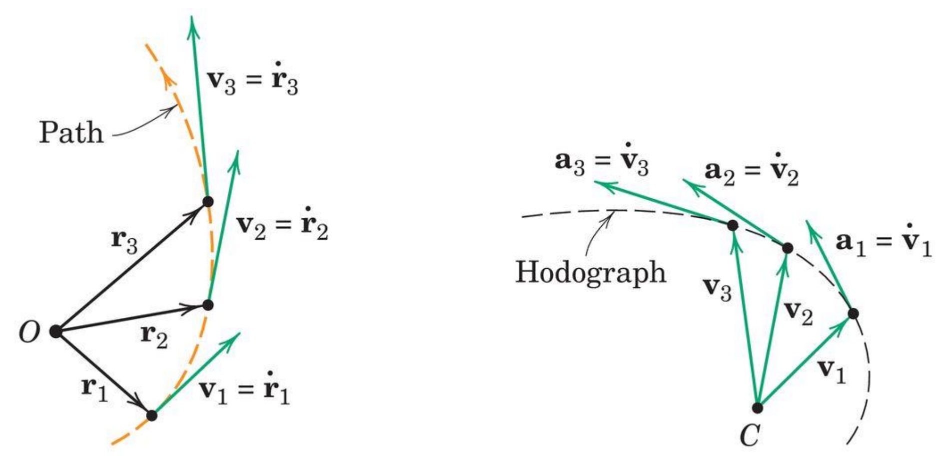
2.3 Plane Curvilinear Motion

Motion along a curved path in a single plane.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{\vec{r}}$$





2.3 Plane Curvilinear Motion

Three different coordinate systems to analyze plane curvilinear motion are:

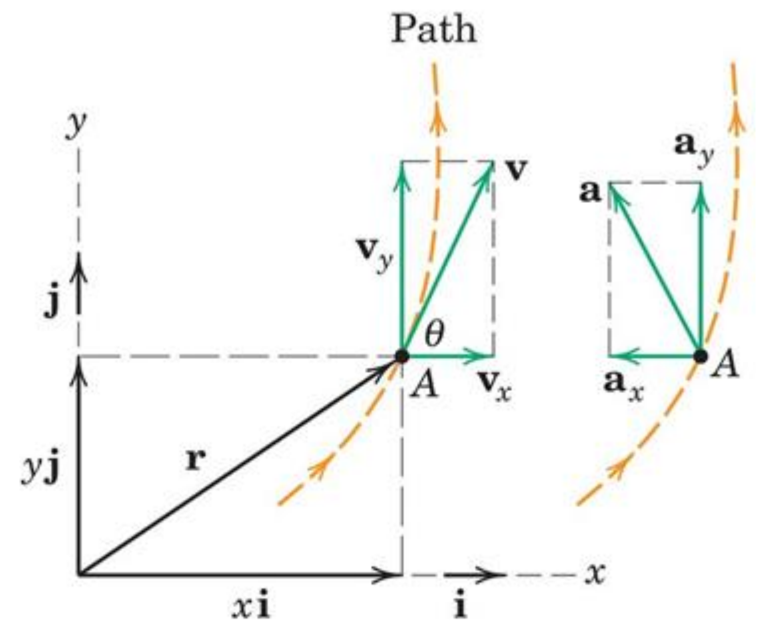
- Rectangular (Cartesian) coordinates
- Normal-tangential (path) coordinates
- Polar coordinates

2.4 Rectangular (Cartesian) Coordinates

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

$$\vec{v} = \dot{\vec{r}} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

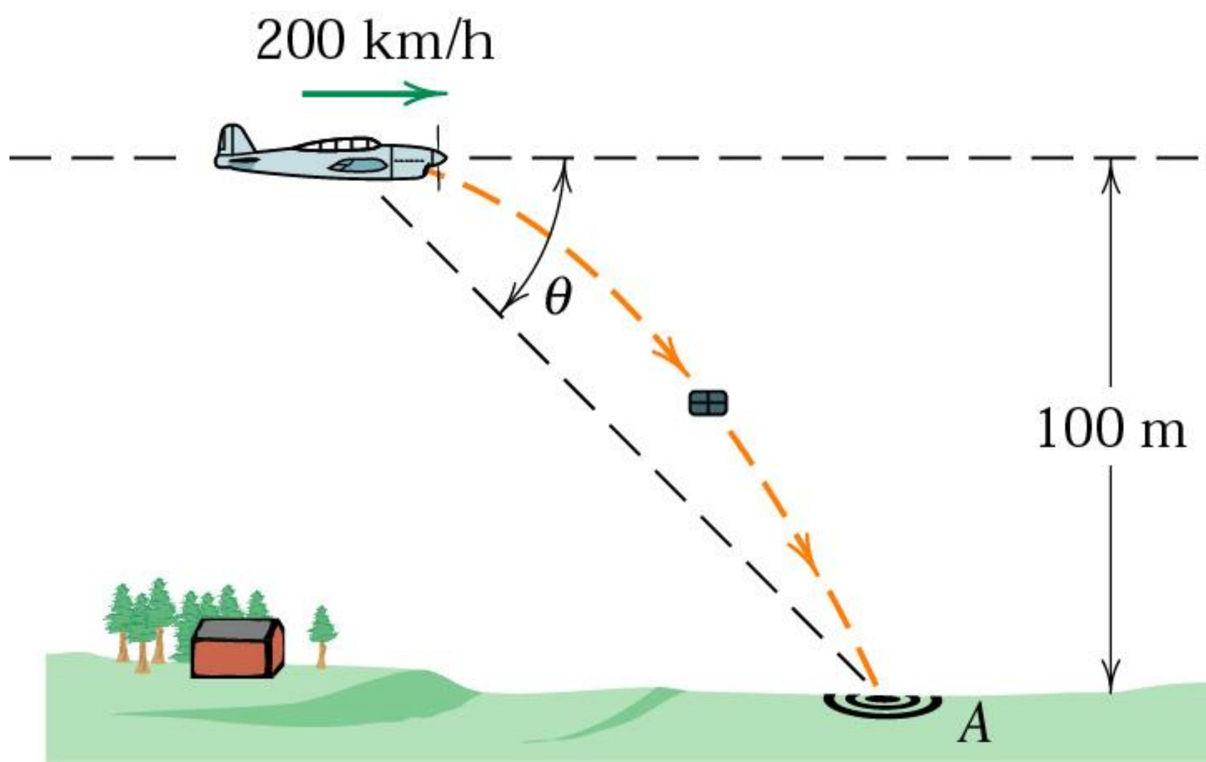


$$|\vec{r}|, \theta = Pol(x, y)$$
$$|\vec{v}|, \theta_v = Pol(v_x, v_y)$$
$$|\vec{a}|, \theta_a = Pol(a_x, a_y)$$

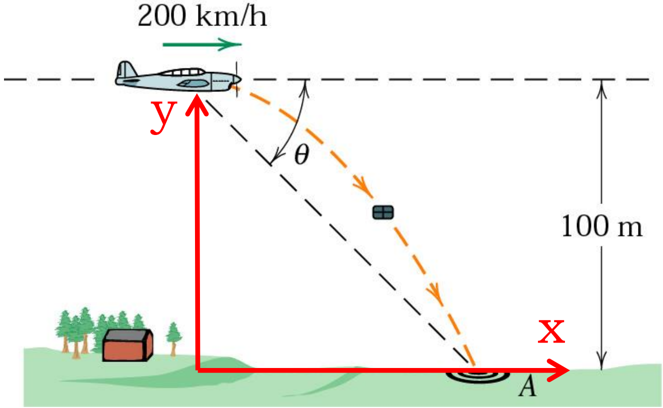
Rectangular coordinates is the superposition of two rectilinear motions in two mutually perpendicular directions, x and y!

2/75 (4th), 2/80 (5th), None (6th), 2/75 (7th), 2/76 (8th)

The pilot of an airplane carrying a package of mail to a remote outpost wishes to release the package at the right moment to hit the recovery location A. At what angle θ with the horizontal should the pilot's line of sight to the target make the instant of release? The airplane is flying horizontally at an altitude of 100 m with a velocity of 200 km/h.



Free-fall (constant acceleration which is g) in vertical (say *negative* y-direction), constant speed travel in horizontal (say x-direction) *when we neglect air friction on the package*.



$y(t) = h - \frac{1}{2}gt^2$

$x(t) = v_0 t$

$0 = 100 - \frac{1}{2}9.81t^2$

$t = \sqrt{\frac{200}{9.81}} = 4.51523641\ s \cong 4.52\ s$

$x = 200\ {}^{km}/_{h} \frac{1000\ {}^{m}/_{km}}{3600\ {}^{s}/_{h}} 4.52\ s = 251\ m$ $4.51523641\ s$

$r, \theta = Pol(251{,}100) = 270\ m, 21.7°$

Numerical accuracy versus round-off error accumulation.

Presenting Numerical Results:

Convention 1:

Always round numbers to three significant figures.

Examples:

$\sqrt{2} \cong 1.414213562 \ldots \to 1.41$

$3164\, m \to 3.16\, km = 3.16 * 10^6\, mm$

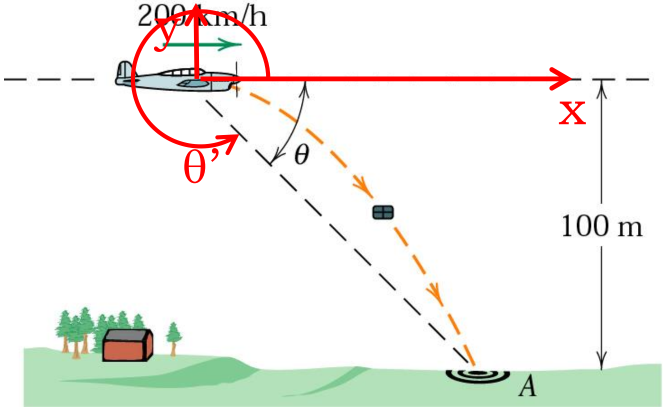
Convention 2:

Numbers starting with a 1, use four significant figures, for other numbers use three significant figures.

Examples:

$\sqrt{2} \cong 1.414213562 \ldots \to 1.412$

$3164\, m \to 3.16\, km = 3.16 * 10^6\, mm$



Free-fall (constant acceleration which is g) in vertical (say *negative* y-direction), constant speed travel in horizontal (say x-direction) *when we neglect air friction on the package*.

$$y(t) = -\frac{1}{2}gt^2$$

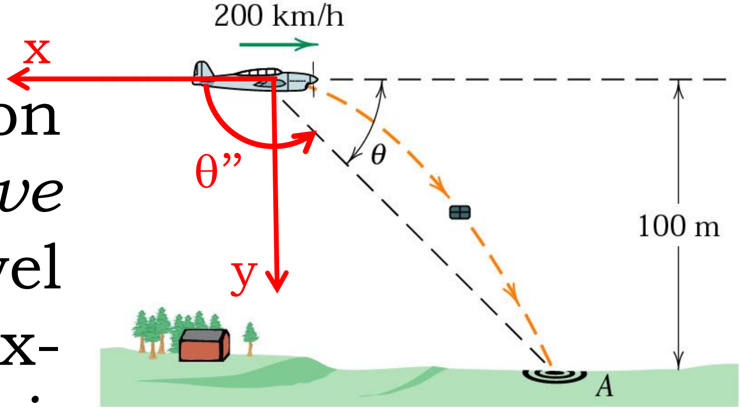
$$x(t) = v_0 t$$

$$-100 = -\frac{1}{2}9.81t^2$$

$$t = \sqrt{\frac{200}{9.81}} = 4.51523641\ s \cong 4.52\ s$$

$$x = 200\ {}^{km}/_{h} \frac{1000\ {}^{m}/_{km}}{3600\ {}^{s}/_{h}} 4.52\ s = 251\ m$$

$$r, \theta' = Pol(251, -100) = 270\ m, -21.7°$$



Free-fall (constant acceleration which is g) in vertical (say *positive* y-direction), constant speed travel in horizontal (say *negative* x-direction) *when we neglect air friction on the package.*

$y(t) = \frac{1}{2}gt^2$

$x(t) = -v_0 t$

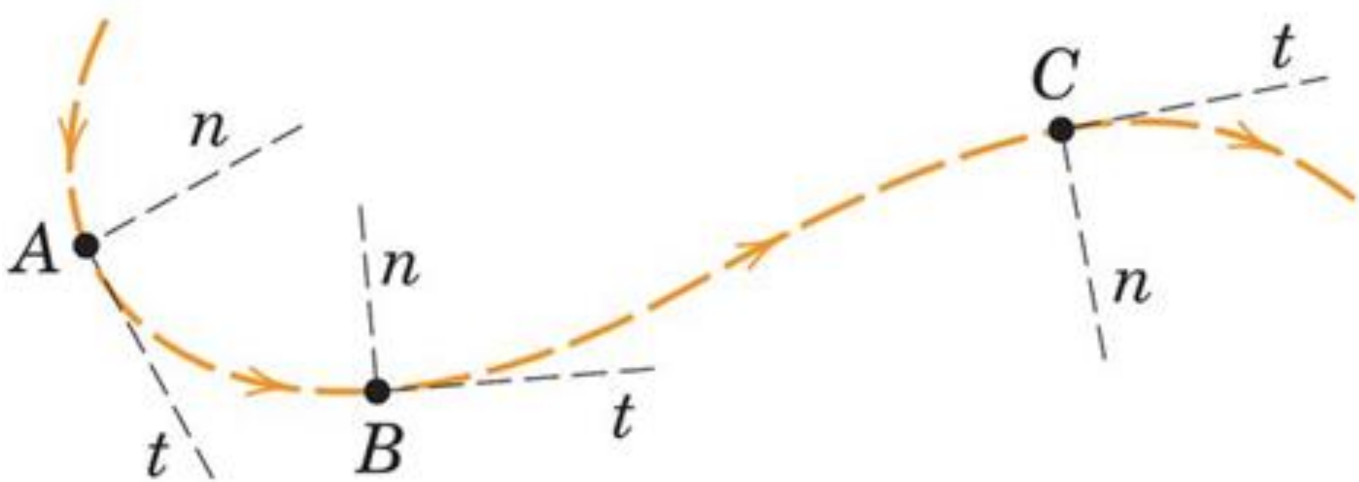
$100 = \frac{1}{2}9.81t^2$

$$t = \sqrt{\frac{200}{9.81}} = 4.51523641\ s \cong 4.52\ s$$

$$x = -200\ {}^{km}/_{h} \frac{1000\ {}^{m}/_{km}}{3600\ {}^{s}/_{h}}\ 4.52\ s = -251\ m$$

$r, \theta'' = Pol(-251{,}100) = 270\ m, 158{,}3°$

2.5 Normal & Tangential Coordinates (Path Coordinates)



The normal and tangential coordinates move with the path, t along the direction of motion, tangent to the path, n normal to t towards center of curvature of the path.

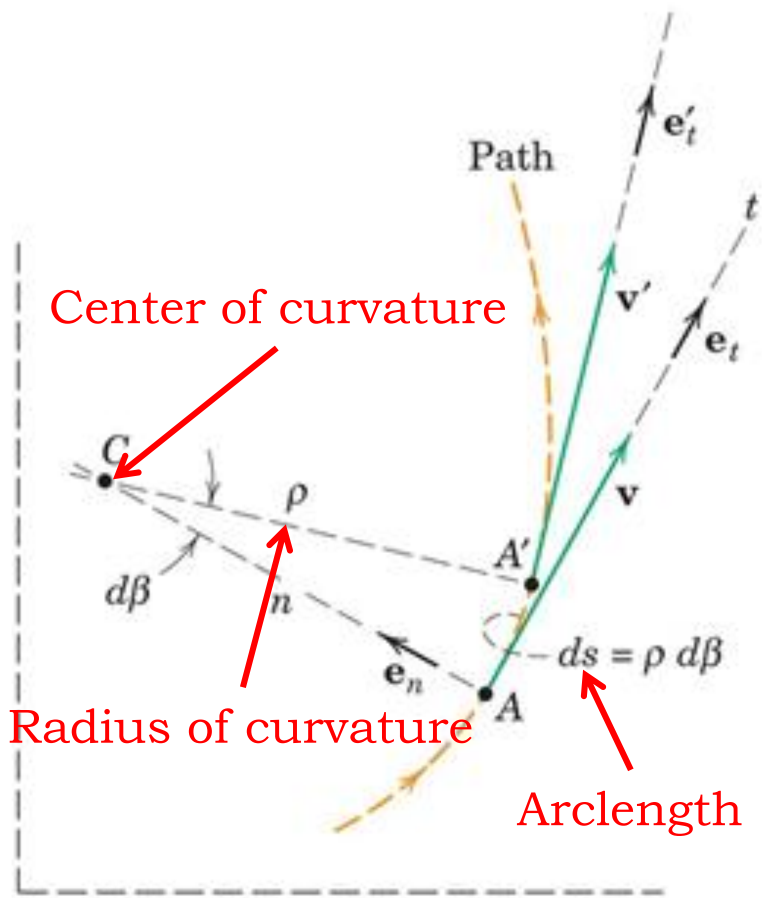
2.5 Normal & Tangential Coordinates (Path Coordinates)

$$ds = \rho d\beta$$

$$v = \frac{ds}{dt} = \rho\frac{d\beta}{dt} + \frac{d\rho}{dt}d\beta = \rho\frac{d\beta}{dt}$$

$$\vec{v} = \rho\dot{\beta}\hat{e}_t$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{e}_t) = \dot{v}\hat{e}_t + v\dot{\hat{e}}_t$$



2.5 Normal & Tangential Coordinates (Path Coordinates)

$$d\hat{e}_t = d\beta\,\hat{e}_n$$

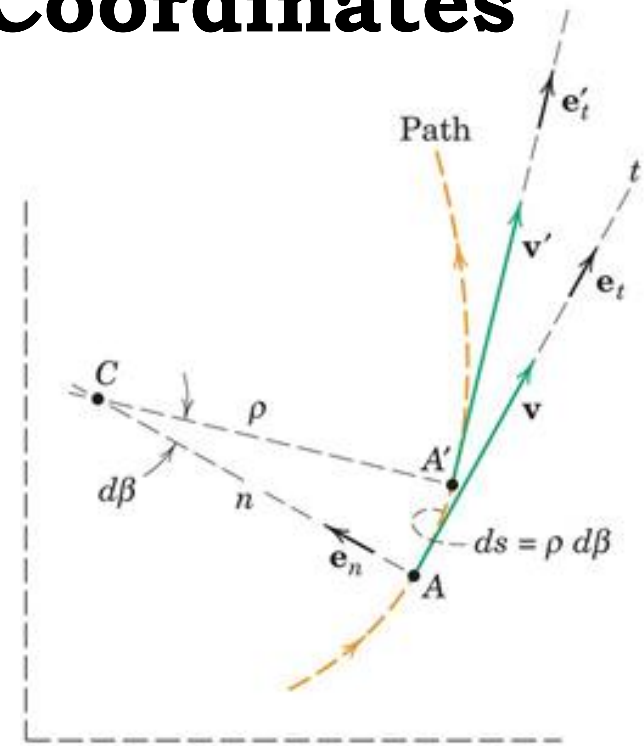
$$\dot{\hat{e}}_t = \frac{d\hat{e}_t}{dt} = \frac{d\beta}{dt}\hat{e}_n = \dot{\beta}\,\hat{e}_n$$

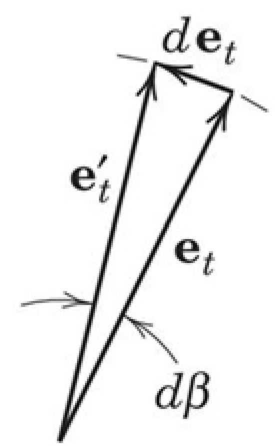
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{e}_t) = \dot{v}\hat{e}_t + v\dot{\hat{e}}_t$$

$$\vec{a} = \dot{v}\hat{e}_t + v\dot{\beta}\hat{e}_n$$

$$v = \rho\dot{\beta}, \dot{\beta} = \frac{v}{\rho}$$

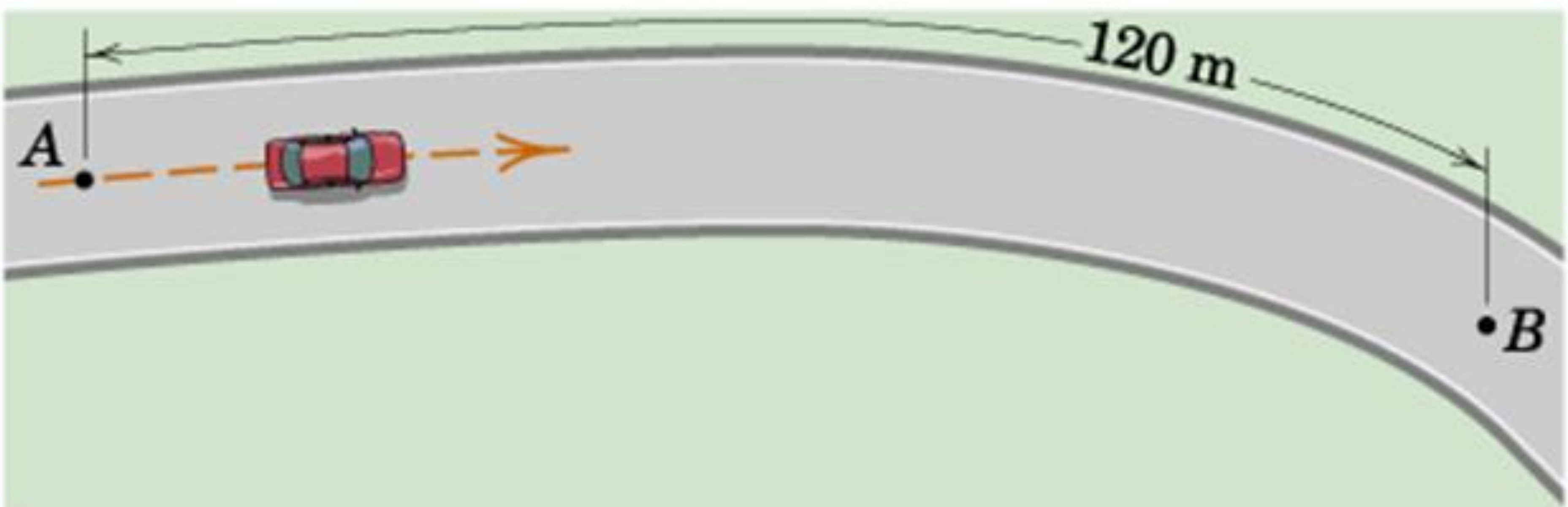
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n = a_t\hat{e}_t + a_n\hat{e}_n$$

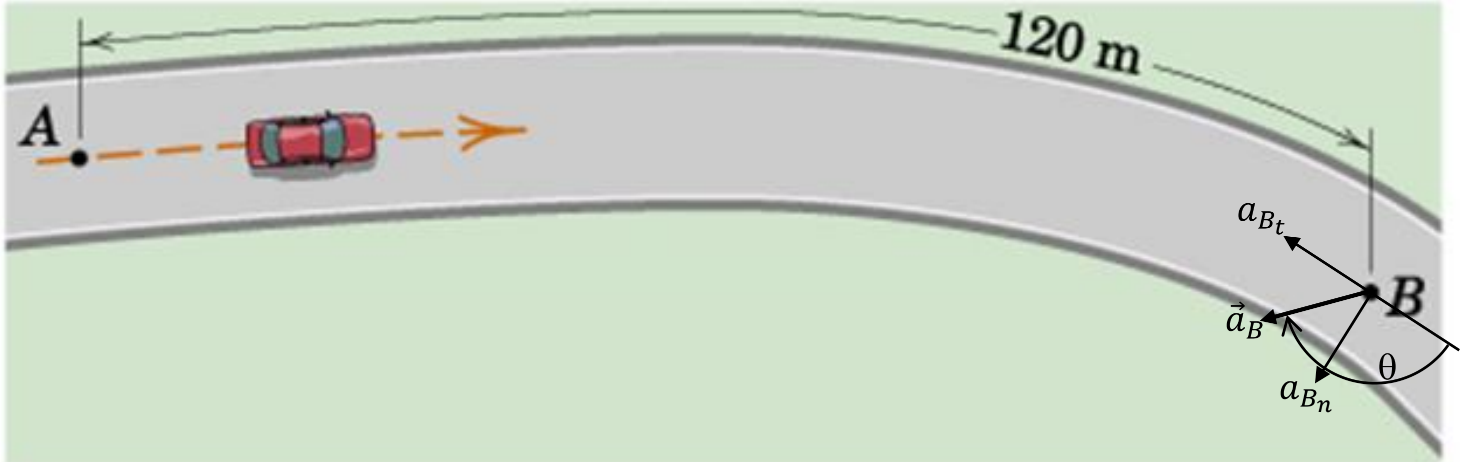




2/116 (4th), None (5th), 2/116 (6th), None (7th), None (8th)

A car travels along the level curved road with a speed which is decreasing at the constant rate of 0.6 m/s each second. The speed of the car as it passes point A is 16 m/s. Calculate the magnitude of the total acceleration of the car as it passes point B which is 120 m along the road from A. The radius of curvature of the road at B is 60 m.





$|\vec{a}_B| = ?$

$\dot{v} = a_t = -0.6\, m/s^2$

$$a_{B_n} = \frac{{v_B}^2}{\rho}$$

${v_B}^2 = {v_A}^2 + 2as = 16^2 + 2 * (-0.6)120 = 112$

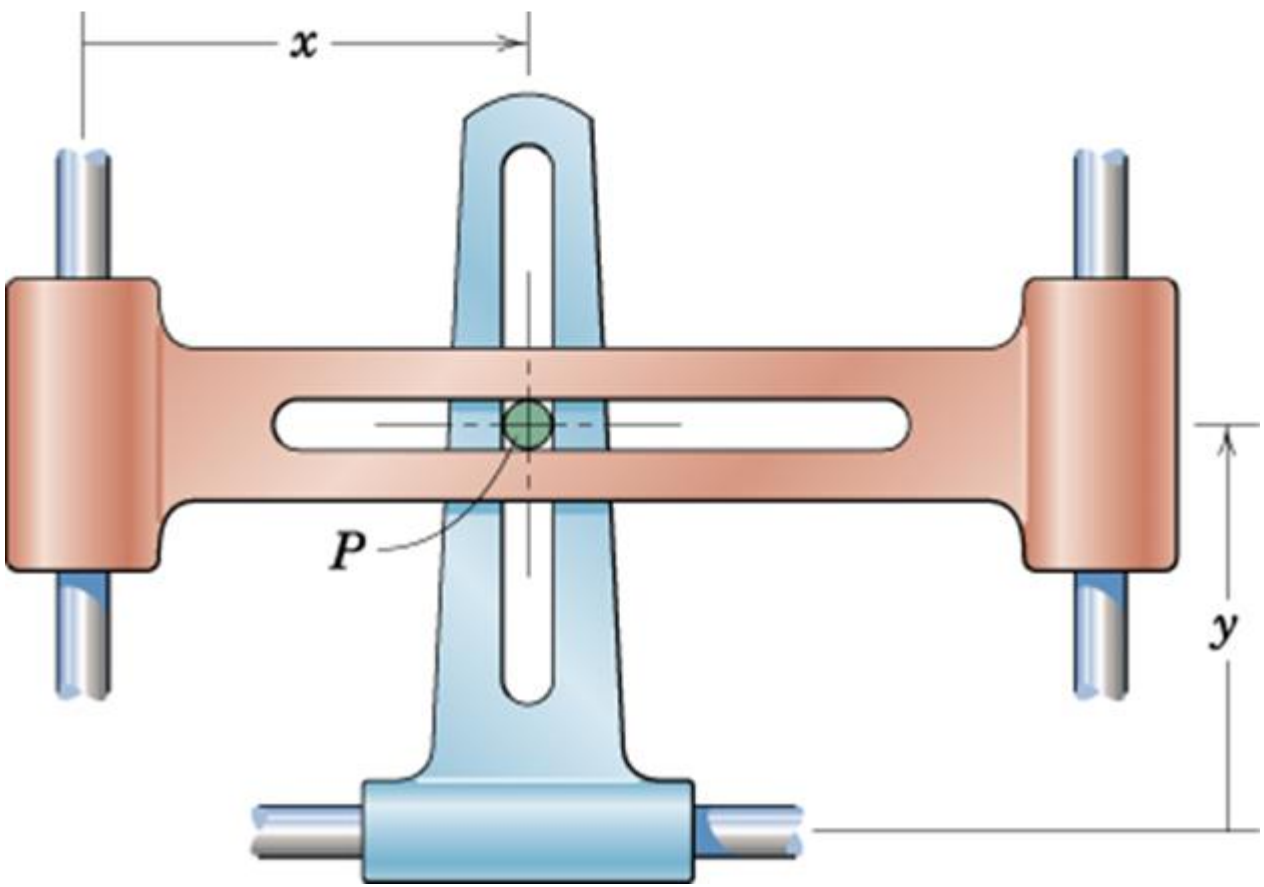
$v_B = 10.58\, m/s$

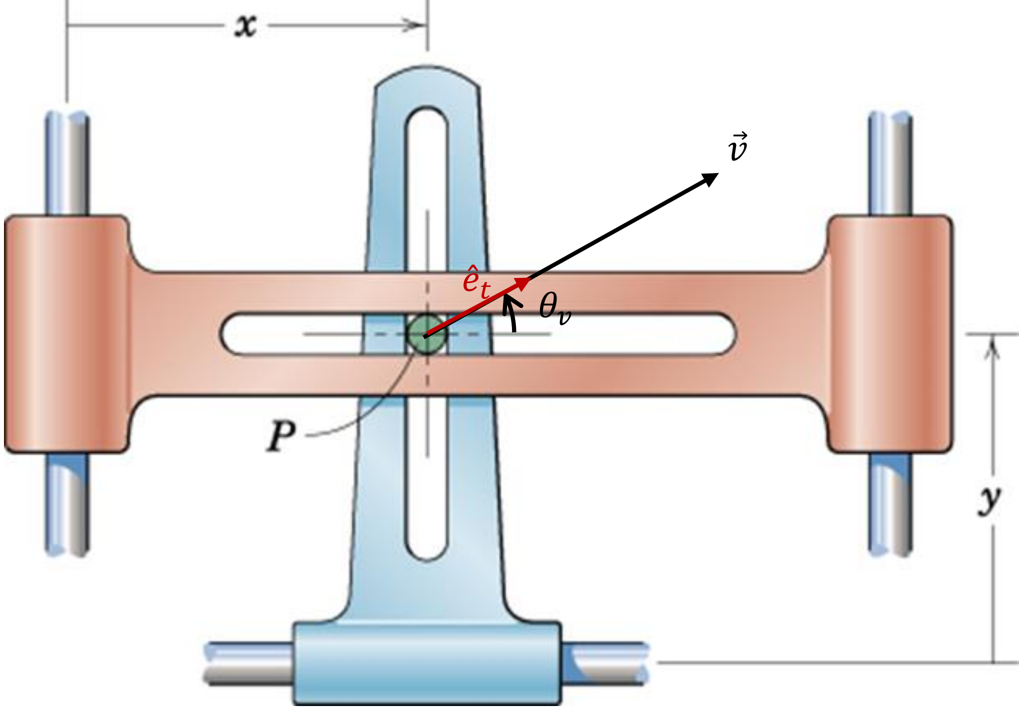
$$a_{B_n} = \frac{{v_B}^2}{\rho} = \frac{10.58^2}{60} = 1.867\, m/s^2$$

$|a_B|, \theta = Pol\left(a_t, a_{B_n}\right) = 1.960\, m/s^2, 162.2°$

2/128 (4th), None (5th), 2/132 (6th), None (7th), 2/123 (8th)

During a short interval the slotted guides are designed to move according to $x = 16 - 12t + 4t^2$ and $y = 2 + 15t - 3t^2$, where x and y are in millimeters and t in seconds. At the instant when t = 2 s, determine the radius of curvature, ρ, of the path of the constrained pin P.





$$x = 16 - 12t + 4t^2$$

$$\dot{x} = -12 + 8t$$

$$\ddot{x} = 8$$

$$y = 2 + 15t - 3t^2$$

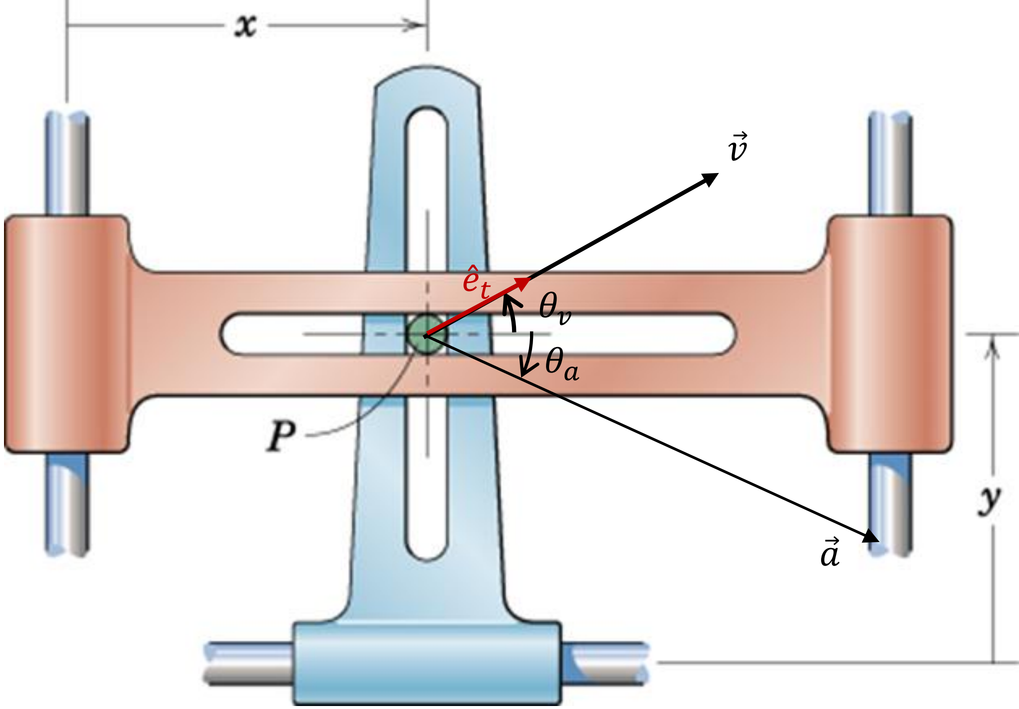
$$\dot{y} = 15 - 6t$$

$$\ddot{y} = -6$$

$$\vec{v}(t = 2s) = [(-12 + 8 * 2)\hat{\imath} + (15 - 6 * 2)\hat{\jmath}] = (4\hat{\imath} + 3\hat{\jmath})mm/s$$

$$\hat{e}_t = \frac{\vec{v}}{|\vec{v}|} = \frac{4\hat{\imath} + 3\hat{\jmath}}{5}$$

$$\theta_v = 36.87°$$



$$x = 16 - 12t + 4t^2$$

$$\dot{x} = -12 + 8t$$

$$\ddot{x} = 8$$

$$y = 2 + 15t - 3t^2$$

$$\dot{y} = 15 - 6t$$

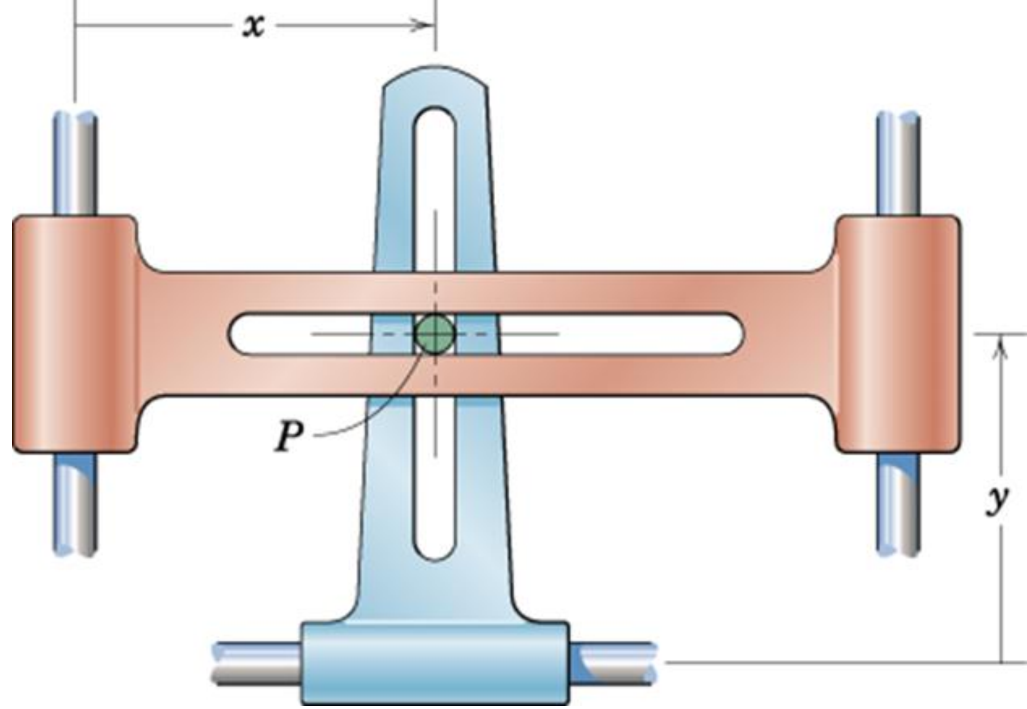
$$\ddot{y} = -6$$

$$\vec{a} = (8\hat{\imath} - 6\hat{\jmath})\ mm/s^2$$

$$\theta_a = -36.87°$$

$$a_n = \frac{v^2}{\rho} = \frac{5^2}{\rho} = 10sin(2 * 36.87°) = 9.60\ mm/s^2$$

$$\rho = 2.60\ mm$$



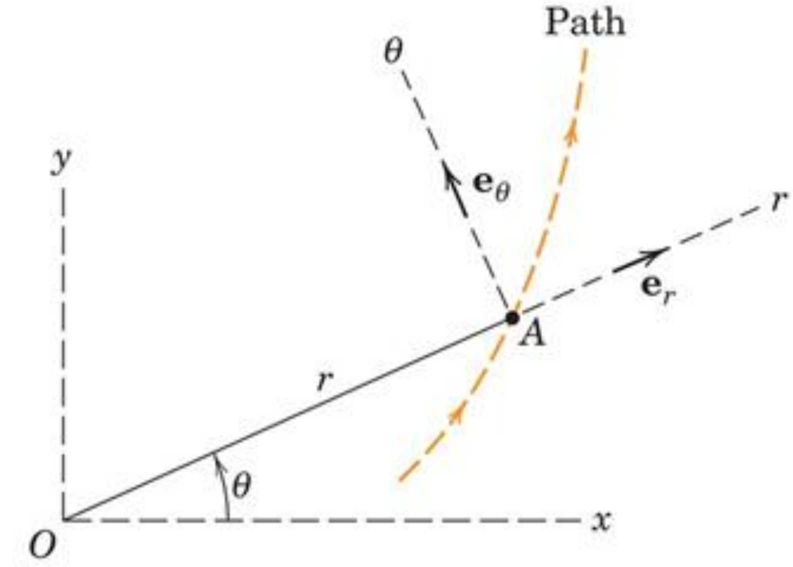
Alternative Method

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Eliminate parameter t from x and y and apply the formula to obtain the same radius of curvature or use:

$$\frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}}$$

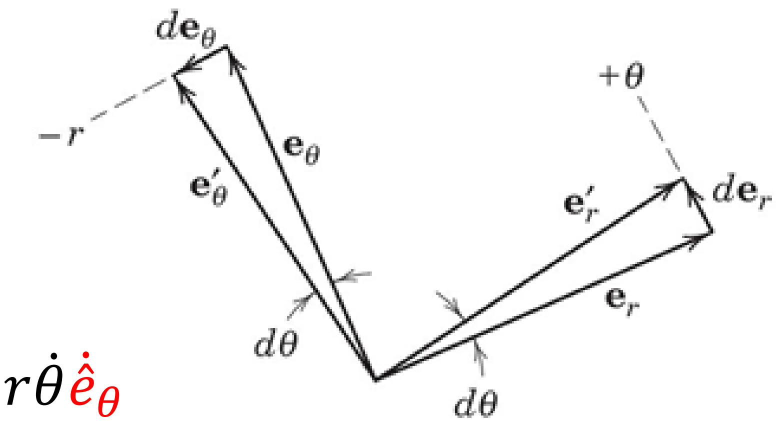
2.6 Polar Coordinates



$\vec{r} = r\hat{e}_r$

$\dot{\vec{r}} = \vec{v} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$

Always tangent to the path!



$d\hat{e}_r = d\theta\hat{e}_\theta$

$\dot{\hat{e}}_r = \frac{d\hat{e}_r}{dt} = \frac{d\theta}{dt}\hat{e}_\theta = \dot{\theta}\hat{e}_\theta$

$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta$

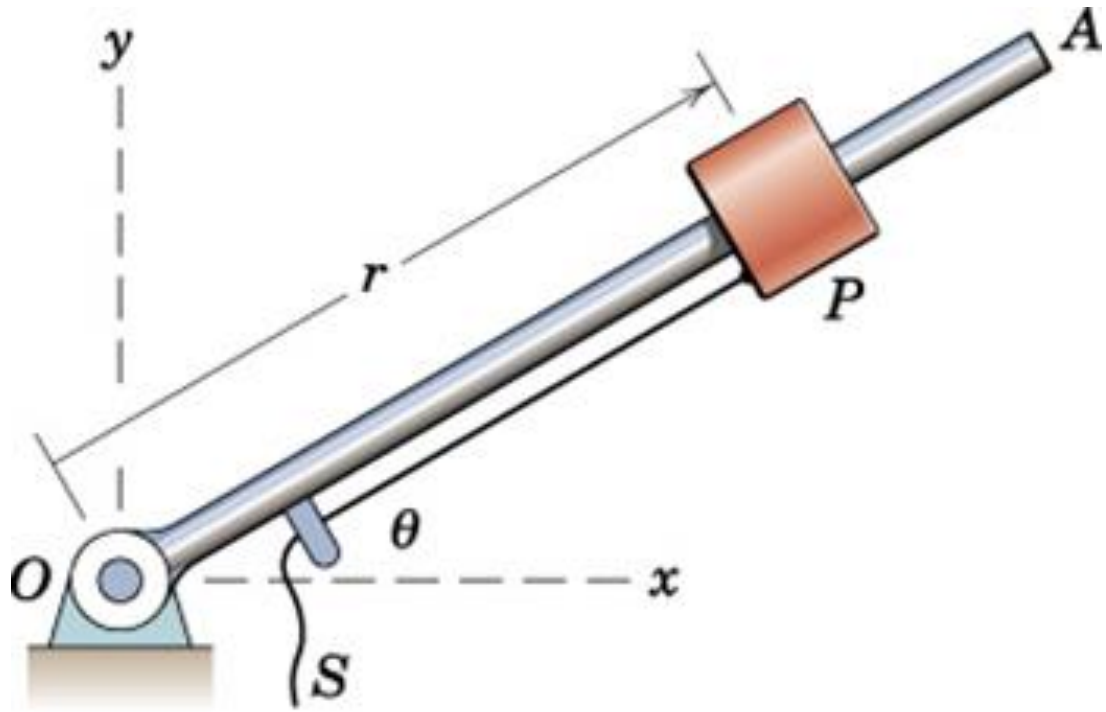
$d\hat{e}_\theta = -d\theta\hat{e}_r$

$\dot{\hat{e}}_\theta = \frac{d\hat{e}_\theta}{dt} = -\frac{d\theta}{dt}\hat{e}_r = \dot{\theta}\hat{e}_r$

$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{e}_\theta = a_r\hat{e}_r + a_\theta\hat{e}_\theta$

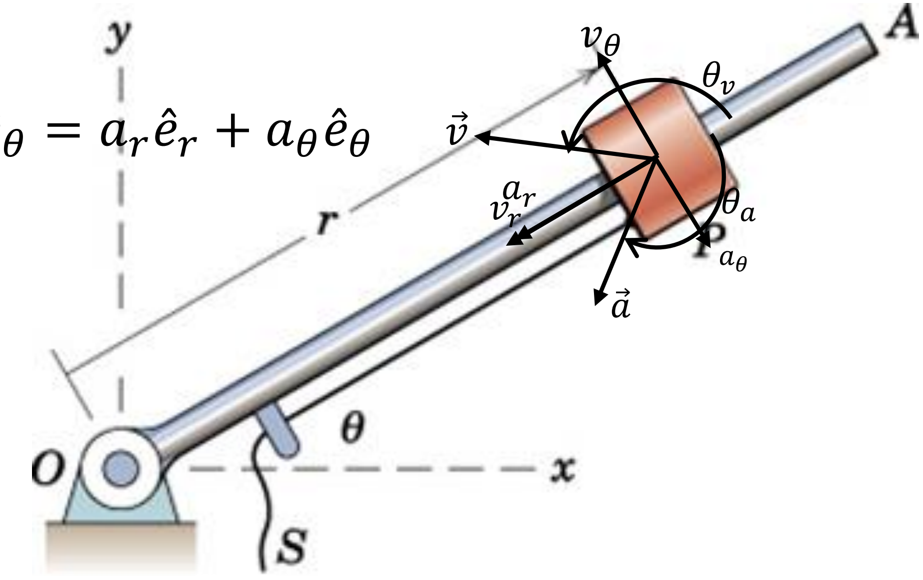
2/142 (4th), 2/144 (5th), 2/155 (6th), None (7th), None (8th)

The slider P can be moved inward by means of the string S as the bar OA rotates about pivot O. The angular position of the bar is given by $\theta = 0.4 - 0.12t + 0.06t^3$ where θ is in radians and t in seconds. The position of the slider is given by $r = 0.8 - 0.1t - 0.05t^2$, where r is in meters and t in seconds. Determine and sketch the velocity and acceleration of the slider at time t = 2 s. Find the angles α and β which **v** and **a** make with positive x-axis.



$$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{e}_\theta = a_r\hat{e}_r + a_\theta\hat{e}_\theta$$



$$\theta(t) = 0.4 - 0.12t + 0.06t^3$$

$$\dot{\theta}(t) = -0.12 + 0.18t^2$$

$$\ddot{\theta}(t) = 0.36t$$

$$\theta(t = 2\ s) = 0.640\ rad \ \equiv 36.7°$$

$$\dot{\theta}(t = 2\ s) = 0.60\ rad/s$$

$$\ddot{\theta}(t = 2\ s) = 0.72\ \ rad/s^2$$

$$r(t) = 0.8 - 0.1t - 0.05t^2$$

$$\dot{r}(t) = -0.1 - 0.1t$$

$$\ddot{r}(t) = -0.1$$

$$r(t = 2\ s) = 0.4\ m$$

$$\dot{r}(t = 2\ s) = -0.3\ m/s$$

$$\ddot{r}(t = 2\ s) = -0.1\ m/s^2$$

$$\vec{v}(t = 2\ s) = -0.3\hat{e}_r + 0.4 * 0.6\hat{e}_\theta = (-0.3\hat{e}_r + 0.24\hat{e}_\theta)\ m/s$$

$$v, \theta_v = Pol(-0.3{,}0.24) = 0.384\ m/s, 141.3°$$

$$\alpha = \theta + \theta_v = 178°$$

$$\vec{a} = (-0.1 - 0.4 * 0.6^2)\hat{e}_r + (2 * -0.3 * 0.6 + 0.4 * 0.72)\hat{e}_\theta$$

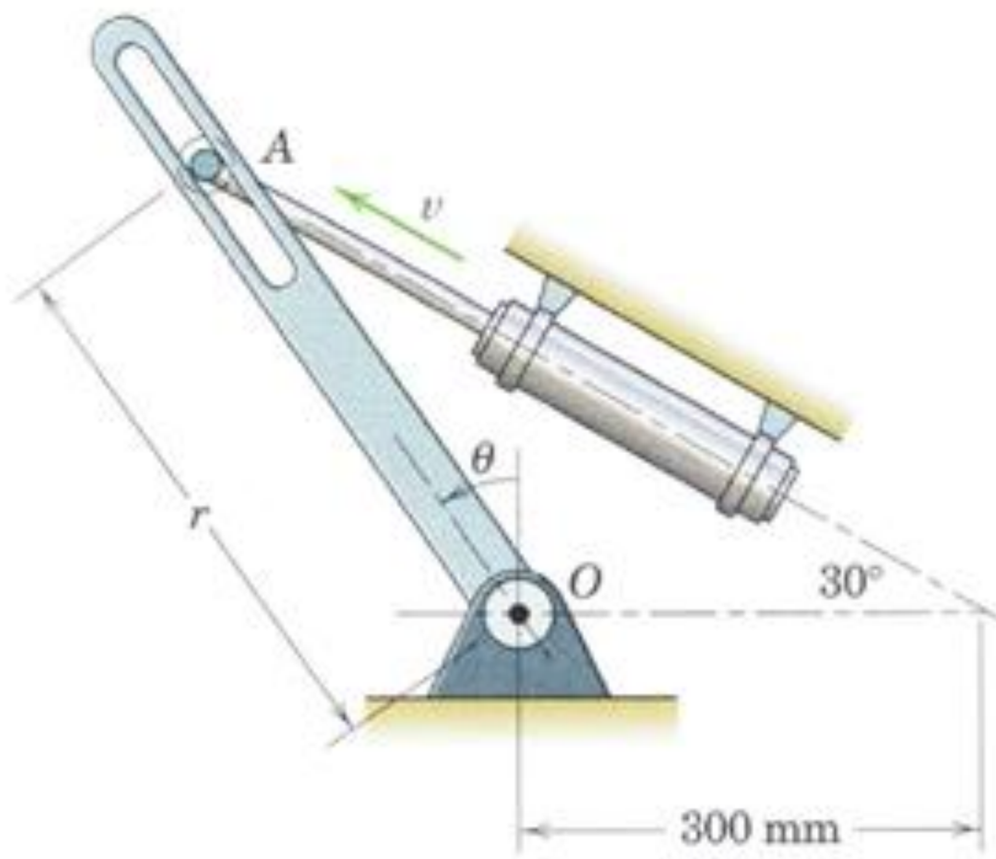
$$= (-0.224\hat{e}_r - 0.072\hat{e}_\theta)\ m/s^2$$

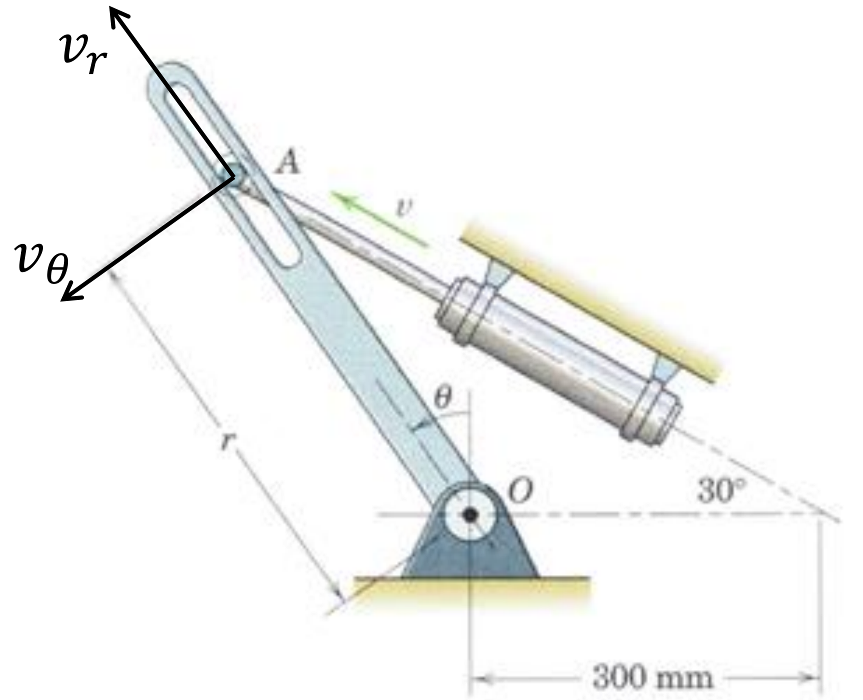
$$a, \theta_a = Pol(-0.224, -0.072) = 0.235\ m/s^2, -162.2°$$

$$\beta = \theta + \theta_a = -125.5°\ or\ 234{,}5°$$

2/142 (4^{th})

The hydraulic cylinder gives pin A a constant velocity v = 2 m/s along its axis for an interval of motion and, in turn, causes the slotted arm to rotate about O. Determine the values of $\dot{r}$, $\ddot{r}$ and $\ddot{\theta}$ for the instant when θ = 30°.





$$r = 0.3\ m$$

$$v_r = \dot{r} = v\cos(30°) = 1.732\ m/s$$

$$v_\theta = r\dot{\theta} = v\sin(30°) = 1\ m/s$$

$$\dot{\theta} = \frac{v_\theta}{r} = \frac{1}{0.3} = 3.33\ rad/s$$

$\vec{a} = \vec{0} \rightarrow a_r = 0$ **AND** $a_\theta = 0$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$\ddot{r} = r\dot{\theta}^2 = 0.3 * 3.33^2 = 3.33\ m/s^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{-2\dot{r}\dot{\theta}}{r} = \frac{-2 * 1.732 * 3.33}{0.3} = -38.5\ rad/s^2$$

2.7 Space Curvilinear (3-D) Motion

Rectangular (Cartesian) Coordinates (x-y-z)

$\vec{R} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\vec{v} = \dot{\vec{R}} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$

$\vec{a} = \ddot{\vec{R}} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

Cylindrical Coordinates (r-θ-z)

$\vec{R} = r\hat{e}_r + z\hat{k}$

$\vec{v} = \dot{\vec{R}} = \dot{r}\hat{e}_r + \dot{r}\theta\hat{e}_\theta + \dot{z}\hat{k}$

$\vec{a} = \ddot{\vec{R}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta + \ddot{z}\hat{k}$

Spherical Coordinates (R-θ-ϕ)

$\vec{R} = R\hat{e}_R$

Normal-Tangential (Path) Coordinates (n-t-b)

$\hat{e}_b = \hat{e}_t \times \hat{e}_n$