ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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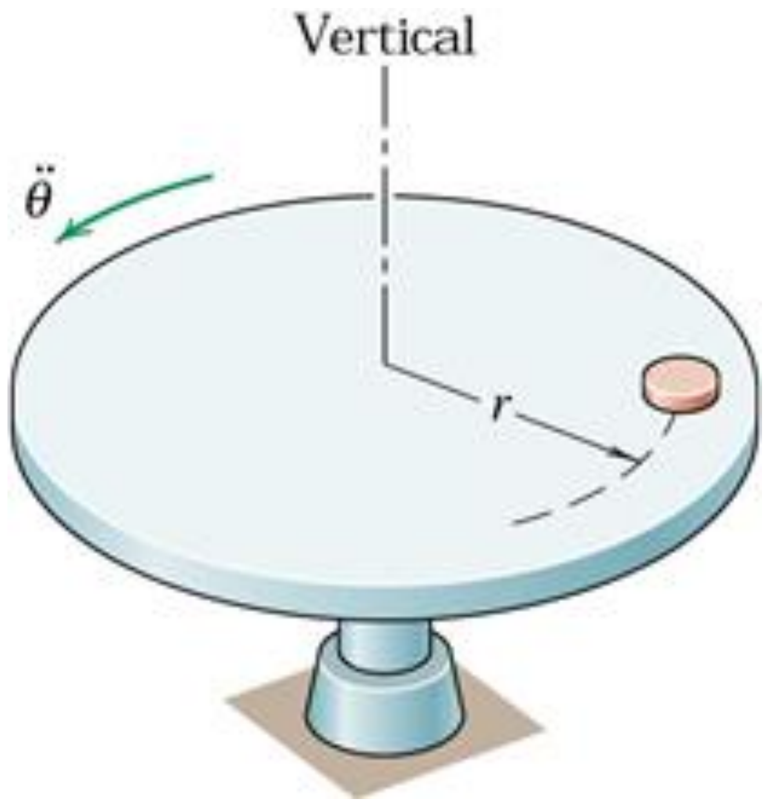
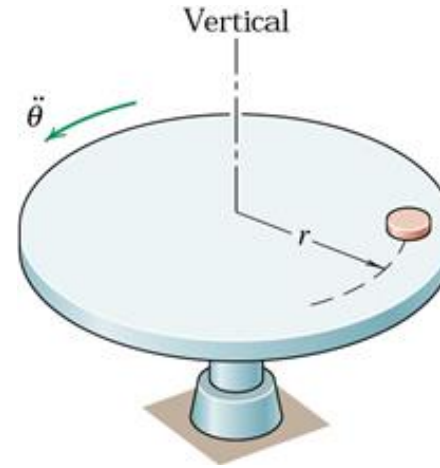
3/86 (4th), 3/88 (5th), None (6th), 3/81 (7th), 3/85 (8th)

A small coin is placed on the horizontal surface of the rotating disk. If the disk starts from rest and is given a constant angular acceleration, α, determine an expression for the number of revolutions, N, through which the disk turns before the coin slips. The coefficient of static friction between the coin and the disk is μ_s.



One may use n-t or r-θ coordinates. For circular motion since $\dot{r} = \ddot{r} = 0$ they yield equivalent results (t = θ, r = -n). Let us use n-t:



$$\sum F_t = ma_t = mr\alpha$$

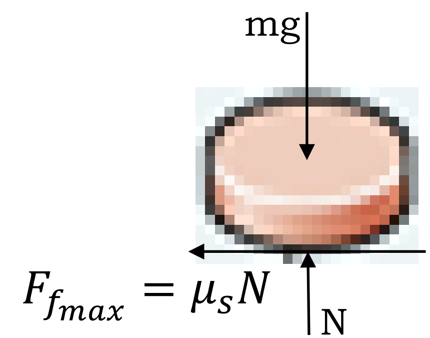
$$\sum F_n = ma_n = mr\omega^2 = m\frac{v^2}{r}$$

$$F_{f_{max}} = \mu_s N = \sqrt{{F_n}^2 + {F_t}^2}$$

$$\mu_s mg = mr\sqrt{\alpha^2 + \omega^4}$$

$$(\mu_s mg)^2 = m^2r^2(\alpha^2 + \omega^4)$$

$$\omega^2 = \frac{\sqrt{{\mu_s}^2 g^2 + r^2\alpha^2}}{r}$$

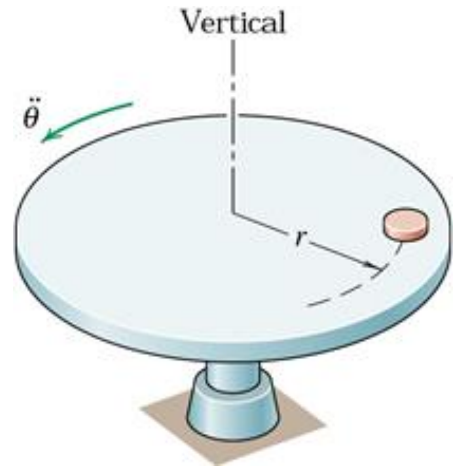


For constant α similar to linear motion $\omega d\omega = \alpha d\theta$ can be integrated to get $\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$ and $\theta = 2\pi N$ so

$$\frac{\sqrt{{\mu_s}^2 g^2 + r^2\alpha^2}}{r} = 4\pi\alpha N$$

$$N = \frac{1}{4\pi}\sqrt{\left(\frac{\mu_s g}{r\alpha}\right)^2 - 1}$$

$$N = \frac{1}{4\pi}\sqrt{\left(\frac{\mu_s g}{r\alpha}\right)^2 - 1}$$



$$\left(\frac{\mu_s g}{r\alpha}\right)^2 - 1 \geq 0 \rightarrow \begin{cases} \alpha_{max} \\ r_{max} \end{cases}$$

PART I: PARTICLES

Chapter 3: Kinetics of Particles

There are three different approaches to the kinetics problems:

A. Direct application of Newton's Second Law/Force-Mass Acceleration Method

B. Work – Energy Principles (integration of second law with respect to displacement)

C. Impulse and Momentum Methods (integration of second law with respect to time)

PART I: PARTICLES
Chapter 3: Kinetics of Particles
B. Work – Energy Principles

3.6 & 3.7 Work, Kinetic and Potential Energies

Newton's second law establishes an instantaneous relation between the net force and acceleration. To determine the change in velocity or displacement, the computed acceleration has to be integrated using appropriate kinematic relations.

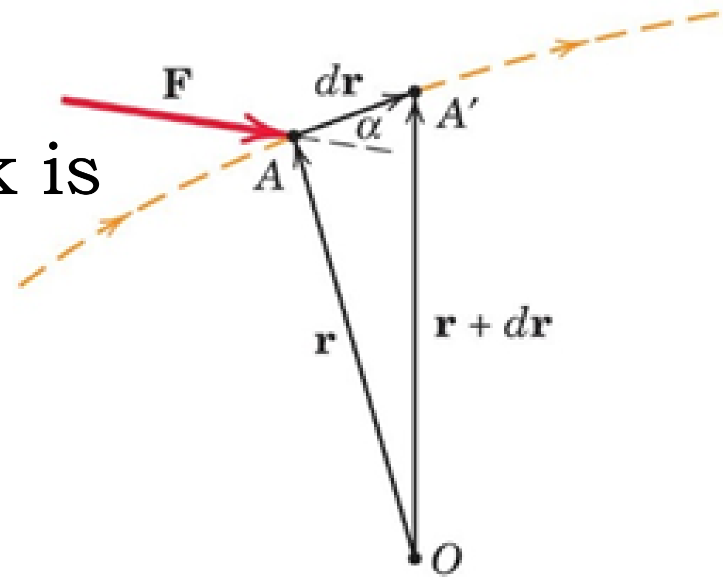
Integration of Newton's second law with respect to displacement yields work-energy relations where the cumulative effect of net force on change of velocity of the particle is directly obtained.

Work

By definition infinitesimal work is

$$dU = \vec{F} \cdot d\vec{r}$$

$$dU = F ds cos\alpha = F_t ds$$

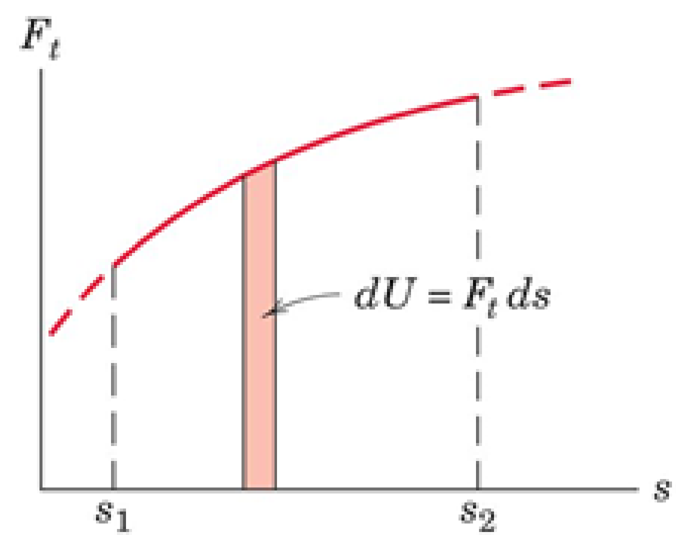


$$dU \begin{cases} > 0, F_t \text{ in the direction of displacement, work done on particle} \\ < 0, F_t \text{ in the opposite direction of displacement, work by particle} \\ = 0, F \text{ is perpendicular to displacement, no work} \end{cases}$$

$$U = \int dU = \int_{\vec{r}_0}^{\vec{r}} \vec{F} . d\vec{r}$$

$$[U] = [F \cdot L] = [N \cdot m] \equiv [J]$$

$$U = \int dU = \int F_x dx + F_y dy + F_z dz$$



Work done on a particle: Kinetic Energy

$$U_{1\to 2} = \int_1^2 \vec{F} \cdot d\vec{r}$$

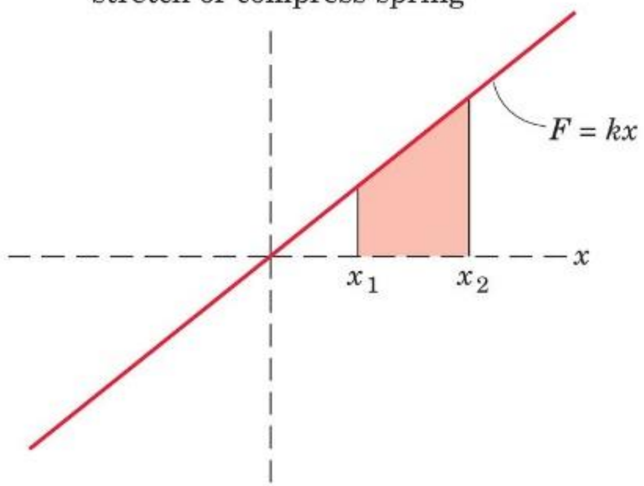
$$U_{1\to 2} = \int_1^2 m\vec{a} \cdot d\vec{r}$$

$$U_{1\to 2} = \int_{v_1}^{v_2} m\vec{v} \cdot d\vec{v} = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

Recall first law of thermodynamics stating conservation of energy. The work done on a particle of mass m is stored as kinetic energy and can be fully recovered by bringing a moving mass to a rest. Kinetic energy of a moving mass is therefore defined as:

$$T = \frac{1}{2} m v^2$$

Work done when deforming a Spring: Elastic Potential Energy



$$U_{1\rightarrow 2} = \int_{x_1}^{x_2} F_s dx$$

$$U_{1\rightarrow 2} = \int_{x_1}^{x_2} kx dx$$

$$U_{1\rightarrow 2} = \frac{1}{2} k({x_2}^2 - {x_1}^2)$$

This work is stored in an ideal elastic spring as *elastic potential energy* and can fully be recovered. Therefore elastic potential energy stored in a spring having deformation x (compression or tension does not matter) is defined as:

$$V_e = \frac{1}{2} kx^2$$

Change in elastic potential energy of an elastic spring is:

$$\Delta V_e = \frac{1}{2} k({x_2}^2 - {x_1}^2)$$

Work done against Gravity: Gravitational Potential Energy

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} -mg\hat{k} \cdot \left(dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right)$$

$$U_{1\to 2} = -mg\int_{z_1}^{z_2} dz = -mg(z_2 - z_1)$$

$$U_{1\to 2} \; = mg\Delta h$$

Height change is positive when up (storing potential energy).

This energy stored due to height change is defined as the gravitational potential energy:

$$\Delta V_g = mg\Delta h$$

Principle of Conservation of Energy (First Law of Thermodynamics)

$$U_{1\to 2} = \Delta T + \Delta V_g + \Delta V_e$$

Power

Power is defined as rate of doing work or delivering energy:

$$\mathbb{P} = \frac{dU}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot d\vec{v}$$

$$[J/s] \equiv [W]$$

Mechanical Efficiency

$$e_m = \frac{\mathbb{P}_{out}}{\mathbb{P}_{in}} \leq 1$$

Actually it is *always* less than 1 due to second law of thermodynamics.

Conservative Force Fields

In conservative force fields the work done is independent of path but only a function of end positions. Some examples of conservative force fields are gravitational and elastic forces. In that case

$$U = \oint \vec{F} \cdot d\vec{r} = 0$$

for *any* closed path. Further for conservative fields $\vec{F} \cdot d\vec{r}$ is an exact differential of a function, $\vec{F} \cdot d\vec{r} = -dV$ then

$$F_x = -\frac{dV}{dx}, F_y = -\frac{dV}{dy} \text{ and } F_z = -\frac{dV}{dz} \text{ or } \vec{F} = -\vec{\nabla}V$$

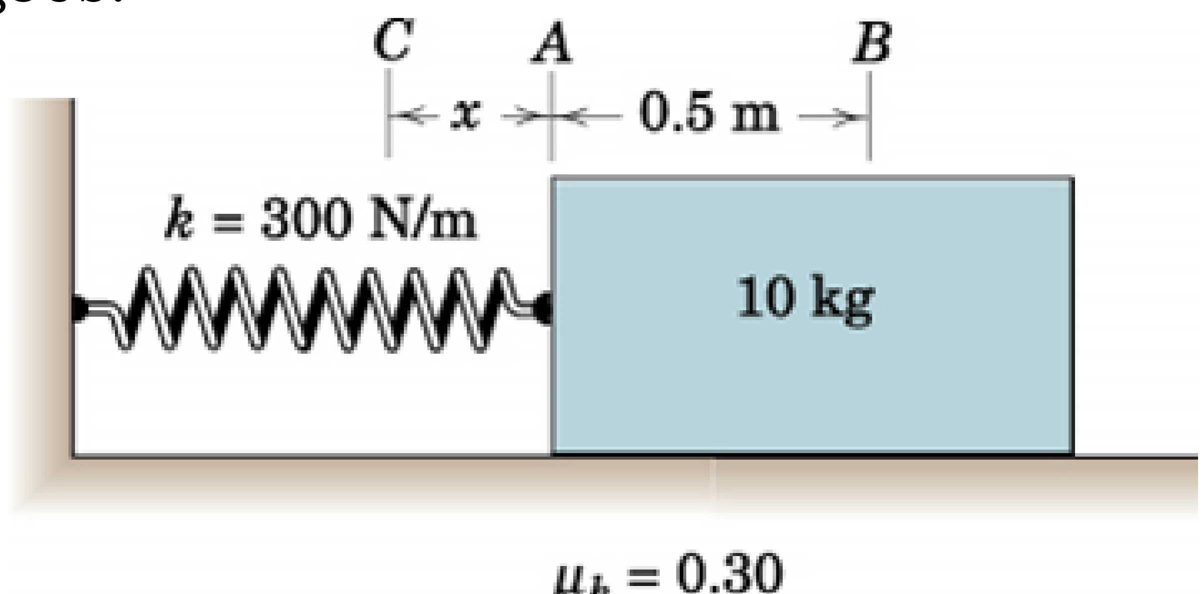
where $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\boldsymbol{j}}\frac{\partial}{\partial y} + \widehat{\boldsymbol{k}}\frac{\partial}{\partial z}$

V is known as potential function (V_g is gravitational potential and V_e is elastic potential) and $\vec{\nabla}V$ is the gradient of the potential function.

3/140 (4th), None (5th), 3/145 (6th), None (7th), None (8th)

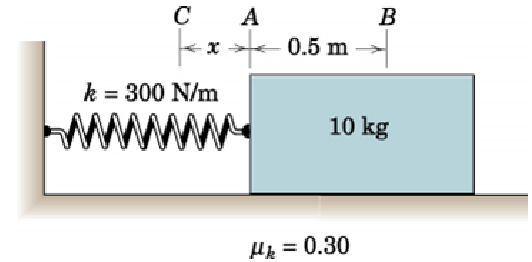
The 10 kg block is released from rest on the horizontal surface at point B, where the spring has been stretched a distance of 0.5 m from its neutral position A. The coefficient of kinetic friction between the block and the plane is 0.30. Calculate:

a. the velocity of the block as it passes point A,
b. the maximum distance x to the left of A which the block goes.

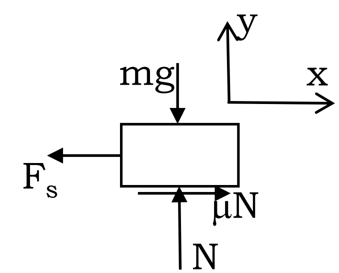


a.

$$\sum F_y = 0, N - mg = 0$$



$$N = mg = 10 * 9.81 = 98.1\ N$$



$$\mu N = 0.3 * 98.1 = 29.4\ N$$

$$U_{B\rightarrow A} = \Delta T + \Delta V_e$$

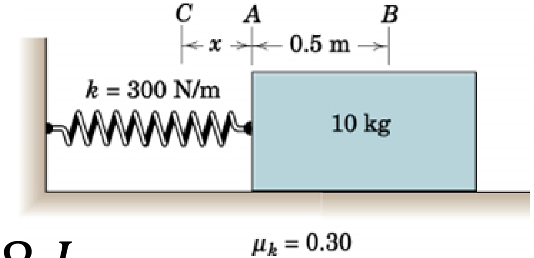
$$U_{B\rightarrow A} = -29.4 * 0.5 = -14.715\ J$$

$$\Delta T = \frac{1}{2}m({v_A}^2 - {v_B}^2) = \frac{1}{2}10({v_A}^2 - 0^2)$$

$$\Delta V_e = \frac{1}{2}k({x_A}^2 - {x_B}^2) = \frac{1}{2}300(0^2 - 0.5^2) = -37.5\ J$$

$$v_A = 2.13\ m/s$$

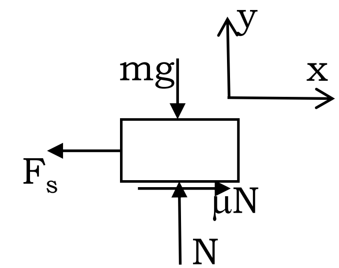
b.



$$U_{A\to C} = \Delta T + \Delta V_e \; or \; U_{B\to C} = \Delta V_e$$

$$\Delta T = \frac{1}{2} m({v_C}^2 - {v_A}^2) = \frac{1}{2} 10(0^2 - 2.13^2) = -22.8\, J$$

$$\Delta V_e = \frac{1}{2} k({x_C}^2 - {x_A}^2) = \frac{1}{2} 300({x_C}^2 - 0^2)$$



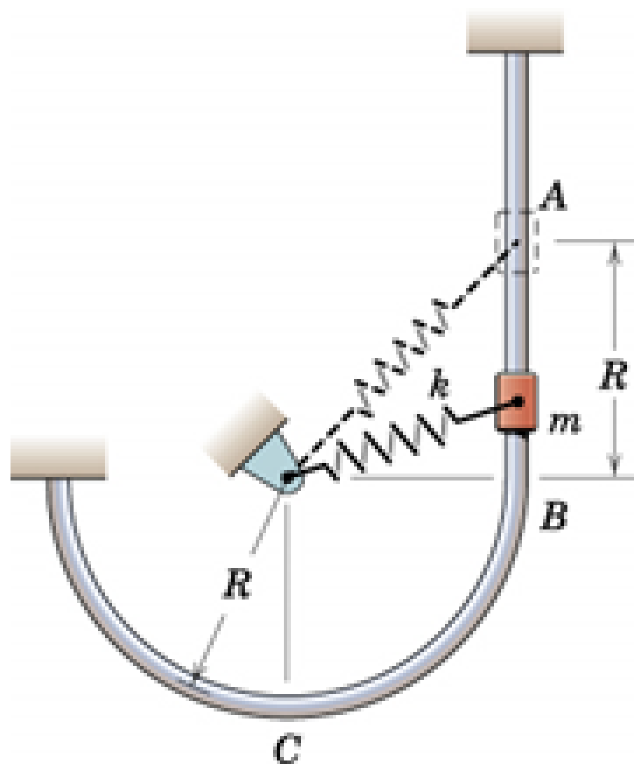
$$150x^2 + 29.43x - 22.785 = 0, x = \begin{cases} -0.5\, m \\ 0.304\, m \end{cases}$$

Since physics of friction force is not included in the formulation of the problem as $\vec{F}_f = -\mu N sgn(v)\hat{\imath}$ the treatment is having a constant force in a fixed direction. Therefore under the action of this constant force the mass has two positions where it stops! It does **not** consider the change in direction of the friction force when the direction of motion of the block changes.

You could solve the same problem by applying Newton's second law directly, determining net force on the mass and resulting acceleration and integrating.

3/148 (4th), 3/149 (5th), 3/153 (6th), 3/144 (7th), None (8th)

The spring of constant k is unstretched when the slider of mass m passes position B. If the slider is released from rest in position A, determine its speed as it passes points B and C. What is the normal force exerted by the guide on the slider at position C? Neglect friction between the mass and the circular guide, which lies in the vertical plane.



$$U_{A\to B} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{A\to B} = 0$$

$$\Delta T = \frac{1}{2}m({v_B}^2 - {v_A}^2) = \frac{1}{2}m({v_B}^2 - 0^2)$$

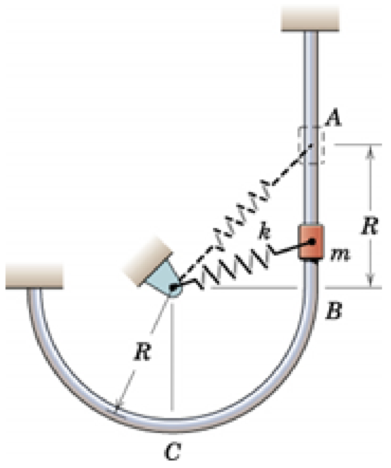
$$\Delta V_g = -mgR$$

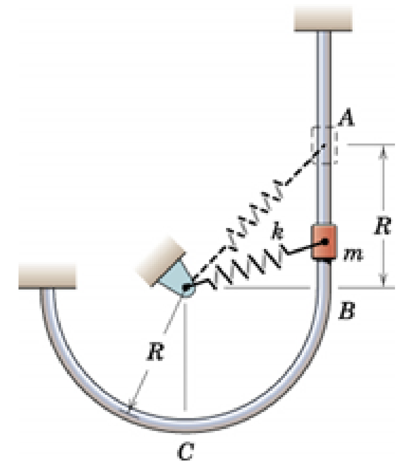
$$\Delta V_e = \frac{1}{2}k({x_B}^2 - {x_A}^2)$$

$$x_A = \sqrt{R^2 + R^2} - R = R\left(\sqrt{2} - 1\right)$$

$$\Delta V_e = \frac{1}{2}k\left(0^2 - \left[R\left(\sqrt{2} - 1\right)\right]^2\right)$$

$$v_B = \sqrt{2gR + \frac{kR^2}{m}\left(\sqrt{2} - 1\right)^2}$$





$$U_{A\to C} = \Delta T + \Delta V_g + \Delta V_e \text{ or } U_{B\to C} = \Delta T + \Delta V_g$$

$$\Delta V_g = -mgR$$

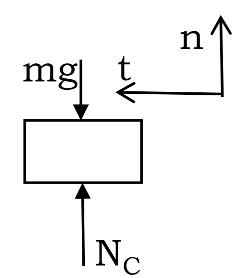
$$\Delta T = \frac{1}{2} m({v_C}^2 - {v_B}^2)$$

$$v_C = \sqrt{4gR + \frac{kR^2}{m}\left(\sqrt{2}-1\right)^2}$$

$$\sum F_n = ma_n = m\frac{{v_C}^2}{R}$$

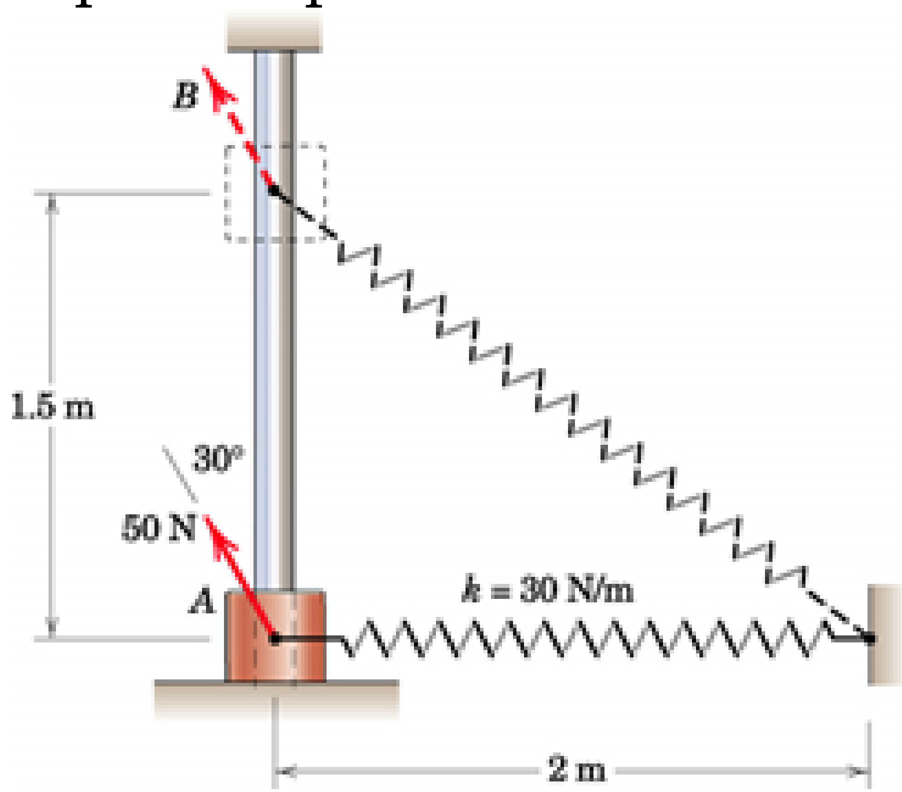
$$N_C - mg = m\left[4g + \frac{kR}{m}\left(\sqrt{2}-1\right)^2\right]$$

$$N_C = 5mg + kR\left(\sqrt{2}-1\right)^2$$



3/153 (4th), None (5th), 3/166 (6th), 3/158 (7th), None (8th)

The collar has a mass of 2 kg and is attached to the light spring which has a stiffness of 30 N/m and an unstretched length of 1.5 m. The collar is released from rest at A and slides up the smooth rod under the action of the constant 50 N force. Calculate the velocity of the collar as it passes position B.



$$U_{A\to B} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{A\to B} = 50 * 1.5 * cos30° = 64.95\ J$$

$$\Delta T = \frac{1}{2}m({v_B}^2 - {v_A}^2) = \frac{1}{2}2({v_B}^2 - 0^2)$$

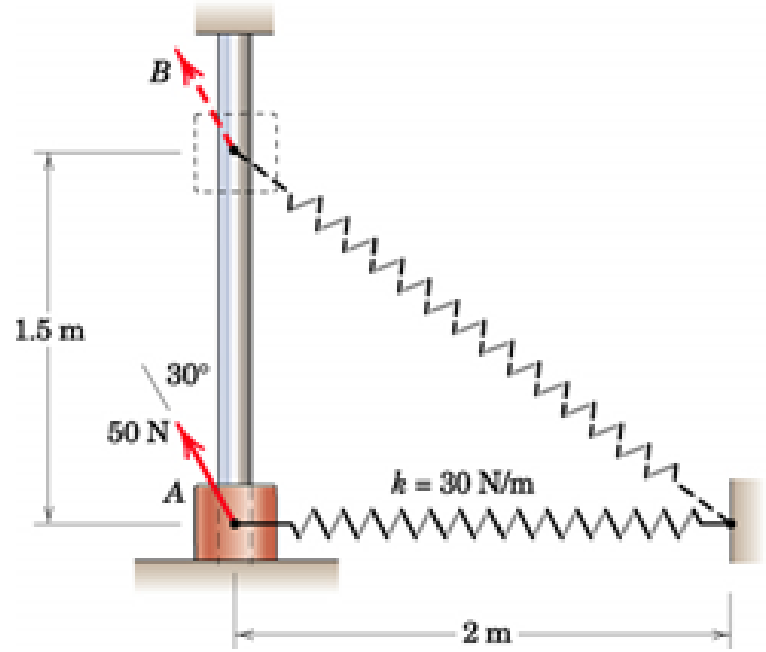
$$\Delta V_g = mg\Delta h = 2 * 9.81 * 1.5 = 29.4\ J$$

$$\Delta V_e = \frac{1}{2}k({x_B}^2 - {x_A}^2) = \frac{1}{2}30({x_B}^2 - 0.5^2)$$

$$x_B = \ell_B - \ell_0 = \sqrt{2^2 + 1.5^2} - 1.5 = 1\ m$$

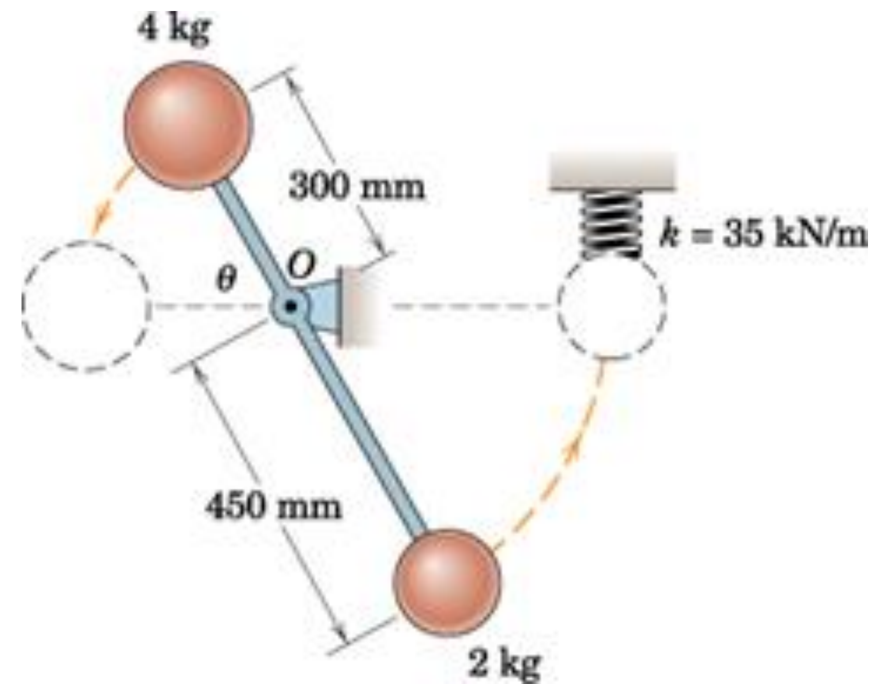
$$\Delta V_e = \frac{1}{2}30(1^2 - 0.5^2) = 11.25\ J$$

$$v_B = 4.93\ m/s$$

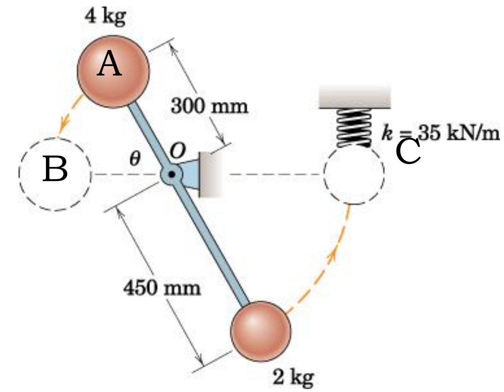


3/149 (4th), 3/151 (5th), 3/155 (6th), 3/149 (7th), None (8th)

The light rod is pivoted at O and carries the 2 and 4 kg particles. If the rod is released from rest at θ = 60° and swings in the vertical plane, calculate (a) the velocity of the 2 kg particle just before it hits the spring in the dashed position and (b) the maximum compression of the spring. Assume that the compression is small so the position of the rod when the spring is compressed is essentially horizontal.



Here please recognize that we have two particles whose motions are dependent (i.e. we have a constraint between them which is the rod).



$$U_{A\to B} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{A\to B} = 0$$

$$\Delta T = \frac{1}{2} m_{2kg} \left({v_{2kg}}_B{}^2 - {v_{2kg}}_A{}^2 \right) + \frac{1}{2} m_{4kg} \left({v_{4kg}}_B{}^2 - {v_{4kg}}_A{}^2 \right)$$

$$0.3 v_{2kg} = 0.45 v_{4kg}$$

$$v_{2kg} = 1.5 v_{4kg}$$

$$\Delta T = \frac{1}{2} 2 \left({v_{2kg}}_B{}^2 - 0^2 \right) + \frac{1}{2} 4 \left(\left(\frac{2}{3} v_{2kg} \right)^2 - 0^2 \right)$$

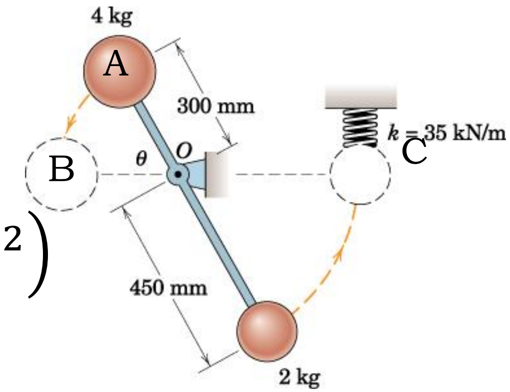
$$\Delta V_g = m_{2kg} g \Delta h_{2kg} + m_{4kg} g \Delta h_{4kg}$$

$$\Delta h_{2kg} = 0.45 \sin 60°$$

$$\Delta h_{4kg} = -0.3 \sin 60°$$

$$\Delta V_e = 0$$

$${v_{2kg}}_B = 1.162\ m/s$$



$$U_{B\rightarrow C} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{B\rightarrow C} = 0$$

$$\Delta T = \frac{1}{2}m_{2kg}\left({v_{2kg}}_{BC}{}^2 - {v_{2kg}}_B{}^2\right) + \frac{1}{2}m_{4kg}\left({v_{4kg}}_C{}^2 - {v_{4kg}}_B{}^2\right)$$

$$0.3v_{4kg} = 0.45v_{2kg}$$

$$v_{4kg} = 1.5v_{2kg}$$

$$\Delta T = \frac{1}{2}2\left(0^2 - {v_{2kg}}_B{}^2\right) + \frac{1}{2}4\left(0^2 - \left(\frac{2}{3}v_{2kg}\right)^2\right)$$

$$\Delta V_g \cong 0$$

$$\Delta V_e = \frac{1}{2}kx^2$$

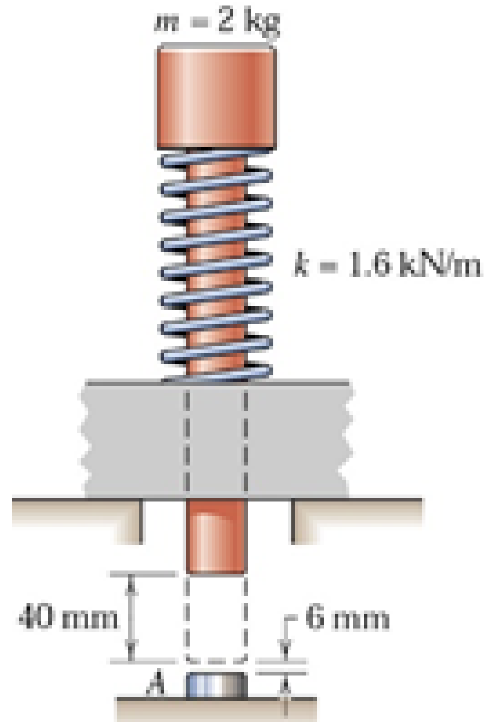
$$x = 12.07\ mm$$

ΔV_g could have been included as well (which is linear in x) so the energy equation would have been a quadratic in the unknown x.

$$\left(\Delta V_g = m_{2kg}g\Delta h_{2kg} + m_{4kg}g\Delta h_{4kg} = 2 * 9.81 * x - 4 * 9.81 * \frac{x}{1.5} = -6.64x\right)$$

None (4th), 3/164 (5th), None (6th), 3/159 (7th), None (8th)

The shank of the 2 kg vertical plunger occupies the dashed position when resting in equilibrium against the spring of stiffness k = 1.6 kN/m. The upper end of the spring is welded to the plunger, and the lower end is welded to the base plate. If the plunger is lifted 40 mm above its equilibrium position and released from rest, calculate its velocity v as it strikes the button A. Friction is negligible.



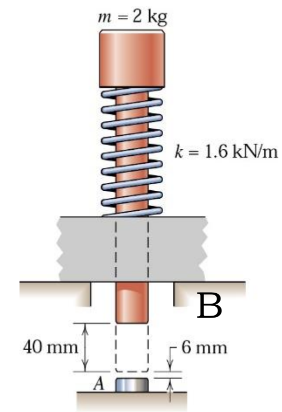
$$U_{A\to B} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{A\to B} = 0$$

$$\Delta T = \frac{1}{2}m({v_A}^2 - {v_B}^2) = \frac{1}{2}2({v_A}^2 - 0^2)$$

$$\Delta V_g = mg\Delta h = -2 * 9.81 * 0.046 = -0.903\ J$$

$$\Delta V_e = \frac{1}{2}k({x_A}^2 - {x_B}^2)$$



During static equilibrium, weight causes a static deflection in the spring: $mg = kx_s$

$$x_s = \frac{mg}{k} = \frac{2 * 9.81}{1600} = 0.0123\ m$$

So unstretched length of the spring where it has zero elastic potential energy is 12.3 mm above the static equilibrium position.

$$x_A = 0.0123 + 0.006 = 0.01286\ mm$$

$$x_B = 0.040 - 0.0123 = 0.0277\ mm$$

$$v_A = 1.119\ m/s$$

PART I: PARTICLES

Chapter 3: Kinetics of Particles

There are three different approaches to the kinetics problems:

A. Direct application of Newton's Second Law/Force-Mass Acceleration Method

B. Work – Energy Principles (integration of second law with respect to displacement)

C. Impulse and Momentum Methods (integration of second law with respect to time)

PART I: PARTICLES
Chapter 3: Kinetics of Particles
C. Impulse and Momentum Methods
3/9 Linear Impulse and Linear Momentum

Linear momentum is defined as

$$\vec{G} \equiv m\vec{v}$$

Actually (*in words because in Newton's era mathematical notation was not developed*) second law is $\sum \vec{F} = \dot{\vec{G}} = \frac{d}{dt}(m\vec{v})$ which boils down to $\sum \vec{F} = m\vec{a}$ for constant mass particles because $\dot{m} = \frac{dm}{dt} = 0$.

Linear impulse momentum principle states that

$$\int_{t_1}^{t2} \sum \vec{F}\, dt = \vec{G}(t_2) - \vec{G}(t_1) = \Delta\vec{G}$$

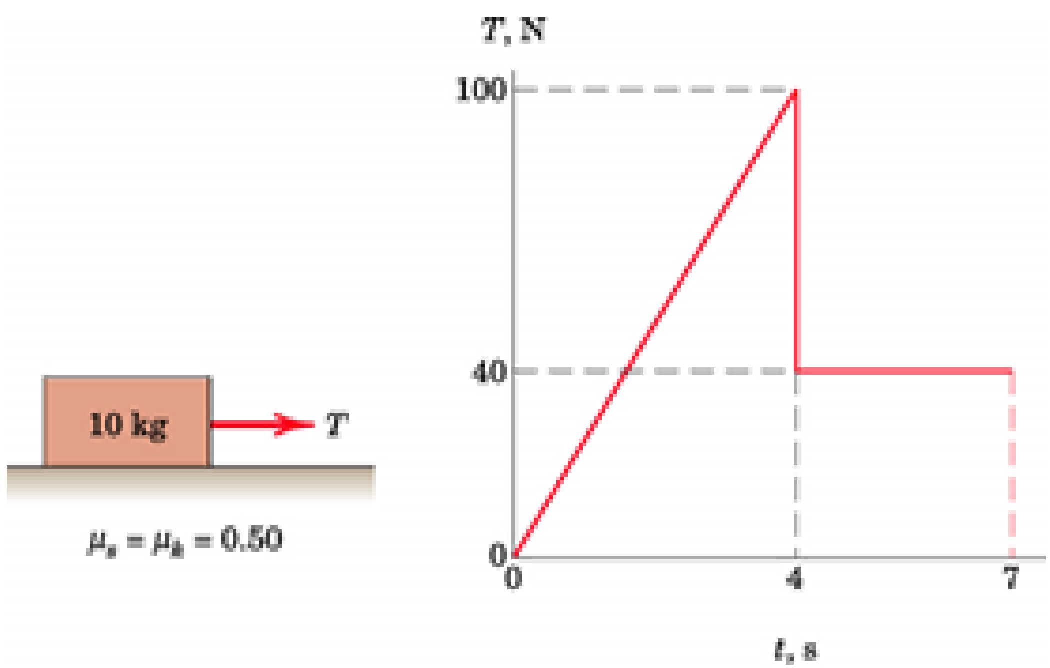
PART I: PARTICLES
Chapter 3: Kinetics of Particles
C. Impulse and Momentum Methods
3/9 Linear Impulse and Linear Momentum

Conservation of Linear Momentum

During certain period of time if net force on a particle is zero then during that time interval momentum does not change and is conserved. It is possible that the linear momentum (as a vector) may be conserved or it may be conserved in one particular direction whereas it may change in other directions where forces act.

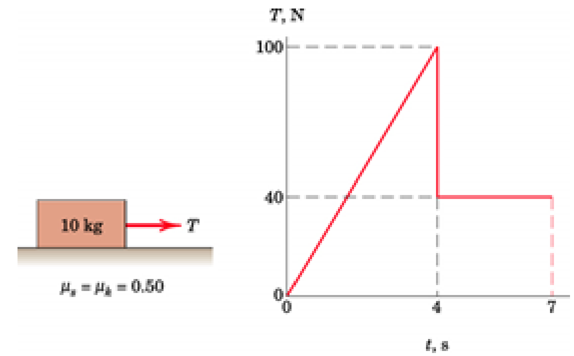
3/215 (4th), None (5th), 3/217 (6th), None (7th), None (8th)

The 10 kg block is resting on the horizontal surface when the force T is applied to it for 7 seconds. The variation of T with time is shown. Calculate the maximum velocity reached by the block and the total time Δt during which the block is in motion. The coefficients of static and kinetic friction are both 0.50.

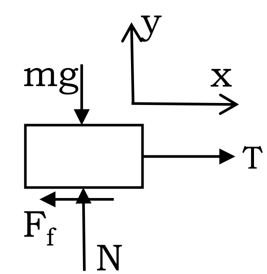


Recall that

$$F_f = \begin{cases} F_{f\,s} \leq \mu_s N \; if \; v_{rel} = 0 \\ F_{f\,k} = \mu_k N \; if \; v_{rel} \neq 0 \end{cases}$$



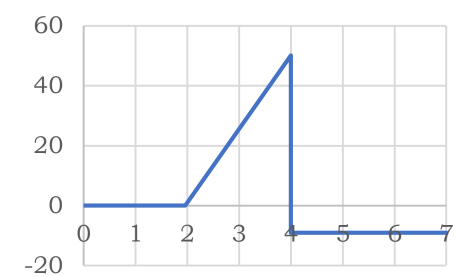
Since block is initially stationary until T > μ_sN it will stay stationary and F_f = T. As soon as T > F_f the *unbalanced* force starts the motion and block starts to slide. F_f is constant and μ_kN during sliding. Using this information one may obtain a plot showing F_{net} versus time. However we do not know if the block stops before t = 7 s or not.



Maximum speed at t = 4s

$$\int F_{net} dt = \frac{4 - 1.96}{2} 50.15 = 10v$$

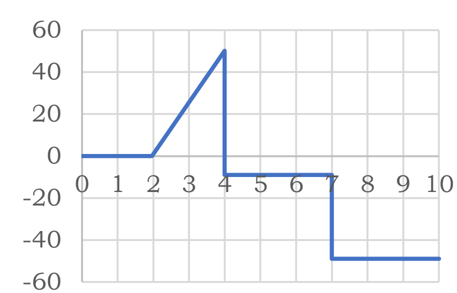
$$v = 5.12 \; m/s$$



Assume it stops before t = 7 s.

$$\int_{t=0}^{t} F_{net} dt = 0$$

$$\frac{4 - 1.96}{2} 50.15 - 9.05(t - 4) = 0, t = 9.74 \; s > 7 \; s$$

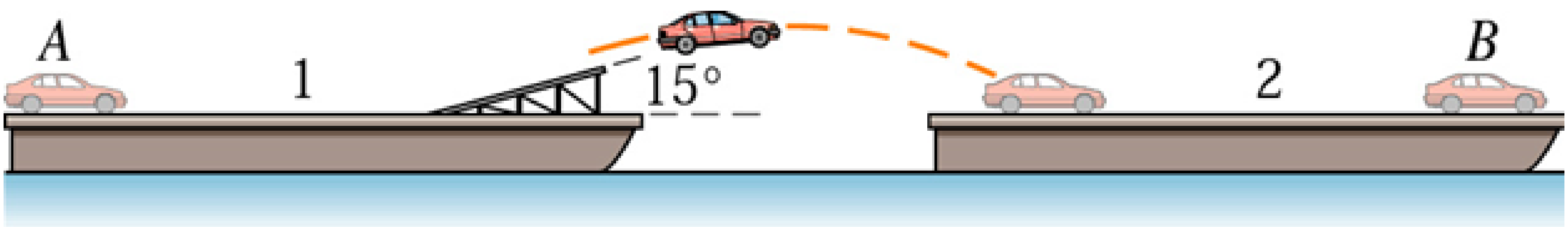


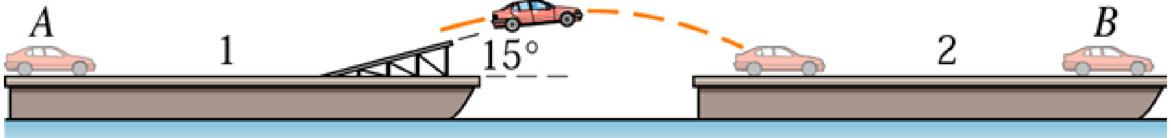
Therefore it is still moving when t = 7s

$$\frac{4 - 1.96}{2} 50.15 - 9.05(7 - 4) - 49.05(t - 7), t = 7.5 \; s$$

3/220 (4th), 3/218 (5th), None (6th), 3/214 (7th), 3/214 (8th)

Two barges each with a displacement (mass) of 500 metric tons are loosely moored in calm water. A stunt driver starts his 1500 kg car from rest at A, drives along the deck, and leaves the end of 15° ramp at a speed of 50 km/h relative to the barge and the ramp. The driver successfully jumps the gap and brings his car to rest relative to barge 2 at B. Calculate the velocity imparted to barge 2 just after the car has come to a rest on the barge. Neglect the resistance of water to motion at low velocities involved.





Till the instant car jumps off barge 1, the momentum of barge 1 and car together is conserved in horizontal direction since there is no horizontal external force.

$$0 = m_1 v_1' + m_C v_{C_{horiz}}$$

Since the car started moving 50 km/h speed is relative to the barge 1 and using relative velocity:

$$\vec{v}_C = \vec{v}_1 + \vec{v}_{C/1}, \vec{v}_{C/1} = 50(cos15°\hat{\imath} + sin15°\hat{\jmath})km/h \equiv (13.42\hat{\imath} + 3.59\hat{\jmath})m/s$$

$$\vec{v}_C = [(13.42 - v_1)\hat{\imath} + 3.59\hat{\jmath}]m/s$$

$$v_{C_{horiz}} = (13.42 - v_1)m/s$$

$$0 = 500\,000 v_1' + 1500\,(13.42 - v_1), v_1 = -40.1\,mm/s$$

During *flight* of the car, neglecting air resistance, horizontal component of its velocity is conserved so initial momentum of barge 2 and car is that of the car which is shared by the barge and the car when the car comes to a rest relative to the barge.

$$m_C v_{C_{horiz}} = (m_2 + m_C)v_2$$

$$1500 * 13.38 = (500\,000 + 1500)v_2$$

$$v_2 = 40\,mm/s$$