ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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2.2 Rectilinear Motion

2.3 Plane Curvilinear Motion

2.4 Rectangular (Cartesian) Coordinates

2.5 Normal & Tangential Coordinates

2.6 Polar Coordinates

2.7 Space Curvilinear (3-D) Motion

All were with respect to a "fixed" coordinate system (i.e. absolute motion)

2/8 Relative Motion (Translating Axes)

So far every motion was analyzed with respect to a "*fixed*(!)" coordinate system. Therefore velocities and accelerations were absolute. However sometimes analyzing the motion using a moving (*but not rotating*) coordinate system may bring certain advantages.

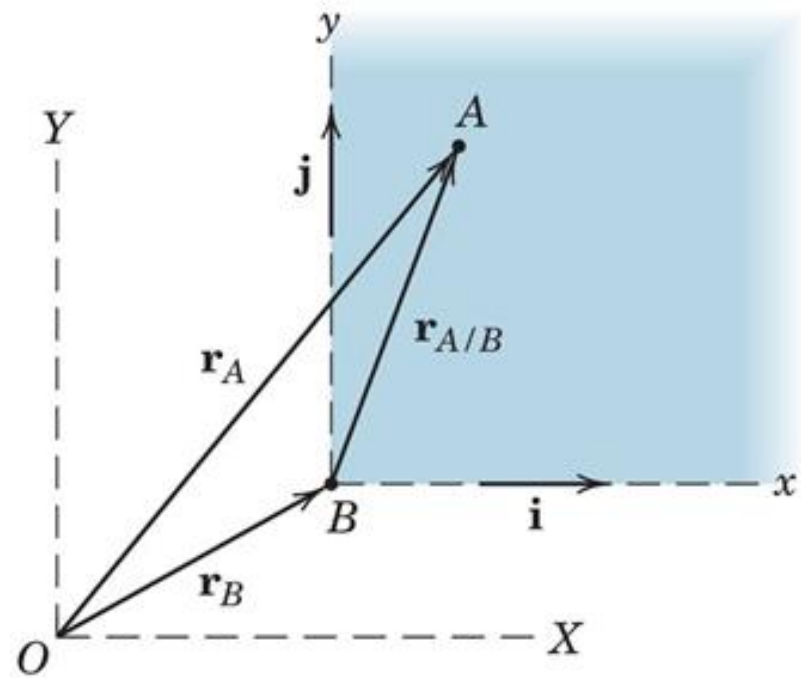
$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \dot{\vec{r}}_{A/B}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$\ddot{\vec{r}}_A = \ddot{\vec{r}}_B + \ddot{\vec{r}}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$



2/8 Relative Motion (Translating Axes)

If one replaces A with B

$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

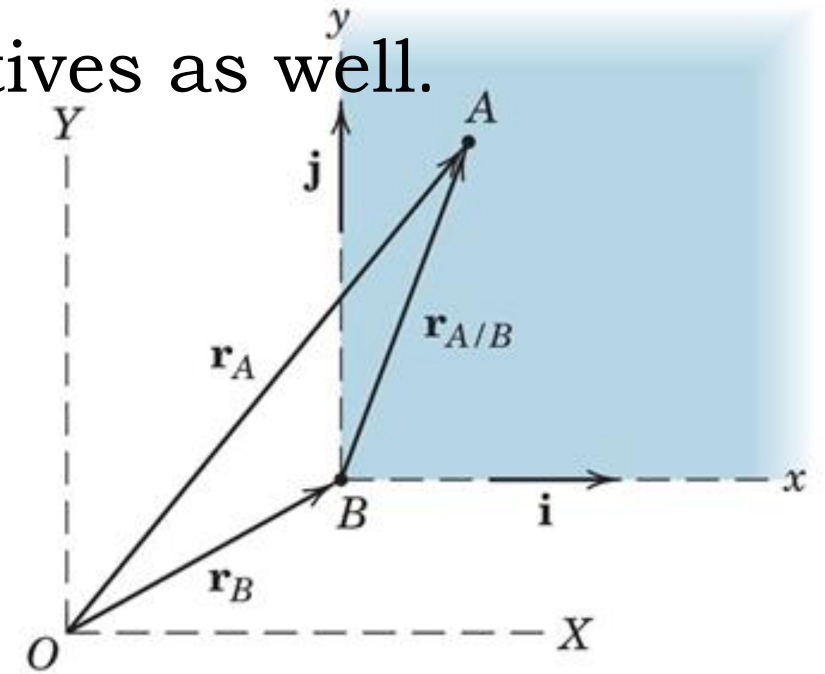
therefore

$\vec{r}_{B/A} = -\vec{r}_{A/B}$

which is true for time derivatives as well.

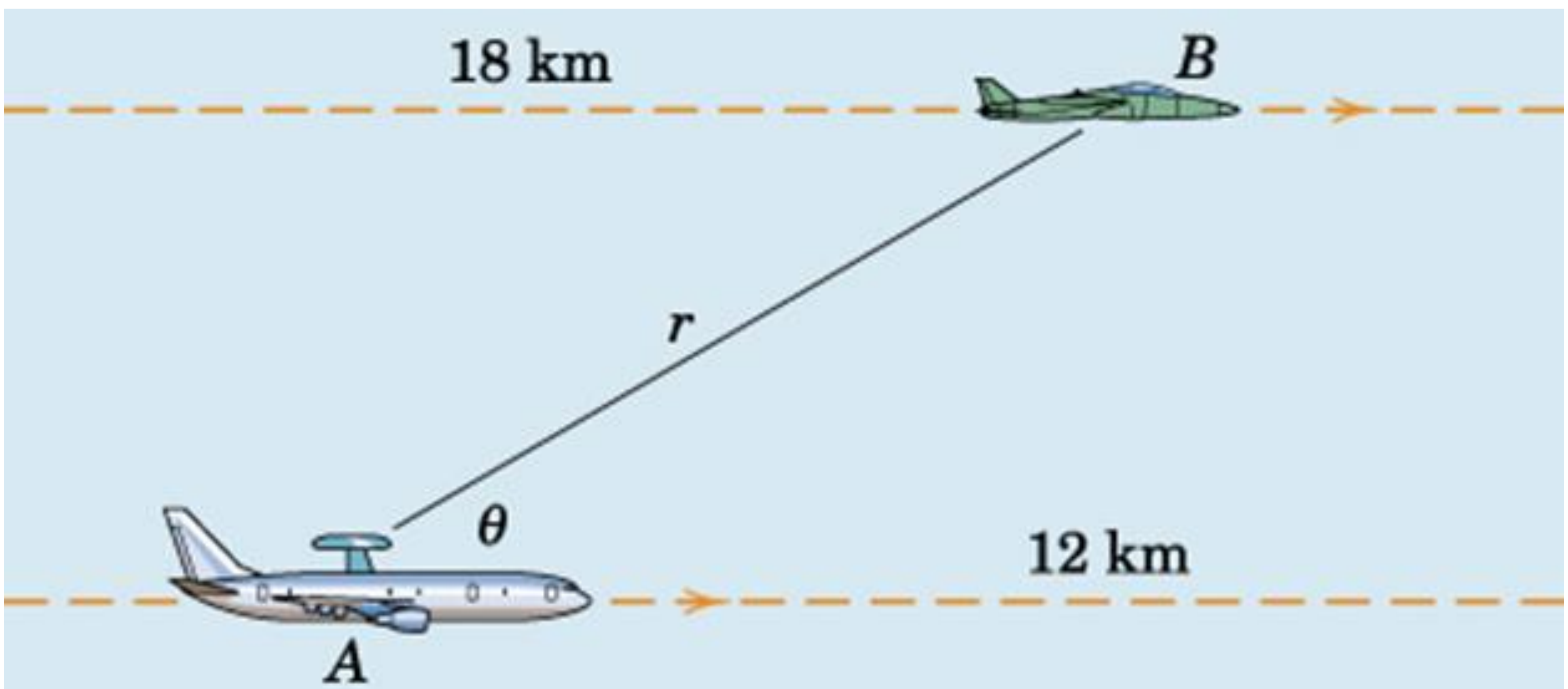
$\vec{v}_{B/A} = -\vec{v}_{A/B}$

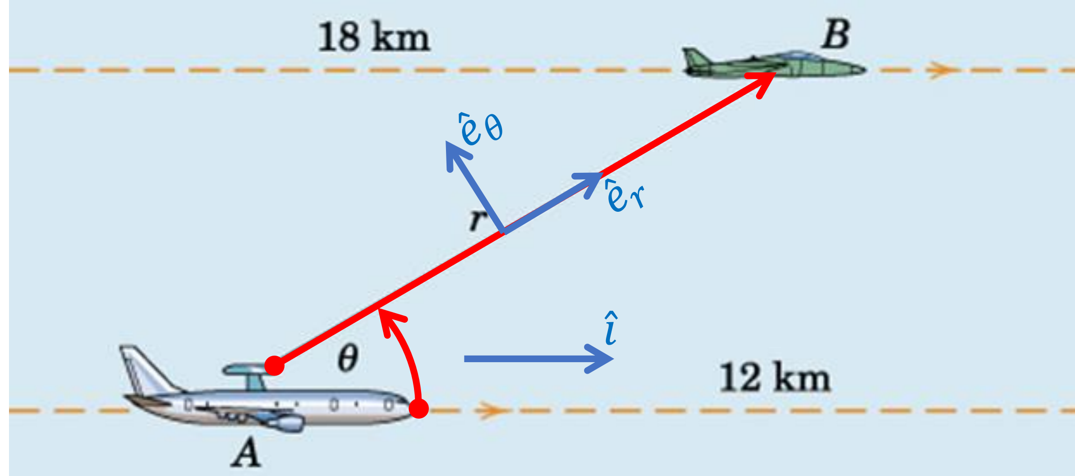
$\vec{a}_{B/A} = -\vec{a}_{A/B}$



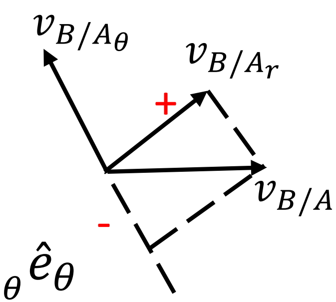
2/205 (4th), 2/204 (5th), 2/210 (6th), 2/204 (7th), 2/205 (8th)

The aircraft A with a radar detection equipment is flying horizontally at an altitude of 12 km and is increasing its speed at a rate of 1.2 m/s each second. Its radar locks onto an aircraft B flying in the same direction and in the same vertical plane at an altitude of 18 km. If A has a speed of 1000 km/h at the instant when θ = 30°, determine the values of $\ddot{r}$ and $\ddot{\theta}$ at the same instant when B has a constant speed of 1500 km/h.





$\ddot{r} = ?\quad \ddot{\theta} = ?$



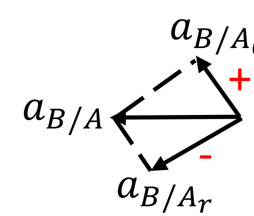
$\vec{v}_A = 1000\hat{\imath}\ km/h \equiv 278\hat{\imath}\ m/s$

$\vec{v}_B = 1500\hat{\imath}\ km/h \equiv 417\hat{\imath}\ m/s$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}, \vec{v}_{B/A} = 139.0\hat{\imath}\ m/s = v_{B/A_r}\hat{e}_r + v_{B/A_\theta}\hat{e}_\theta$

$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$\dot{r} = 139cos30° = 120.3\ m/s$



$\dot{\theta} = \frac{-139sin30°}{6000/sin30°} = -0.00579\ rad/s$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}, \vec{a}_{B/A} = -1.2\hat{\imath}\ m/s^2 = a_{B/A_r}\hat{e}_r + a_{B/A_\theta}\hat{e}_\theta$

$= -1.2cos30°\hat{e}_r + 1.2sin30°\hat{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$\ddot{r} = -0.637\ m/s^2$

$\ddot{\theta} = 0.0001667\ rad/s^2$

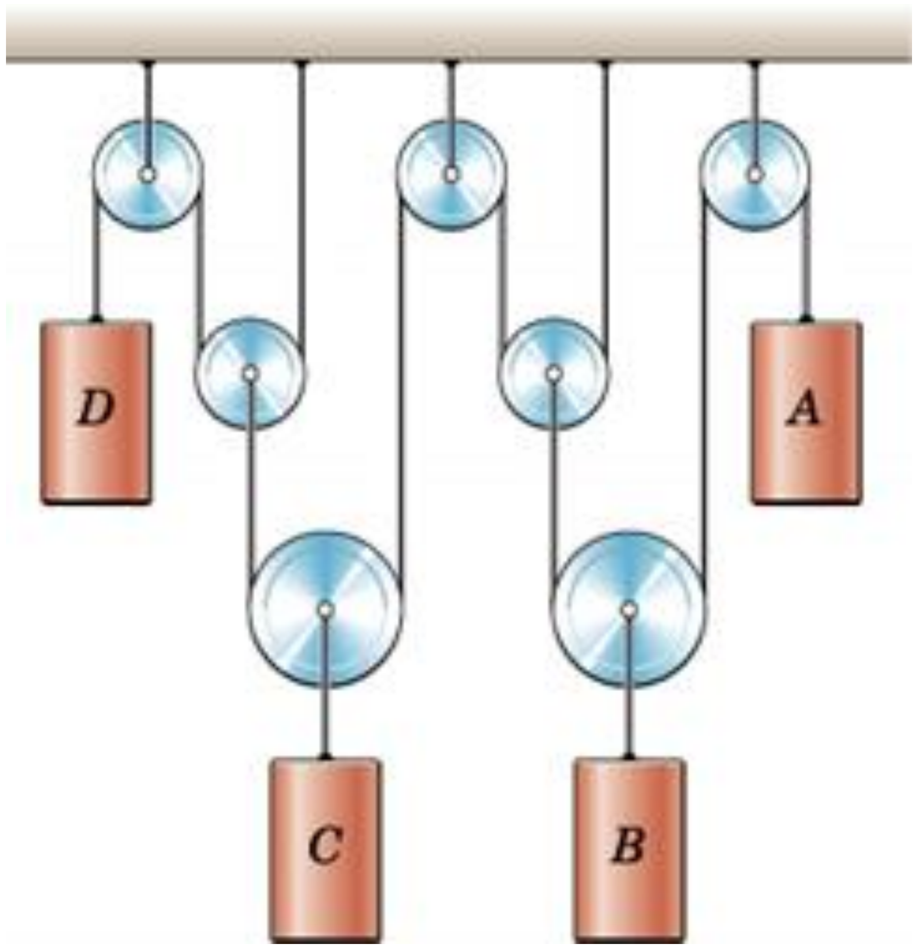
2/9 Constrained Motion of Connected Particles

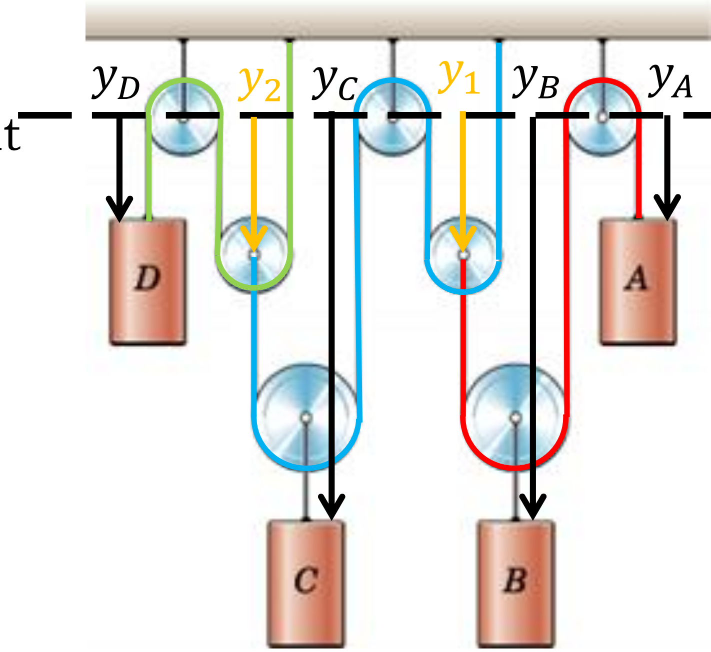
For some systems motions of particles may be interrelated by a geometric constraint. Mathematical expression for this geometric constraint which is the constraint equation relates the positions of particles. First time derivative of constrain equation relates velocities and second time derivative relates accelerations.

Degree of Freedom (DOF, DoF or F) is the **total number of <u>independent</u> parameters** to determine the position (also velocity and acceleration) of the system.

None (4^{th}), *2/213* (5^{th}), *2/219* (6^{th}), *None* (7^{th}), *2/216* (8^{th})

Determine the relationship which governs the velocities of the four cylinders. Express all velocities as positive down. How many degrees of freedom are there?





Please observe that there are three different ropes of constant lengths, L_1, L_2 and L_3.

$L_1 = y_A + 2y_B - y_1 + C_1$

$L_2 = 2y_1 + 2y_C - y_2 + C_2$

$L_3 = 2y_2 + y_D + C_3$

Parameters: y_A, y_B, y_C, y_D, y_1 and y_2: 6

Equations: 3

DOF = P. – Eq. = # Indep. Param. = 6 – 3 = **3**

Taking time derivatives of constraint equations:

$$0 = \dot{y}_A + \dot{y}_B - \dot{y}_1 + 0 \;\rightarrow \dot{y}_1 = \dot{y}_A + 2\dot{y}_B$$

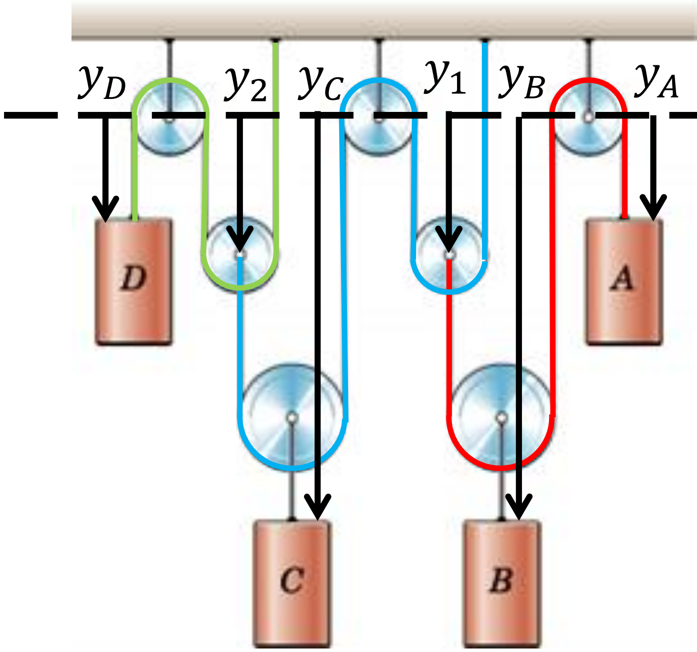
$$0 = 2\dot{y}_1 + 2\dot{y}_C - \dot{y}_2 + 0$$

$$0 = 2\dot{y}_2 + \dot{y}_D + 0, \rightarrow \dot{y}_2 = \frac{-\dot{y}_D}{2}$$

$$\dot{y}_1 = \dot{y}_A + 2\dot{y}_B$$

$$0 = 2\dot{y}_1 + 2\dot{y}_C - \dot{y}_2 + 0$$

$$\dot{y}_2 = \frac{-\dot{y}_D}{2}$$

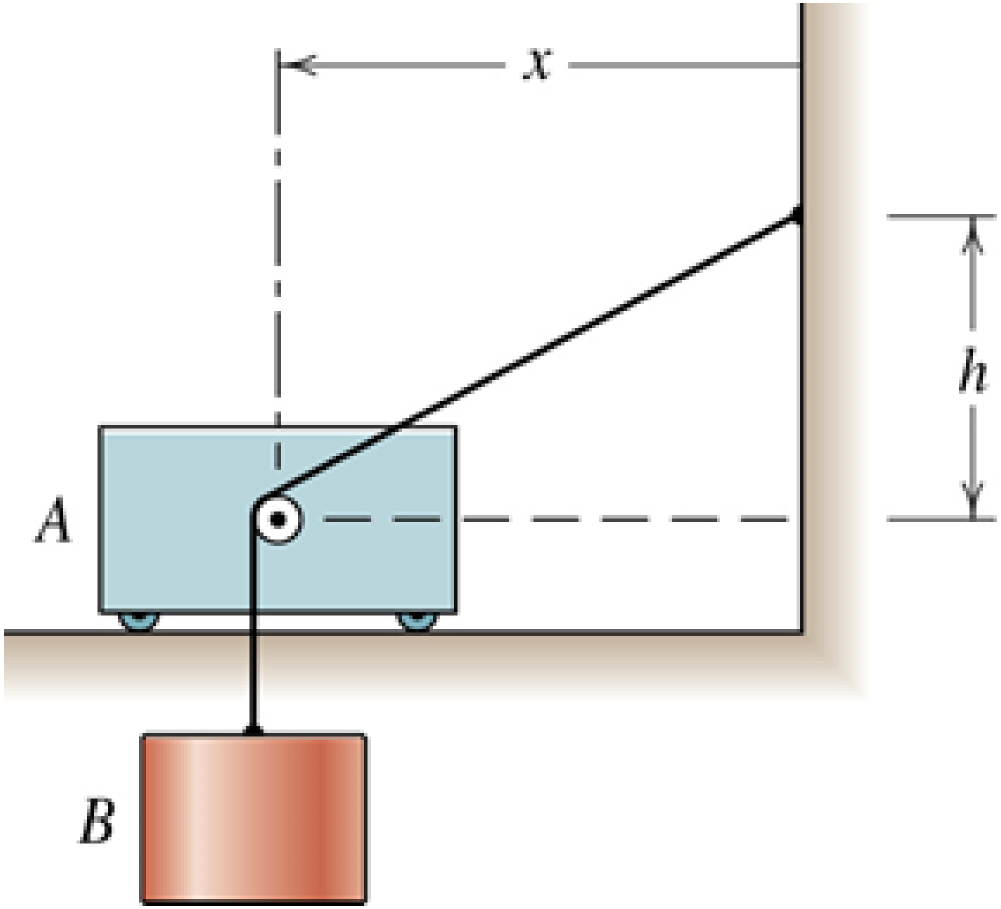


Substituting $\dot{y}_1$ and $\dot{y}_2$ into second equation yields:

$$4\dot{y}_A + 8\dot{y}_B + 4\dot{y}_C + \dot{y}_D = 0$$

Four parameters ($\dot{y}_A$, $\dot{y}_B$, $\dot{y}_C$ and $\dot{y}_D$) and one equation.
Still three of these parameters can be selected freely and remaining one depends on the other three so DOF = **3**
Please note that displacements are increasing down so downward velocities are positive!
Can you relate accelerations?

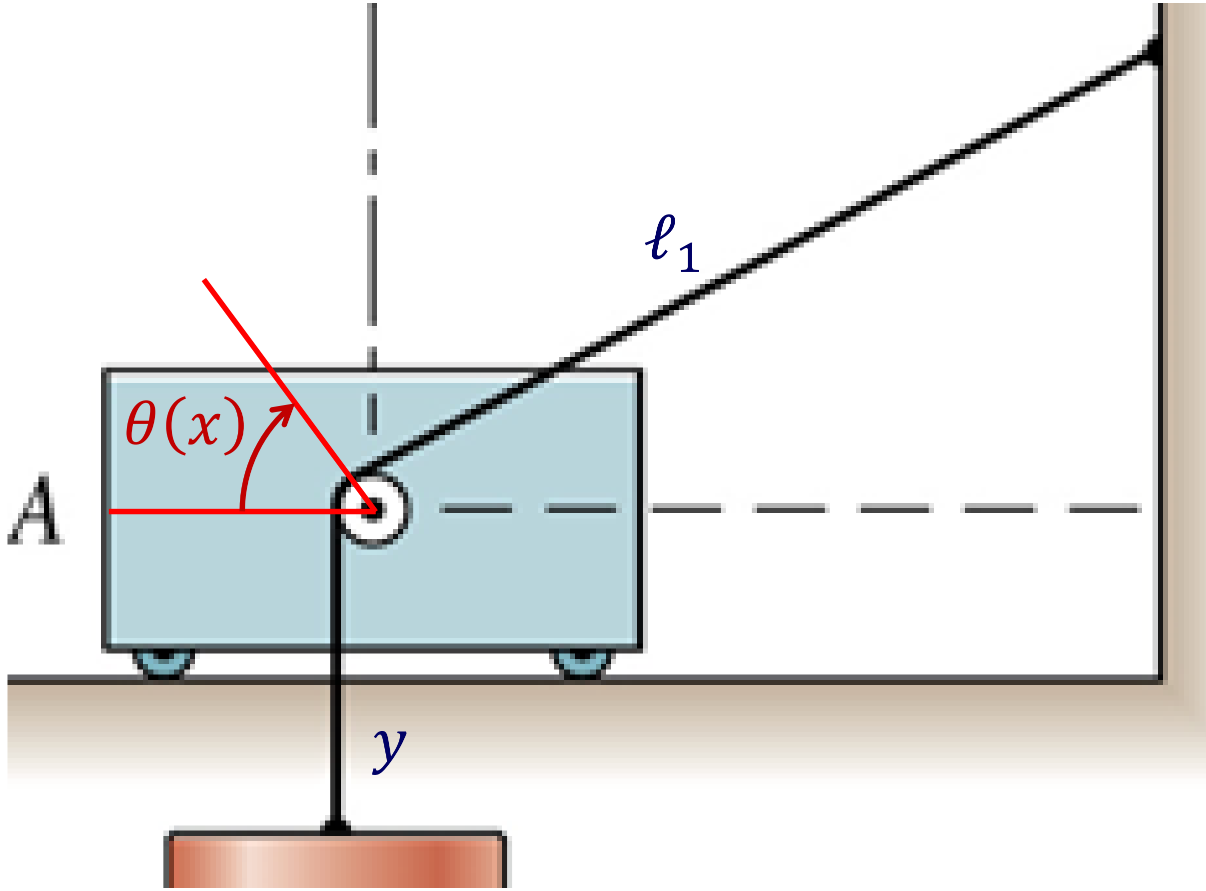
Example 2/222 (4th), 2/221 (5th), None (6th), 226 (7th), None (8th)
Neglect the diameter of the small pulley attached to body A and determine the magnitude of the total velocity of B in terms of the velocity v_A which body A has to the right. Assume that the cable between B and the pulley remains vertical and solve for a given value of x.

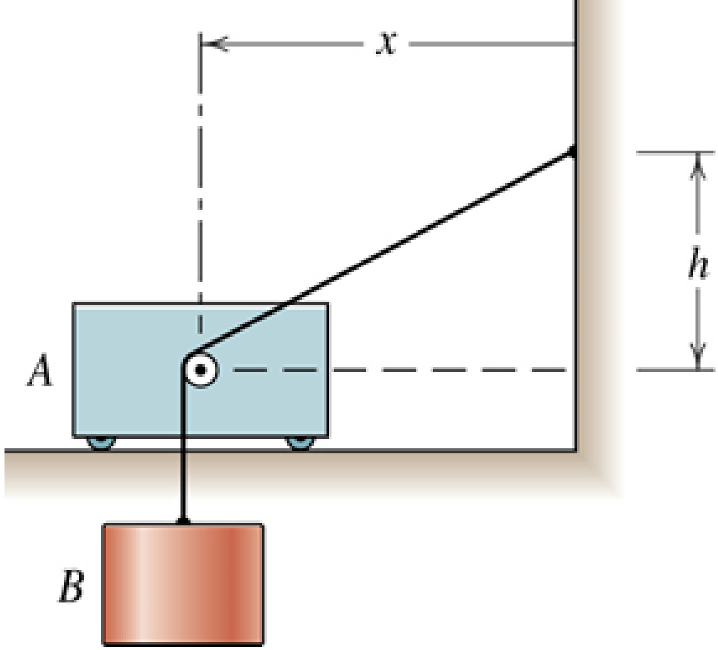


Example 2/222 (4th), 2/221 (5th), None (6th), 226 (7th), None (8th)

Neglect the diameter of the small pulley attached to body A…

$$L = \ell_1 + r\theta + y$$





$$L = \ell_1 + r\theta + y$$

$$L = y + \sqrt{x^2 + h^2} = y + (x^2 + h^2)^{1/2}$$

$$0 = \dot{y} + \frac{1}{2}(x^2 + h^2)^{-1/2} 2x\dot{x}$$

$$\dot{y} = \frac{-x}{\sqrt{x^2 + h^2}}\dot{x} = v_{B_y}$$

$$v_{B_x} = -\dot{x}$$

$$v_B = \sqrt{{v_{B_x}}^2 + {v_{B_y}}^2} = \sqrt{\dot{x}^2 + \frac{x^2}{x^2 + h^2}\dot{x}^2} = \sqrt{\frac{2x^2 + h^2}{x^2 + h^2}}v_A$$

PART I: PARTICLES

Chapter 2: Kinematics of Particles

Chapter 3: Kinetics of Particles

Newton's second law states that any particle, under the action of unbalanced forces will accelerate in the direction of the resultant force.

Kinetics relates the net force to change of motion.

We will use the methods we have developed in Statics to determine the net force and methods we have developed in Chapter 2 Kinematics of Particles to determine the motion.

PART I: PARTICLES

Chapter 3: Kinetics of Particles

There are three different approaches to the kinetics problems:

A. Direct application of Newton's Second Law/Force-Mass Acceleration Method

B. Work – Energy Principles (integration of second law with respect to displacement)

C. Impulse and Momentum Methods (integration of second law with respect to time)

PART I: PARTICLES

Chapter 3: Kinetics of Particles

A. Direct Application of Newton's Second Law – Force Mass Acceleration Method for Constant Mass Particles

3.2 Newton's Second Law

If an ideal experiment is done on a particle in an **inertial** reference frame:

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \frac{F_n}{a_n} = m$$

where m, mass, is a resistance to rate of change of velocity and is an *invariable* property of the particle.

3.3 Equation of Motion and Solution of Problems

Another conclusion of this ideal experiment is that acceleration is always in the direction of the applied force therefore a vector relation can be written as:

$$\vec{F} = m\vec{a}$$

However an ideal experiment cannot be performed because there is no inertial reference frame (that has no acceleration relative to the primary inertial reference frame that is fixed in the universe).

This deficiency in classical mechanics (which was based on the idea that first earth then sun was at the center of the "universe" and is fixed) lead to the idea of relativity.

3.3 Equation of Motion and Solution of Problems

Two typical problems of kinetics are:

1. For known forces, determination of resulting motion (solution may not be trivial especially for problems where force is not constant, the unknown is in differential form, displacement, velocity and acceleration), also known as forward problem.

2. For known motion (i.e. acceleration can be determined from the motion) determination of necessary forces to produce the motion (solution is simpler since unknown forces are in algebraic form) also known as inverse problem.

3.3 Equation of Motion and Solution of Problems

Constrained and Unconstrained Motion

- In unconstrained motion the particle is free of any mechanical guides and follows the path determined by its initial motion and external forces.
- In constrained motion the path is partially or fully determined by restraining guides. There may be reaction forces to keep the particle in the pre-determined path.

In both problems whether there is reaction forces from the existing constraints or not, the particle should obey Newton's Second Law.

Photo by Instructor 15.03.2012
"*Ankara Garı Doğu Makasları*"

3.3 Equation of Motion and Solution of Problems

- Selecting a *convenient* coordinate system is important in solving problems.
- Isolate the particle from its surroundings, replacing the mechanical interaction with the surrounding by appropriate forces (i.e. *draw free body diagram*).
- Write *equation of motion* based on free body diagram.