ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

Dr. Ergin TÖNÜK

Department of Mechanical Engineering

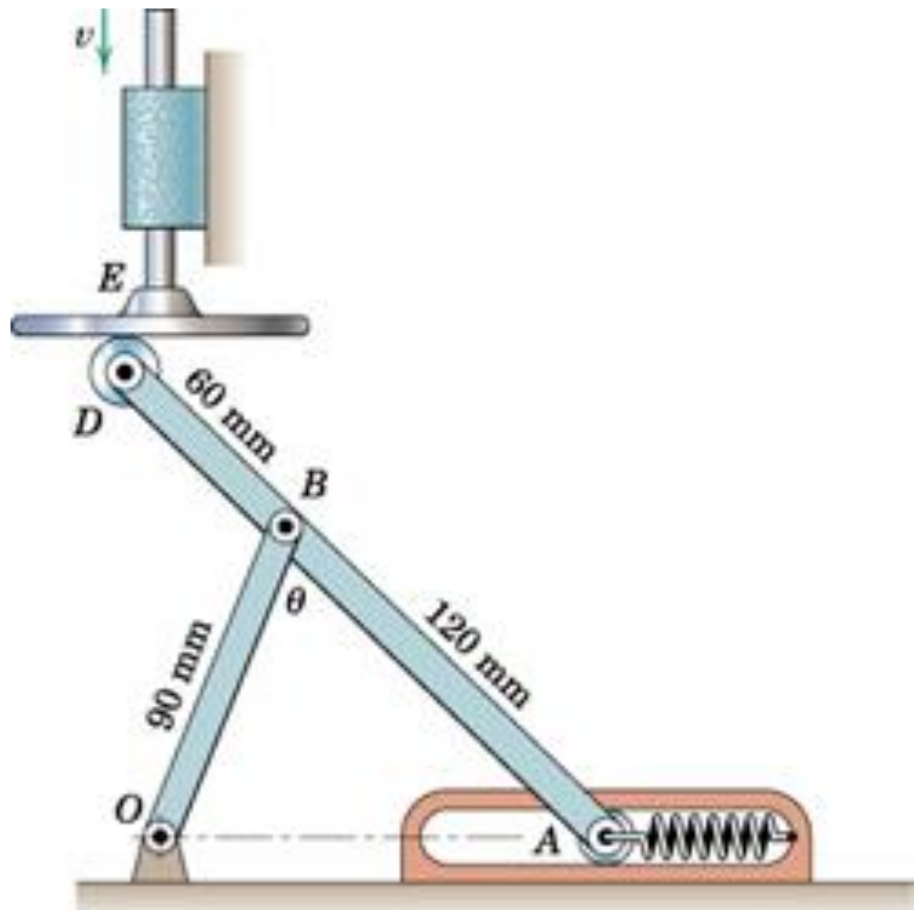
Graduate Program of Biomedical Engineering

tonuk@metu.edu.tr

http://tonuk.me.metu.edu.tr

5/112 (4th), 5/114 (5th), 5/119 (6th), 5/116 (7th), 5/118 (8th)

Motion of the roller A against its restraining spring is controlled by the downward motion of the plunger E. For an interval of motion the velocity of E is v = 0.2 m/s. Determine the velocity of A when θ becomes 90°.



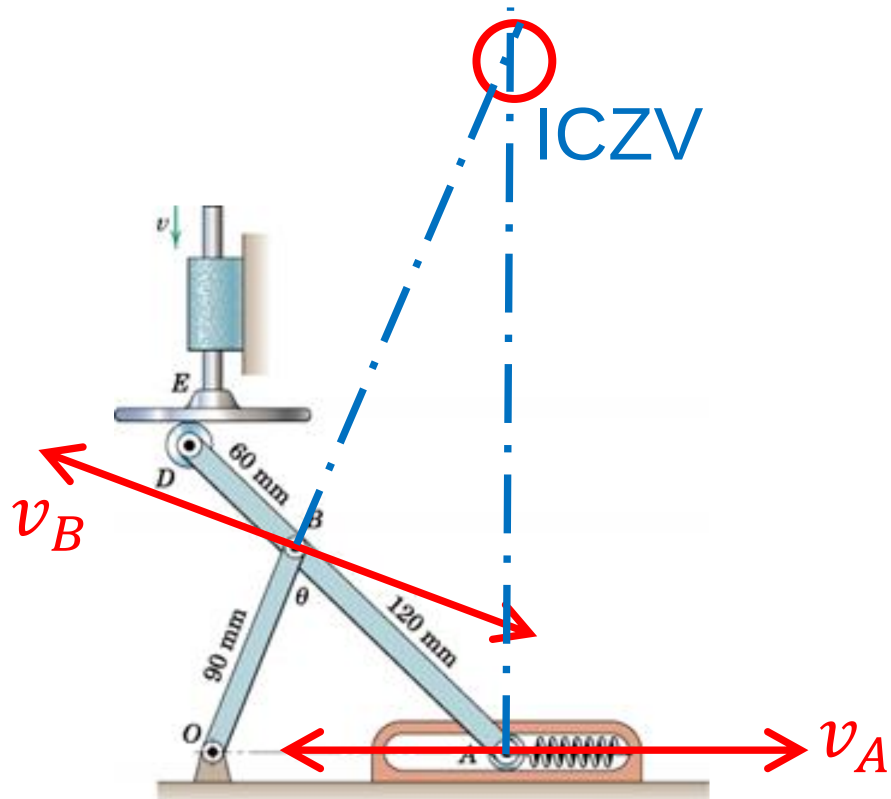
v_A is along the slot,

Consider point B on body OB, it makes fixed axis rotation about O so v_B is $\perp$ to OB.

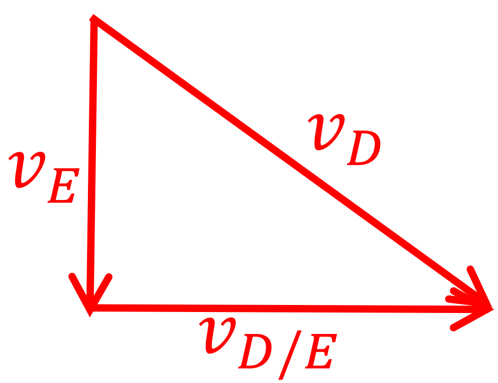
ICZV is on a line $\perp$ to v_A drawn through A

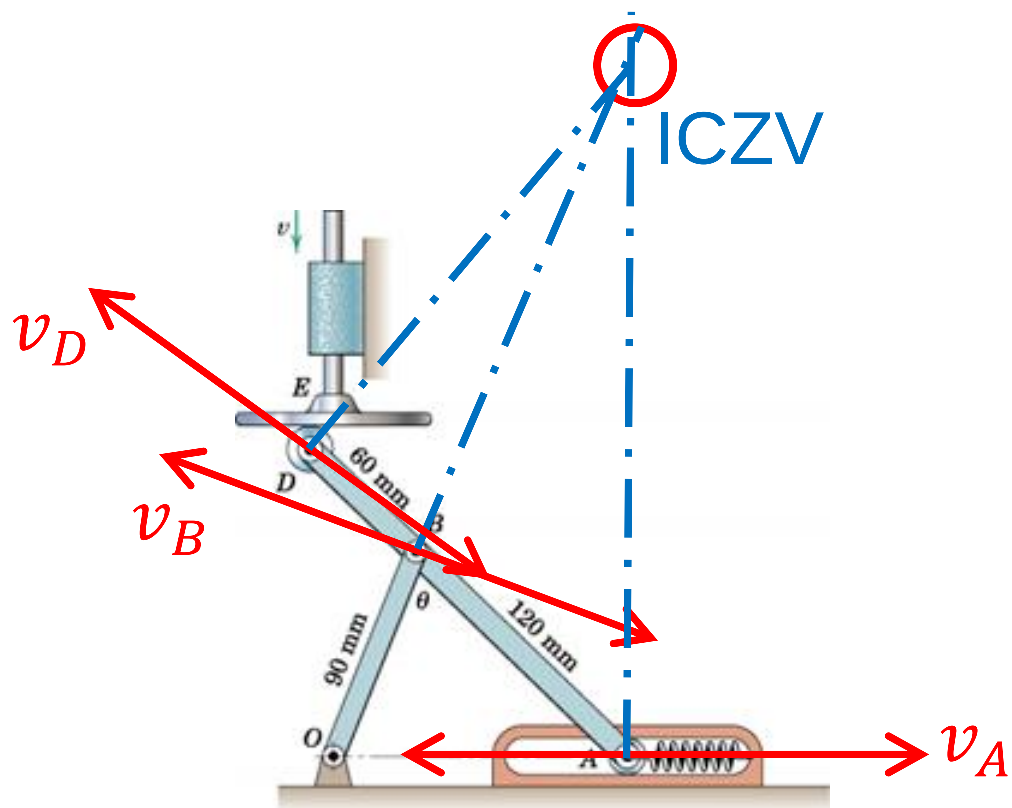
ICZV is on a line $\perp$ to v_B drawn through B

ICZV is at the intersection of these two lines!



v_D is $\perp$ to D ICZV line,
v_E is downwards
therefore the vertical component of v_D should be equal to v_E and there is a relative motion of D with respect to E which is sliding by the roller which is horizontal!





Using trigonometry:

$$|A\ ICZV| = \frac{5}{3} * 120 = 200\ mm$$

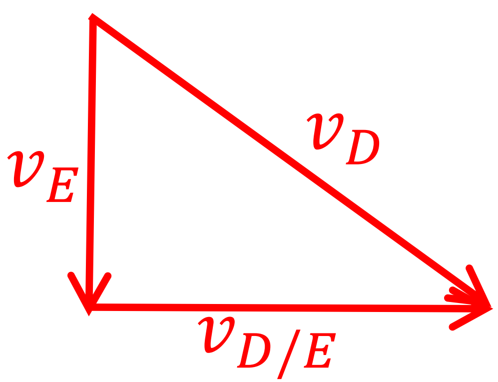
$$|B\ ICZV| = 160\ mm$$

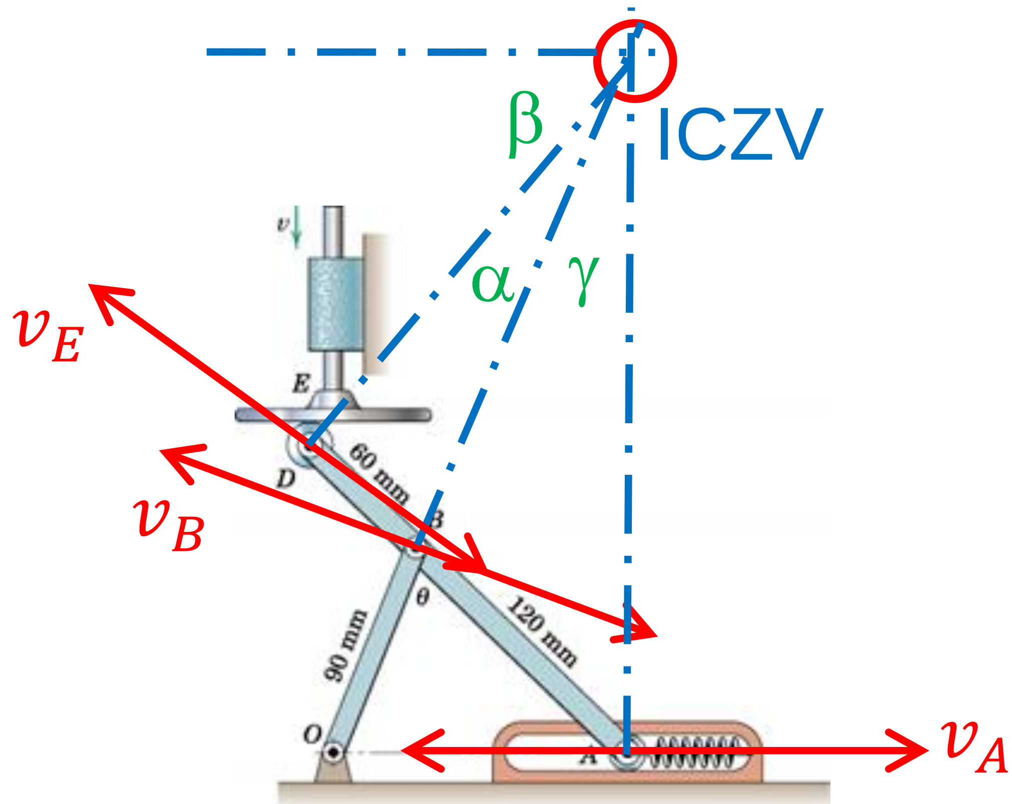
$$|D\ ICZV| = \sqrt{60^2 * 160^2} = 170.9\ mm$$

$$\gamma = sin^{-1}\frac{120}{200} = 36.9°$$

$$\alpha = tan^{-1}\frac{60}{160} = 20.6°$$

$$\beta = 90° - \alpha - \gamma = 32.6°$$

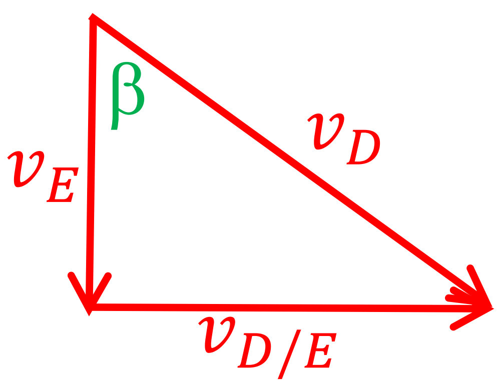


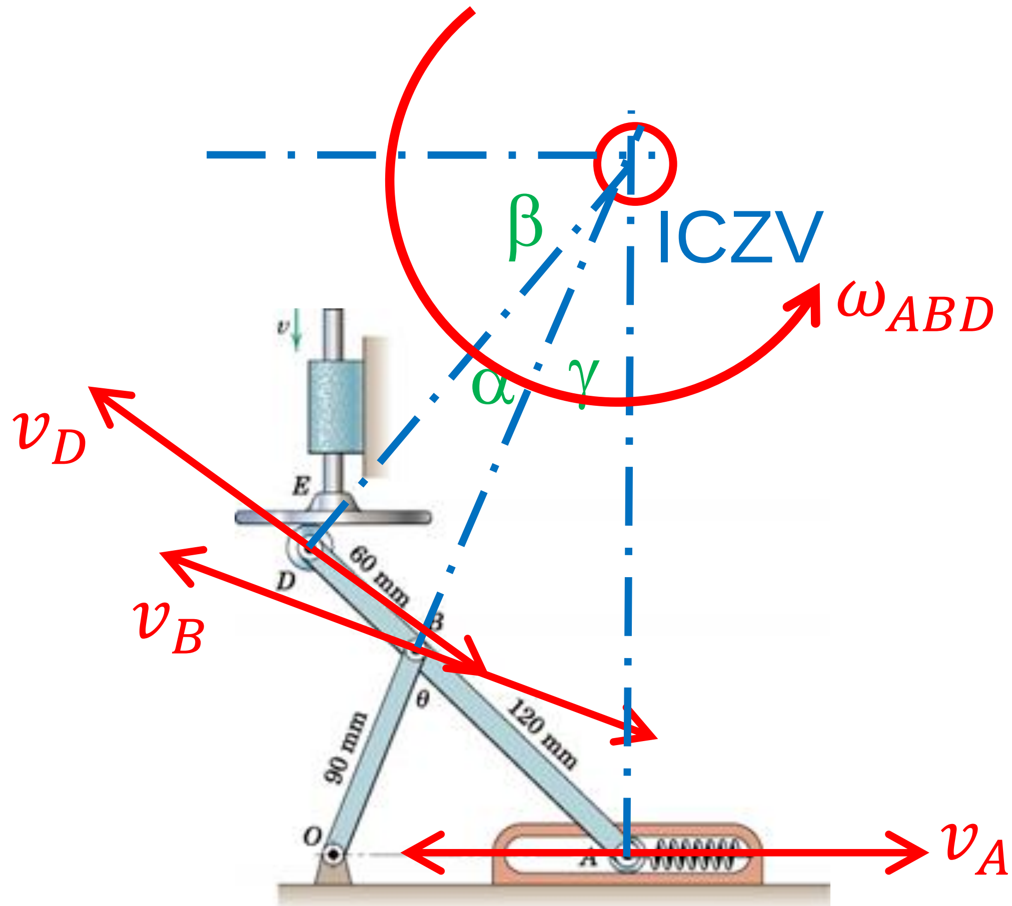


$$v_D = \frac{v}{\cos\beta} = 0.327\ m/s$$

$$\omega_{ABD} = \frac{v_D}{|D\ ICZV|} = \frac{v_A}{|A\ ICZV|}\ (CCW)$$

$$v_A = 0.383\ m/s \rightarrow$$





Analysis of General Plane Motion
Relative Motion Analysis
5/6 Relative Acceleration

Time derivative of relative velocity equation,

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

yields the relative acceleration equation:

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{v}_{A/B} = \vec{\omega} \times \vec{r}_{A/B}$

$$\vec{a}_{A/B} = \frac{d\vec{v}_{A/B}}{dt} = \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{A/B}\right)$$

$\vec{a}_{A/B} = \dot{\vec{\omega}} \times \vec{r}_{A/B} + \vec{\omega} \times \dot{\vec{r}}_{A/B}$

$\vec{a}_{A/B} = \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{A/B}\right)$

$\vec{a}_{A/B} = \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$

$\vec{a}_{A/B} = \quad \vec{a}_{{A/B}_t} \quad + \quad \vec{a}_{{A/B}_n}$

Analysis of General Plane Motion
Relative Motion Analysis
5/6 Relative Acceleration

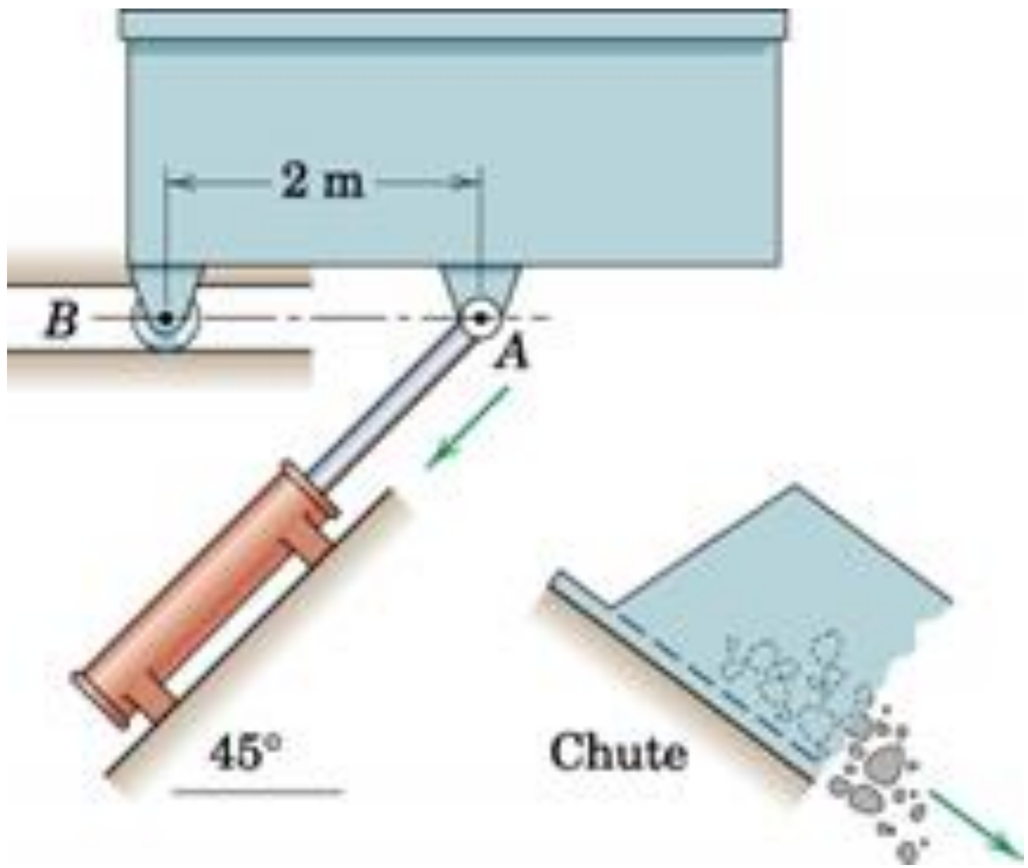
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$
$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$
$$\vec{a}_A = \vec{a}_B + \quad \vec{a}_{A/B_t} \quad + \quad \vec{a}_{A/B_n}$$

Since in general relative acceleration equation contains velocity variables too, it is required to solve relative velocity equation first.

Please remember, instant center of zero velocity *in general* has **non-zero acceleration** therefore cannot be used as an acceleration center!

5/119 (4th), None (5th), 5/126 (6th), None (7th), 5/123 (8th)

A container for waste materials is dumped by the hydraulically activated linkage shown. If the piston rod starts from rest in the position indicated and has an acceleration of 0.5 m/s^2 in the direction shown, compute the initial angular acceleration of the container.



$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B_t} + \vec{a}_{A/B_n}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$

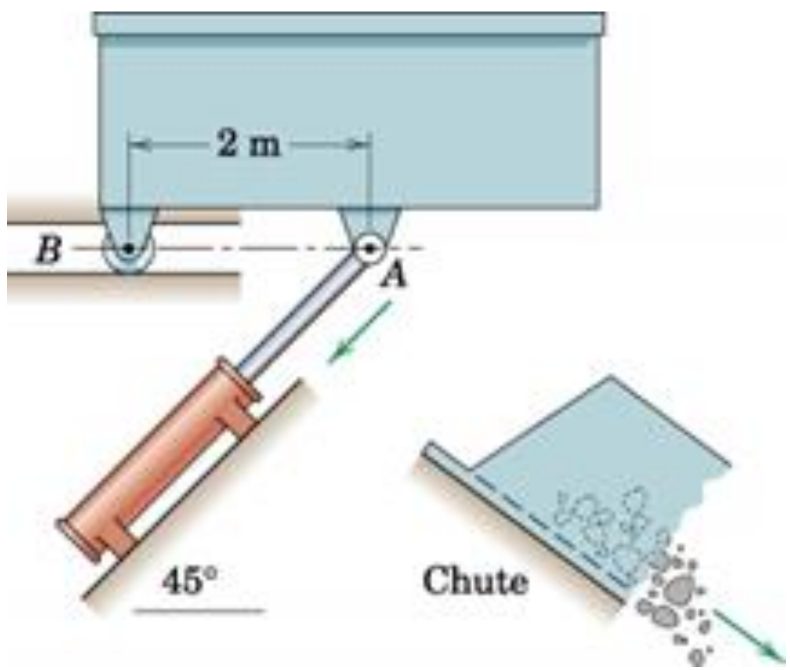
$$\omega = 0$$

$$-0.5(cos45°\hat{\imath} + sin45°\hat{\jmath}) = a_B\hat{\imath} + \alpha\hat{k} \times 2\hat{\imath}$$

$$\hat{\imath}:\ a_B = -0.5cos45° = -0.354\,m/s^2$$

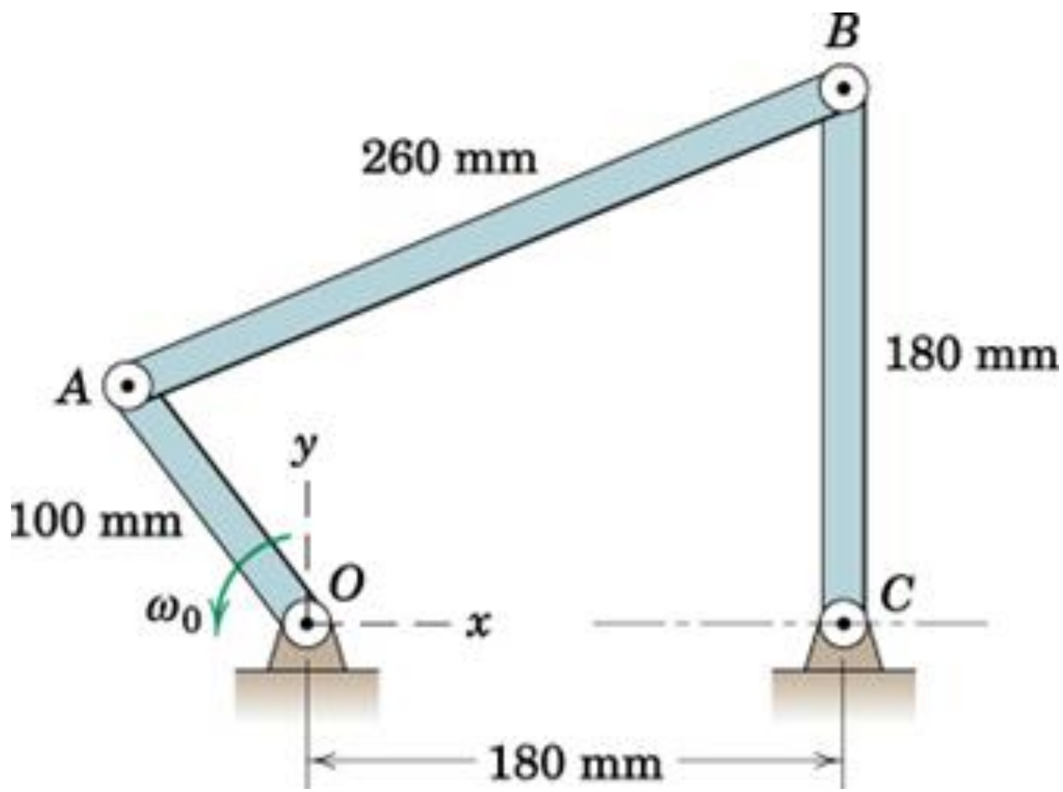
$$\hat{\jmath}:\ -0.5sin45° = 2\alpha$$

$$\alpha = -0.1768\,rad/s^2$$



5/137 (4th), 5/141 (5th), 5/145 (6th), None (7th), None (8th)

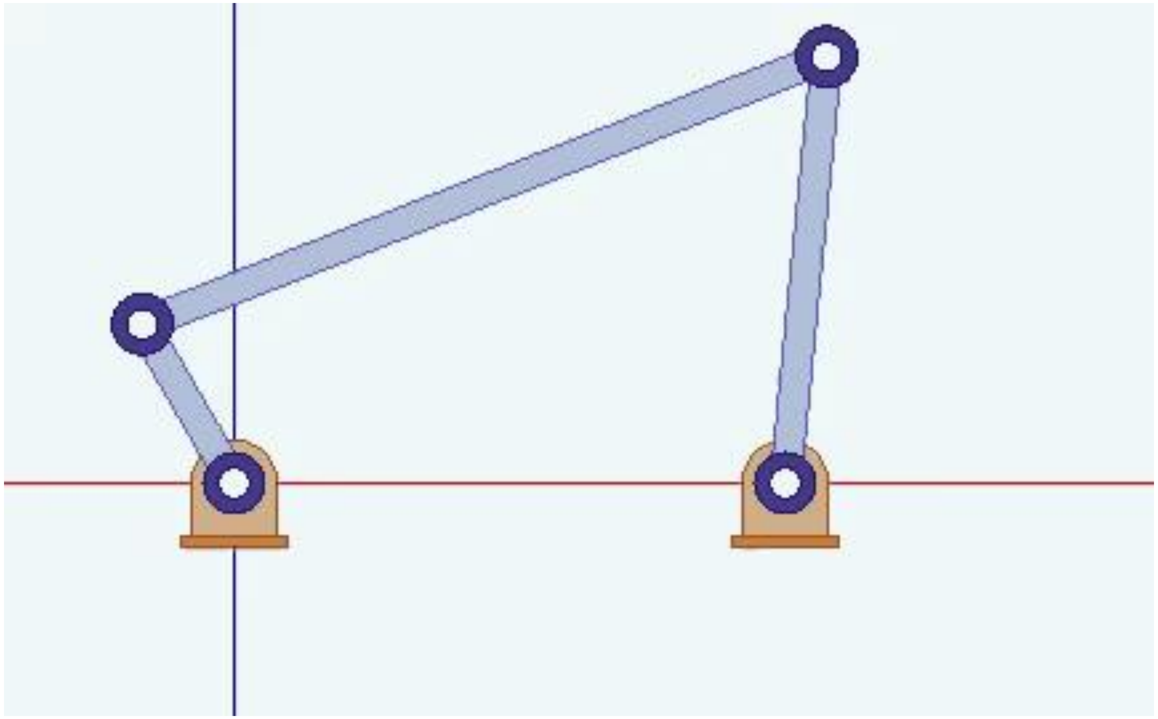
The linkage shown is a four-bar mechanism. If OA has a constant counterclockwise angular velocity $\omega_O = 10$ rad/s, calculate the angular acceleration of link AB for the position where the coordinates of A are x = -60 mm and y = 80 mm. Link BC is vertical for this position. Solve using vector algebra.



Four-bar mechanism is formed by connecting four rigid bodies (one is ground and that's why English people call it a three-bar mechanism) by four revolute (pin) joints.

It is one of the basic building blocks of many mechanical machines.

A crank-rocker four-bar mechanism converts continuous rotation of the crank into swinging (back and forth) motion of the rocker.



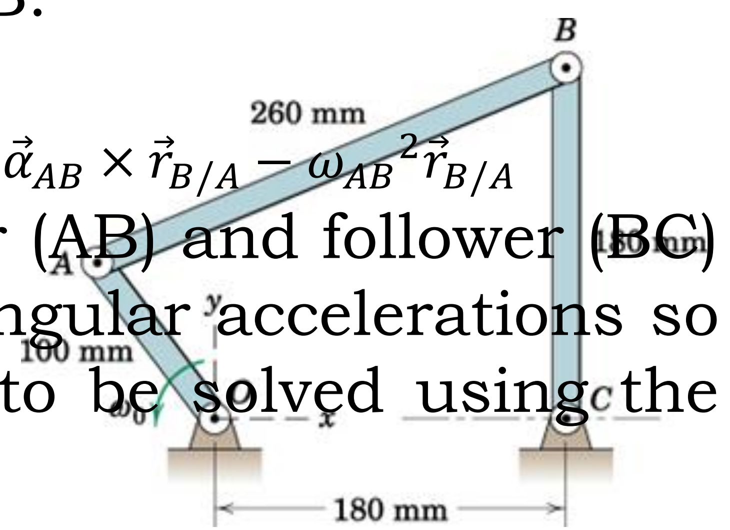
Please recognize that we have a point A on body OA (crank) that makes fixed axis rotation about O and another point A on body AB (coupler). These two points are permanently coincident points both on the axis of the pin (revolute) joint connecting two bodies. Similarly point B on body BC (follower) makes fixed axis rotation about point C and we have a permanently coincident point B on body AB.

In writing absolute accelerations of points A and B we will consider them to be on bodies OA and BC respectively whereas for relative acceleration we will use permanently coincident points A and B on body AB.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A_t} + \vec{a}_{B/A_n}$$

$$\vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}{}^2 \vec{r}_{B/C} = \vec{\alpha}_{OA} \times \vec{r}_{A/O} - \omega_{OA}{}^2 \vec{r}_{A/O} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}{}^2 \vec{r}_{B/A}$$

However angular velocities of coupler (AB) and follower (BC) are not known in addition to their angular accelerations so first relative velocity relation needs to be solved using the same properties of points A and B.



$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{\omega}_{BC} \times \vec{r}_{B/C} = \vec{\omega}_{OA} \times \vec{r}_{A/O} + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

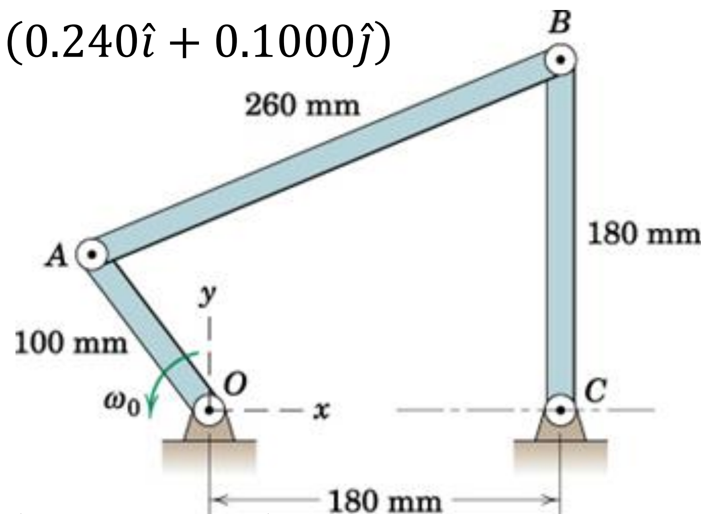
$$\vec{r}_{B/C} = 0.1800\hat{\jmath}\ m$$

$$\vec{r}_{A/O} = (-0.0600\hat{\imath} + 0.0800\hat{\jmath})\ m$$

$$\vec{r}_{B/A} = (0.240\hat{\imath} + 0.1000\hat{\jmath})\ mm$$

$$\omega_{BC}\hat{k} \times 0.1800\hat{\jmath} = 10\hat{k} \times (-0.0600\hat{\imath} + 0.0800\hat{\jmath}) + \omega_{AB}\hat{k} \times (0.240\hat{\imath} + 0.1000\hat{\jmath})$$

$$-0.18\omega_{BC}\hat{\imath} = -0.6\hat{\jmath} - 0.8\hat{\imath} + 0.24\omega_{AB}\hat{\jmath} - 0.1\omega_{AB}\hat{\imath}$$



$$\hat{\jmath}: 0 = -0.6 + 0.24\omega_{AB}, \omega_{AB} = 2.5\ rad/s$$
$$\hat{\imath}: -0.18\omega_{BC} = -0.8 - 0.1 * 2.5, \omega_{BC} = 5.83 rad/s$$

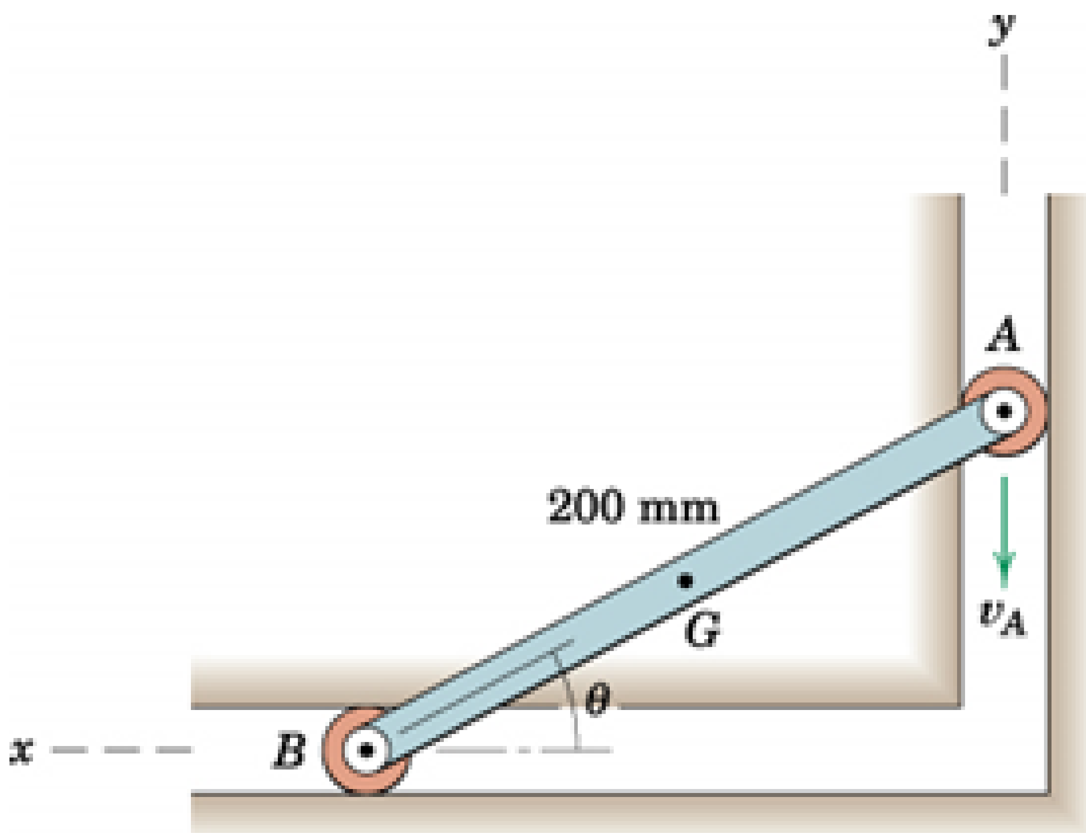
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A_t} + \vec{a}_{B/A_n}$$
$$\alpha_{BC}\hat{k} \times 0.18\hat{\jmath} - 5.83^2 0.18\hat{\jmath}$$
$$= 0\hat{k} \times (-0.06\hat{\imath} + 0.08\hat{\jmath}) - 10^2(-0.06\hat{\imath} + 0.08\hat{\jmath}) + \alpha_{AB}\hat{k} \times (0.24\hat{\imath} + 0.1\hat{\jmath})$$

$$- 2.5^2(0.24\hat{\imath} + 0.1\hat{\jmath})$$
$$-0.18\alpha_{BC}\hat{\imath} - 6.125\hat{\jmath} = 6\hat{\imath} - 8\hat{\jmath} + 0.24\alpha_{AB}\hat{\jmath} - 0.1\alpha_{AB}\hat{\imath} - 1.5\hat{\imath} - 0.625\hat{\jmath}$$
$$\hat{\jmath}:\ -6.125 = -8 + 0.24\alpha_{AB} - 0.625, \alpha_{AB} = 10.42\ rad/s^2$$

$$\hat{\imath}:\ -0.18\alpha_{BC} = 6 - 0.1 * 10.42 - 1.5, \alpha_{BC} = -19.2\ rad/s^2$$

None (4th), 5/146 (5th), 5/154 (6th), None (7th), None (8th)

If end A of the constrained link has a constant downward velocity v_A of 2 m/s as the bar passes the position for which $\theta = 30°$, determine the acceleration of the mass center G in the middle of the link.

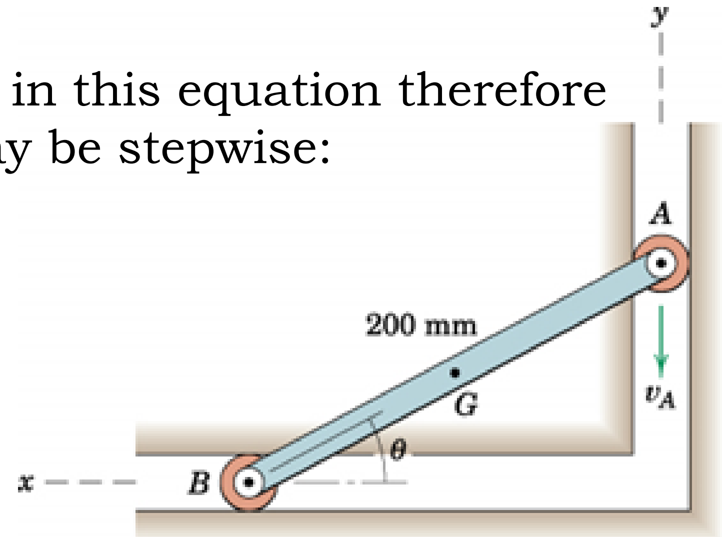


$\vec{a}_G = \vec{a}_A + \vec{a}_{G/A}$

Since neither direction nor magnitude (or none of the two orthogonal components) of a_G are known, the right hand side of the equation should be fully known.

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

angular velocity of AB would be another unknown in this equation therefore starting with the velocity equation the solution may be stepwise:



$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$

$v_B\hat{\imath} = -2\hat{\jmath} + \omega\hat{k} \times -0.2(cos30°\hat{\imath} + sin30°\hat{\jmath})$

$v_B\hat{\imath} = -2\hat{\jmath} - 0.1732\omega\hat{\jmath} + 0.1\omega\hat{\imath}$

$\hat{\jmath}$: $0 = -2 - 0.1732\omega, \omega = -11.55\, rad/s$

$\hat{\imath}$: $v_B = 0.1 * (-11.55) = -1.155\, m/s$

$a_B\hat{\imath} = \vec{0} + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2\vec{r}_{B/A} = \alpha\hat{k} \times -0.2(cos30°\hat{\imath} + sin30°\hat{\jmath}) - 11.55^2 * -0.2(cos30°\hat{\imath} + sin30°\hat{\jmath})$

$a_B\hat{\imath} = -0.1732\alpha\hat{\jmath} + 0.1\alpha\hat{\imath} + 23.1\hat{\imath} + 13.34\hat{\jmath}$

$\hat{\jmath}$: $0 = -0.1732\alpha + 13.34, \alpha = 77.0\, rad/s^2$

$\hat{\imath}$: $a_B = 0.1 * 77.0 + 23.1, a_B = 30.8\, m/s^2$

$\vec{a}_G = \vec{a}_A + \vec{\alpha} \times \vec{r}_{G/A} - \omega^2\vec{r}_{G/A}$

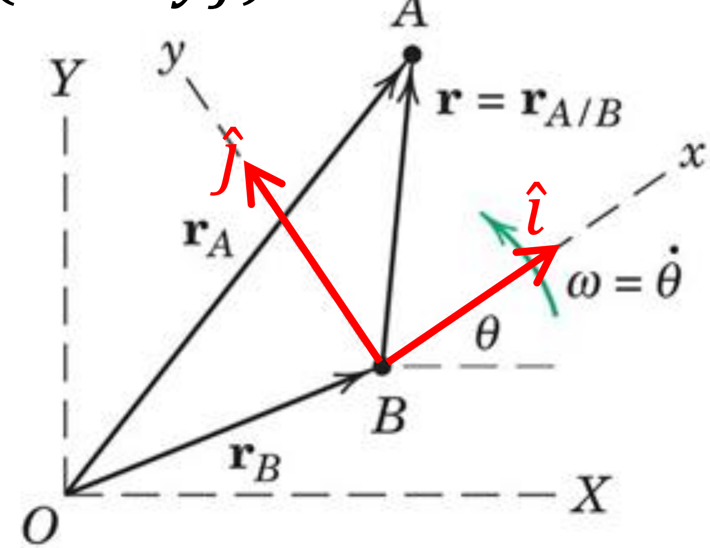
$= 77\hat{k} \times -0.1(cos30°\hat{\imath} + sin30°\hat{\jmath}) + 30.8^2 0.1(cos30°\hat{\imath} + sin30°\hat{\jmath}) = -15.40\hat{\imath}\, m/s^2$

Analysis of General Plane Motion
Relative Motion Analysis
5/7 Motion Relative to Rotating Axes

Up to now we did relative motion analysis using a non-rotating axes. However in many problems a rotating coordinate frame may be necessary.

Similar to previous analysis:

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B} = \vec{r}_B + \vec{r}_{rel} = \vec{r}_B + (x\hat{\imath} + y\hat{\jmath})$$



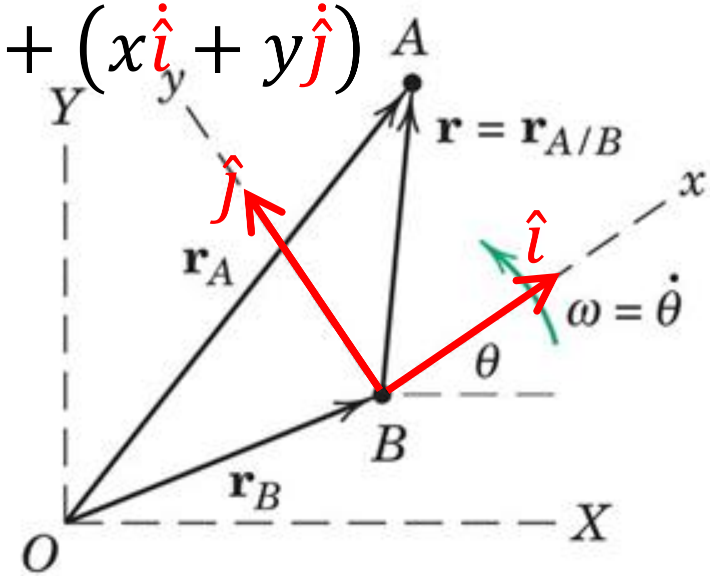
In previous analyses since orientation of x-y coordinate system was fixed relative to the fixed X-Y coordinate system the unit vectors in moving coordinate system, $\hat{\imath}$ and $\hat{\jmath}$, having unit magnitude and fixed directions did not possess time derivatives. In rotating coordinates, despite their fixed magnitudes, since **their directions are changing due to rotation** they possess time derivatives.

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \dot{\vec{r}}_B + (\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}) + \left(x\dot{\hat{\imath}} + y\dot{\hat{\jmath}}\right)$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \left(\boldsymbol{x}\dot{\hat{\boldsymbol{\imath}}} + \boldsymbol{y}\dot{\hat{\boldsymbol{\jmath}}}\right)$$

So the question is:

What are $\dot{\hat{\imath}}$ and $\dot{\hat{\jmath}}$?



So the question is:

What are $\dot{\hat{\imath}}$ and $\dot{\hat{\jmath}}$?

Consider an infinitesimal time interval dt during which the moving coordinate axis rotate by an infinitesimal angle $d\theta$ therefore the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ rotate the same amount.

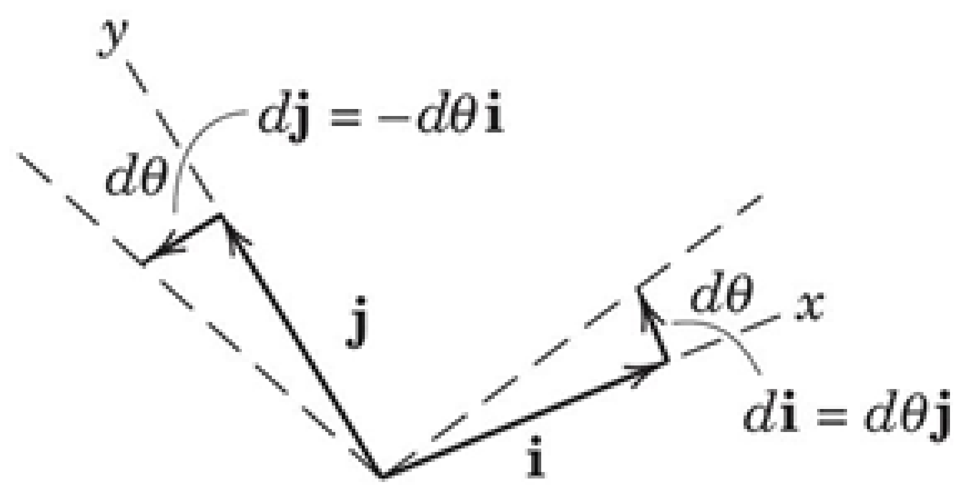
The infinitesimal change in these unit vectors in this infinitesimal time interval is:

$$d\hat{\imath} = d\theta\hat{\jmath}$$

$$d\hat{\jmath} = -d\theta\hat{\imath}$$

$$\frac{d\hat{\imath}}{dt} = \dot{\hat{\imath}}, \frac{d\theta}{dt} = \omega$$

$$\boxed{\begin{aligned} \dot{\hat{\imath}} &= \omega\hat{\jmath} \\ \dot{\hat{\jmath}} &= -\omega\hat{\imath} \end{aligned}}$$



$$\boxed{\begin{aligned}\dot{\hat{\imath}} &= \omega\hat{\jmath}\\ \dot{\hat{\jmath}} &= -\omega\hat{\imath}\end{aligned}}$$

For planar kinematics

$$\vec{\omega} = \omega\hat{k}$$

so

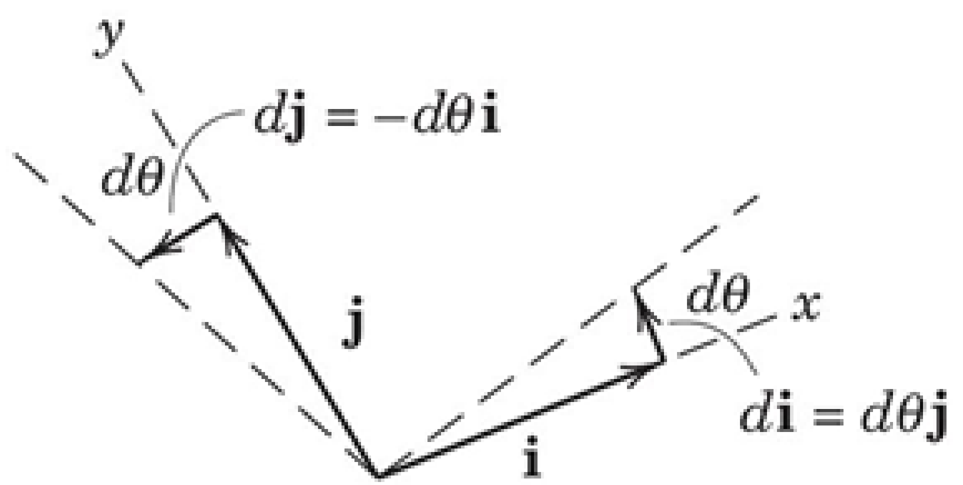
$$\vec{\omega} \times \hat{\imath} = \omega\hat{k} \times \hat{\imath} = \omega\hat{\jmath}$$

and

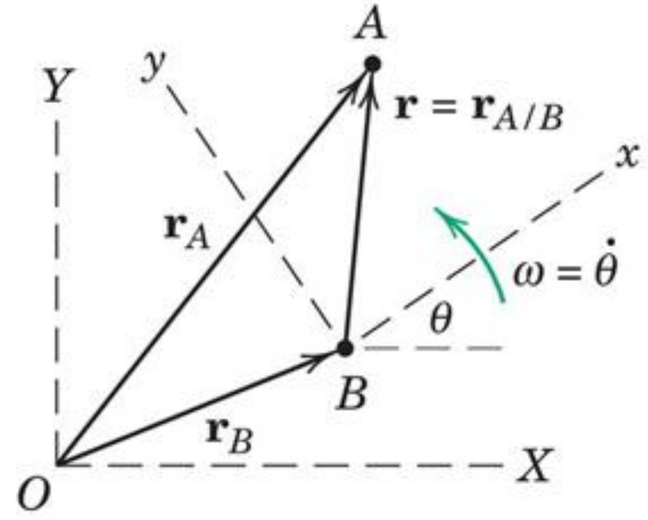
$$\vec{\omega} \times \hat{\jmath} = \omega\hat{k} \times \hat{\jmath} = -\omega\hat{\imath}$$

therefore

$$\boxed{\begin{aligned}\dot{\hat{\imath}} &= \vec{\omega} \times \hat{\imath}\\ \dot{\hat{\jmath}} &= \vec{\omega} \times \hat{\jmath}\end{aligned}}$$



$$\boxed{\begin{aligned}\dot{\hat{\imath}} &= \vec{\omega} \times \hat{\imath} \\ \dot{\hat{\jmath}} &= \vec{\omega} \times \hat{\jmath}\end{aligned}}$$



Relative Velocity

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \left(x\dot{\hat{\imath}} + y\dot{\hat{\jmath}}\right)$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + (x\vec{\omega} \times \hat{\imath} + y\vec{\omega} \times \hat{\jmath})$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \vec{\omega} \times (x\hat{\imath} + y\hat{\jmath}) = \vec{v}_B + \vec{\omega} \times (x\hat{\imath} + y\hat{\jmath}) + \vec{v}_{rel}$$

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

$\vec{v}_{rel}$: velocity of the particle measured in rotating (x-y) coordinates

$\vec{\omega} \times \vec{r}_{rel}$: velocity difference between the rotating (x-y) and non-rotating (X-Y) coordinates

$\vec{v}_B$: absolute velocity of origin of rotating coordinates

$\vec{v}_A$: absolute velocity of the particle

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

Interpretation of Terms

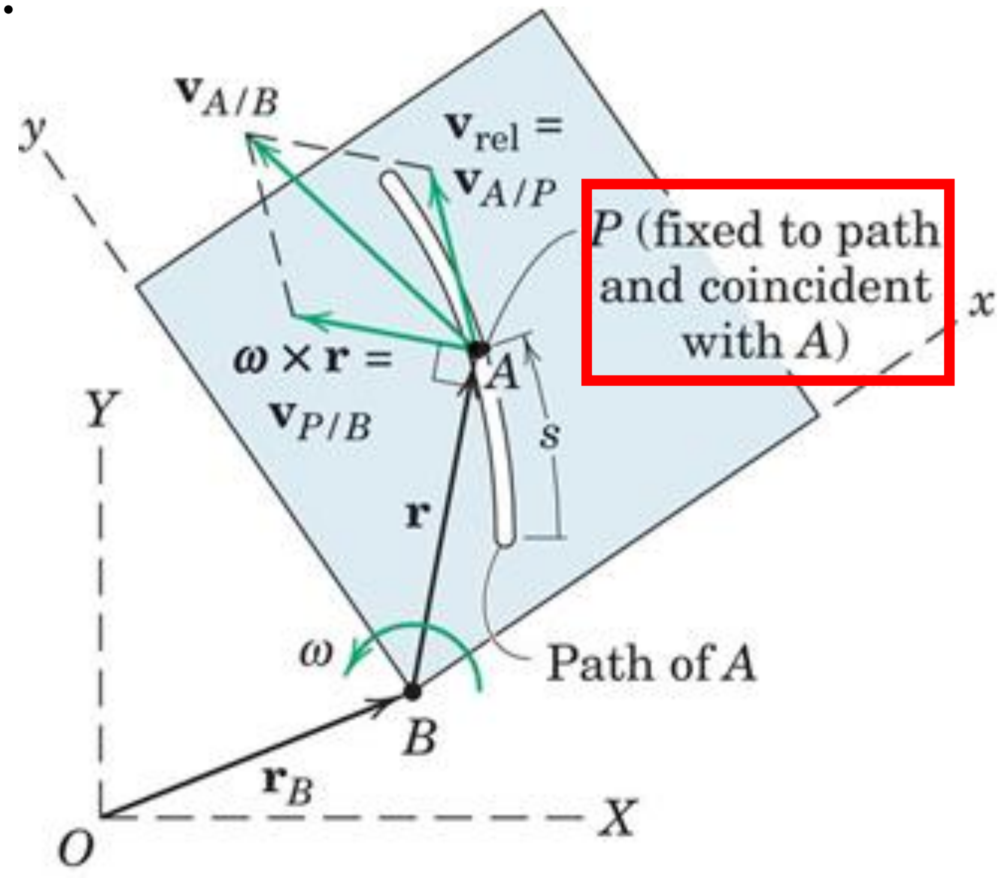
Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A fixed on the moving plane.

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

$$\vec{v}_A = \underbrace{\vec{v}_B + \vec{v}_{P/B}}_{\vec{v}_P} + \vec{v}_{A/P}$$

$$\vec{v}_A = \vec{v}_P + \vec{v}_{A/P}$$



Transformation of a Time Derivative (Transport or Coriolis Theorem)

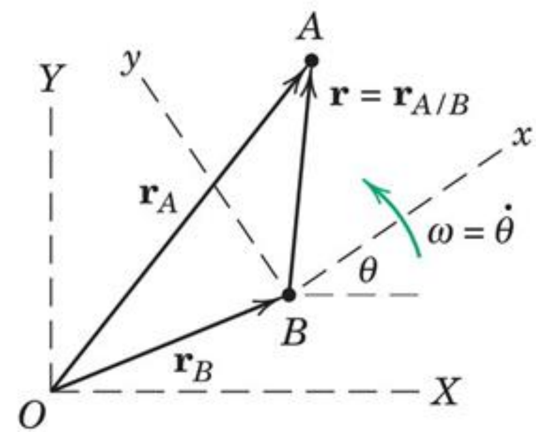
Let $\vec{V}$ be any vector quantity

$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}$

The time derivative of this quantity in X-Y coordinates is

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \underbrace{\left(\dot{V}_x\hat{\imath} + \dot{V}_y\hat{\jmath}\right)}_{} + \underbrace{\left(V_x\dot{\hat{\imath}} + V_y\dot{\hat{\jmath}}\right)}_{}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \left(\frac{d\vec{V}}{dt}\right)_{x-y} + \vec{\omega} \times \vec{V}$$



The term $\vec{\omega} \times \vec{V}$ is the difference between the time derivative of $\vec{V}$ in the rotating (x-y) and non-rotating (X-Y) coordinate frames.

Relative Acceleration

Relative acceleration equation may be obtained either by taking time derivative of relative velocity equation

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

or by applying Transport/Coriolis Theorem.

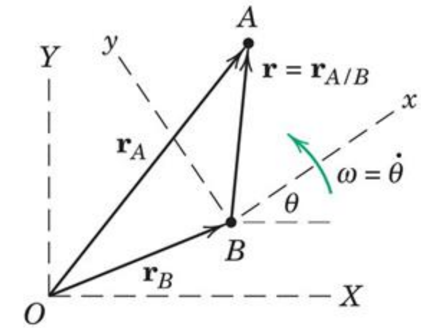
$$\frac{d}{dt}\vec{v}_A = \frac{d}{dt}\vec{v}_B + \frac{d}{dt}(\vec{\omega} \times \vec{r}_{rel}) + \frac{d}{dt}\vec{v}_{rel}$$

$$\boxed{\dot{\hat{\imath}} = \vec{\omega} \times \hat{\imath},\ \dot{\hat{\jmath}} = \vec{\omega} \times \hat{\jmath}}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times \frac{d}{dt}(x\hat{\imath} + y\hat{\jmath}) + \frac{d}{dt}(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath})$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} + \vec{\omega} \times \left[(\dot{x}\hat{\imath} + \dot{y}\hat{\jmath}) + (x\dot{\hat{\imath}} + y\dot{\hat{\jmath}})\right] + (\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}) + (\dot{x}\dot{\hat{\imath}} + \dot{y}\dot{\hat{\jmath}})$$

$$\boxed{\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$$



Relative Acceleration

Relative acceleration equation may be obtained either by taking time derivative of relative velocity equation

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}$$

or by applying Transport/Coriolis Theorem.

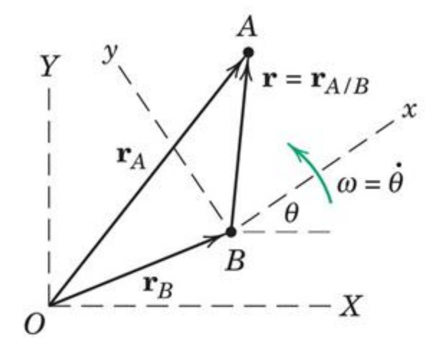
$$\frac{d}{dt}\vec{v}_A = \frac{d}{dt}\vec{v}_B + \frac{d}{dt}(\vec{\omega} \times \vec{r}_{rel}) + \frac{d}{dt}\vec{v}_{rel}$$

$$\left(\frac{d\vec{V}}{dt}\right)_{X-Y} = \vec{\omega} \times \vec{V} + \left(\frac{d\vec{V}}{dt}\right)_{x-y}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}} \times \vec{r}_{rel} + \vec{\omega} \times \dot{\vec{r}}_{rel} + \frac{d}{dt}\vec{v}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{rel} + \vec{v}_{rel}) + (\vec{\omega} \times \vec{v}_{rel} + \dot{\vec{v}}_{rel})$$

$$\boxed{\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha}\times\vec{r}_{rel} - \omega^2\vec{r}_{rel} + 2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}$$

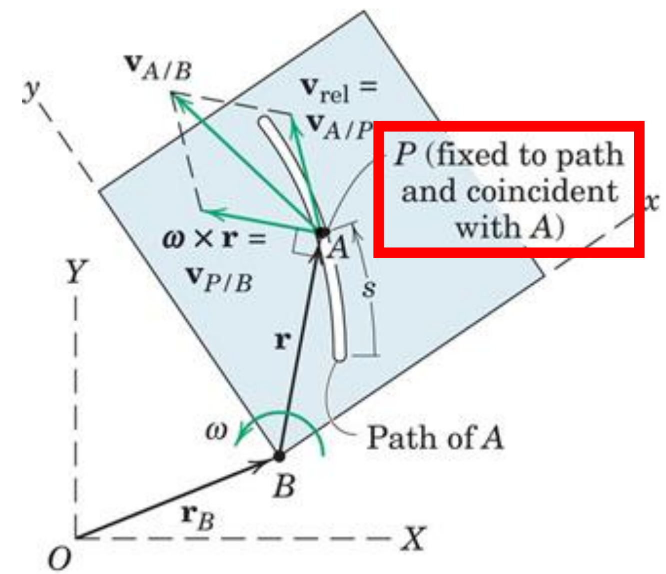
Interpretation of Terms

Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A is fixed on the moving plane.

$$\vec{a}_A = \vec{a}_B + \underbrace{\vec{\alpha}\times\vec{r}_{rel} - \omega^2\vec{r}_{rel}} + \underbrace{2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{P/B} + \vec{a}_{A/P}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

Interpretation of Terms

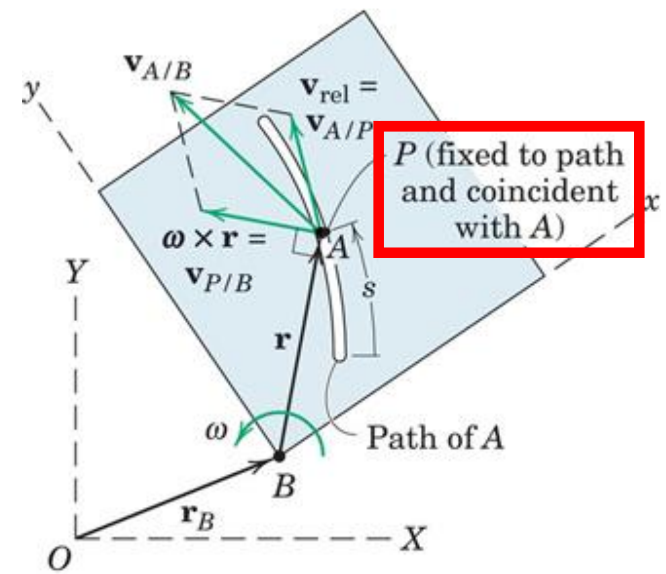
Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A is fixed on the moving plane.

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_A = \underbrace{\vec{a}_B + \vec{a}_{P/B}} + \vec{a}_{A/P}$$

$$\vec{a}_A = \vec{a}_P + \vec{a}_{A/P}$$



$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

Interpretation of Terms

Assume the moving coordinate system x-y be formed of a plate with a slot in it where particle A can move in relative to the moving coordinate system.

Assume a point P, *instantly coincident* with particle A is fixed on the moving plane.

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{rel} - \omega^2 \vec{r}_{rel} + 2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{P/B} + \vec{a}_{A/P}$$

$$\vec{a}_A = \vec{a}_P + \vec{a}_{A/P}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$