ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

ME 208 DYNAMICS

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6/3 Translation

For translation $\omega = \alpha = 0$

$$\sum \vec{F} = m\vec{a}_G$$

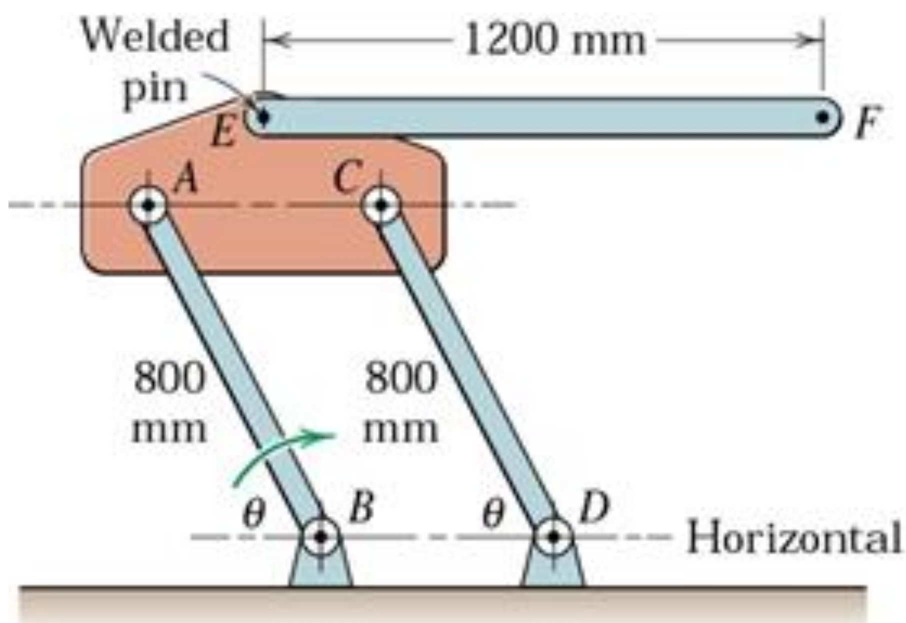
$$\sum M_G = 0$$

However unlike statics since $\vec{a}_G \neq \vec{0}$

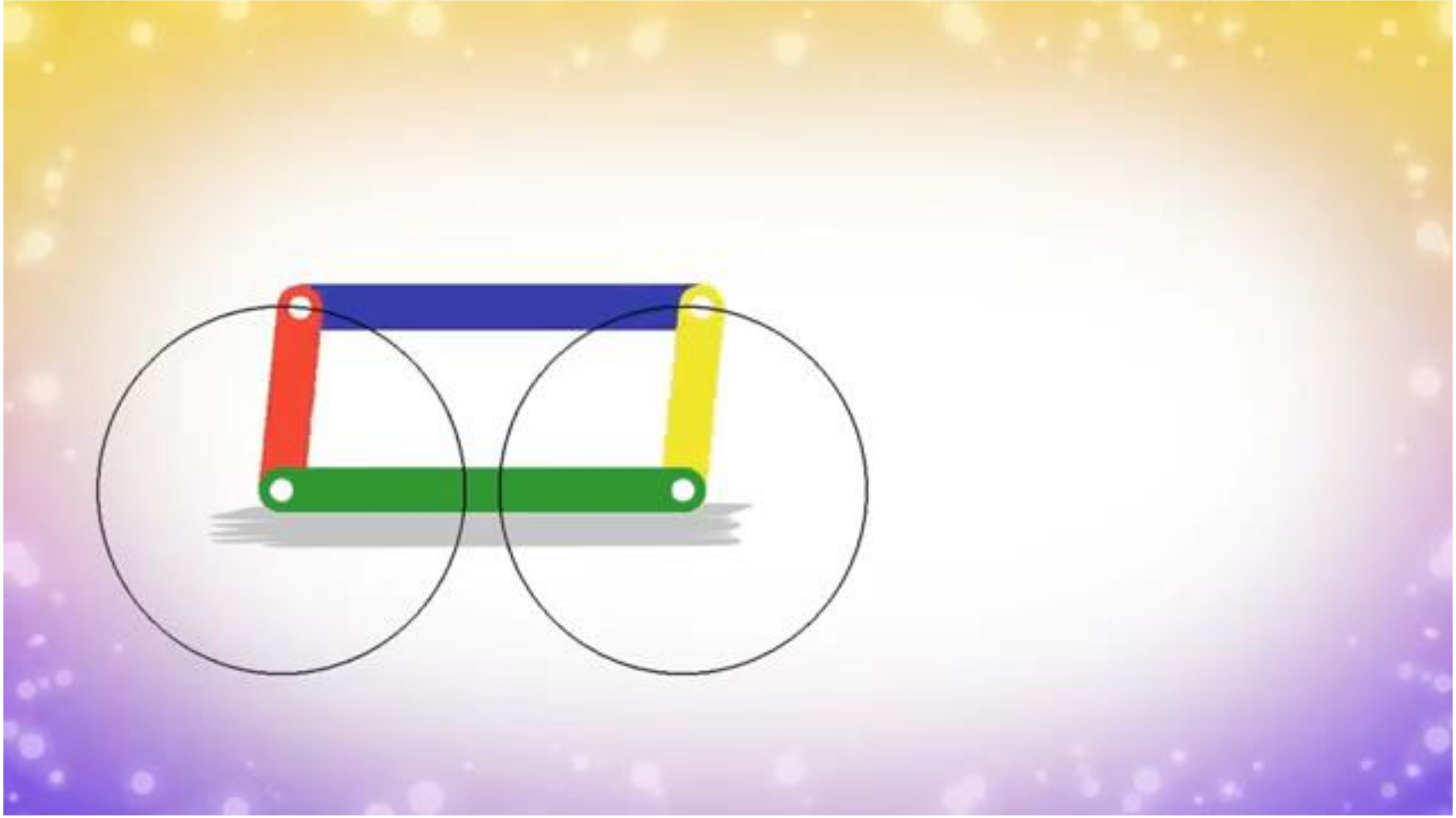
$$\sum M_P = Moment\ of\ m\vec{a}_G\ about\ P \neq 0\ (in\ general)$$

6/24 (4th) 6/23 (5th), None (6th), 6/23 (7th), None (8th)

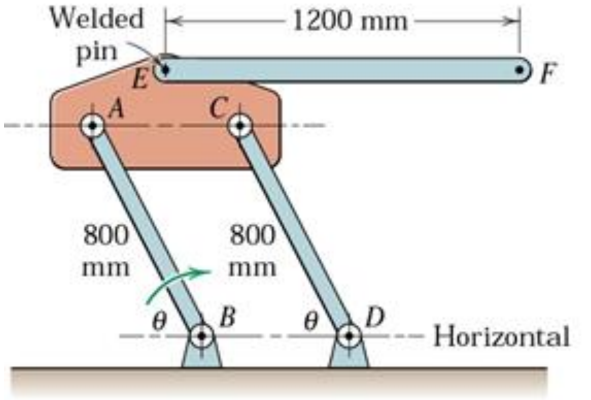
The parallelogram linkage shown moves in the vertical plane with the uniform 8 kg bar EF attached to the plate E by a pin which is welded both to the plate and to the bar. The torque (not shown) is applied to link AB through its lower pin to drive the links in a clockwise direction. When θ reaches 60°, the links have an angular acceleration and angular velocity 6 rad/s^2 and 3 rad/s respectively. For this instant calculate the magnitudes of force F and torque M supported by the pin at E.



In a parallelogram linkage (which is a special type of a four-bar mechanism) the blue body that has no joint with the ground (coupler or floating link) is in translation.



Body ACE therefore EF are in translation without rotation.



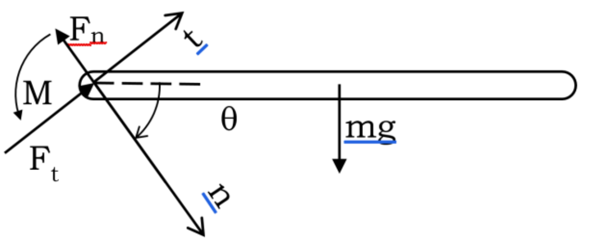
Using n-t (or equally r-θ where r = -n, t = θ)

$$a_{A_n} = r\omega^2 = 0.8 * 3^2 = 7.20\ m/s^2$$

$$a_{A_t} = r\alpha = 0.8 * 6 = 4.80\ m/s^2$$

$$\sum F_t = ma_t$$

$$F_t - 8 * 9.81 cos60° = 8 * 4.8, F_t = 77.6\ N$$



$$\sum F_n = ma_n$$

$$-F_n + 8 * 9.81 sin60° = 8 * 7.2, F_n = -10.36\ N$$

$$F = \sqrt{77.6^2 + 10.36^2} = 78.3\ N$$

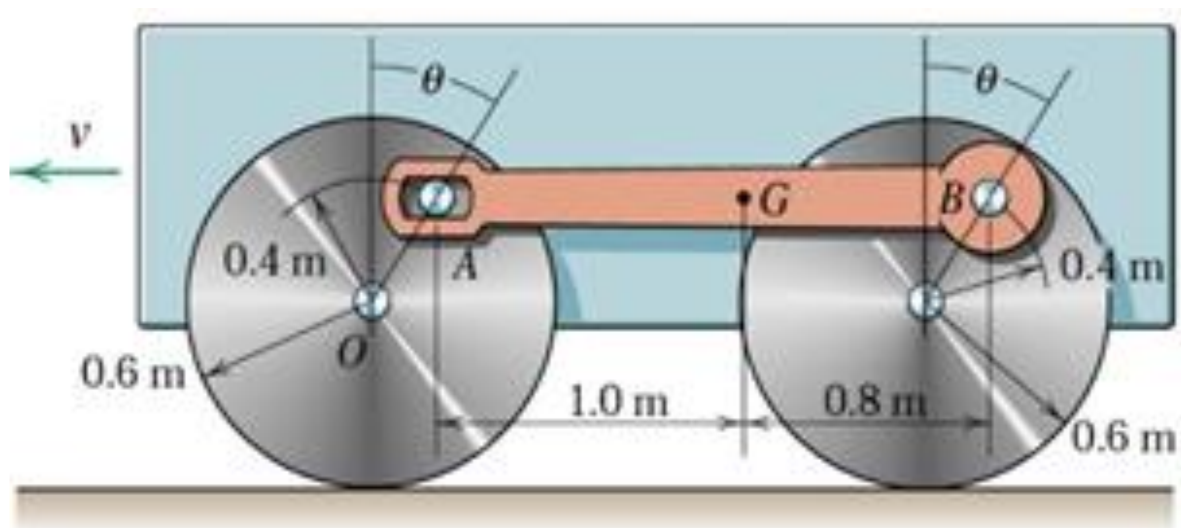
$$\sum M_E = I_G\alpha + moment\ of\ m\vec{a}_G\ about\ E\ \left(or \sum M_G = I_G\alpha = 0\right)$$

$$M - 8 * 9.81 * 0.6 = 8 * 4.8 * 0.6 * cos60° - 8 * 7.2 * 0.6 * sin60°$$

$$M = 28.7\ N.m$$

6/31 (4th, and 5th), None (6th), 6/31 (7th), None (8th)

The two wheels of the vehicle are connected by a 20 kg link AB with center of mass G. The link is pinned to the wheel at B, and the pin A fits into a smooth horizontal slot in the link. If the vehicle has a constant speed of 4 m/s, determine the magnitude of the force supported by pin at B for position $\theta = 30^\circ$.



Original Movie: Buster Keaton – Train (1927)

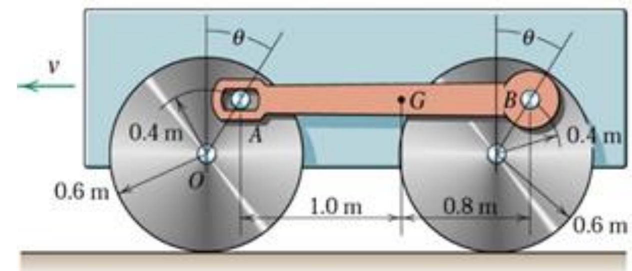
For the wheel

$$\omega = \frac{v}{r} = \frac{4}{0.6} = 6.67\ rad/s\ CCW$$

Point A makes rotation relative to point O and O is in translation with constant velocity therefore $\vec{a}_O = \vec{0}$

Here Cartesian coordinates are simpler to use although n-t (or r-θ) could be used as well.

For the calculation of acceleration of the connecting rod n-t is straight-forward then the calculated acceleration components may be transformed into x-y.



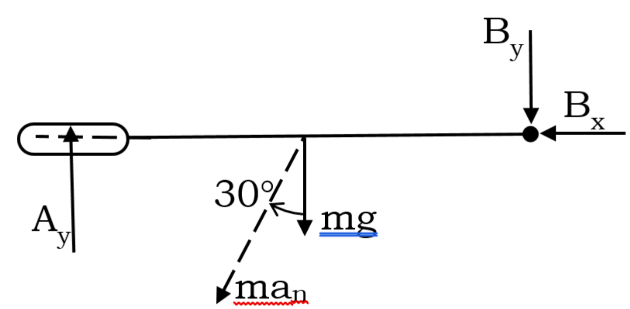
$$a_{A_t} = \alpha r = 0$$

$$a_{A_n} = \omega^2 r = 6.67^2 * 0.4 = 17.78\ m/s^2$$

$$a_x = -17.78 sin30° = -8.89\ m/s^2$$

$$a_y = -17.78 cos30° = -15.40\ m/s^2$$

$$\sum F_x = ma_x$$



$$-B_x = 20 * -8.89, B_x = 177.8\ N$$

$$\sum F_y = ma_y$$

$$A_y - B_y - 9.81 * 20 = 20 * -15.40, A_y - B_y = -111.8\ N$$

$$\sum M_A = I_G\alpha + moment\ of\ m\vec{a}_G\ about\ A\ \left(or \sum M_G = I_G\alpha = 0\right)$$

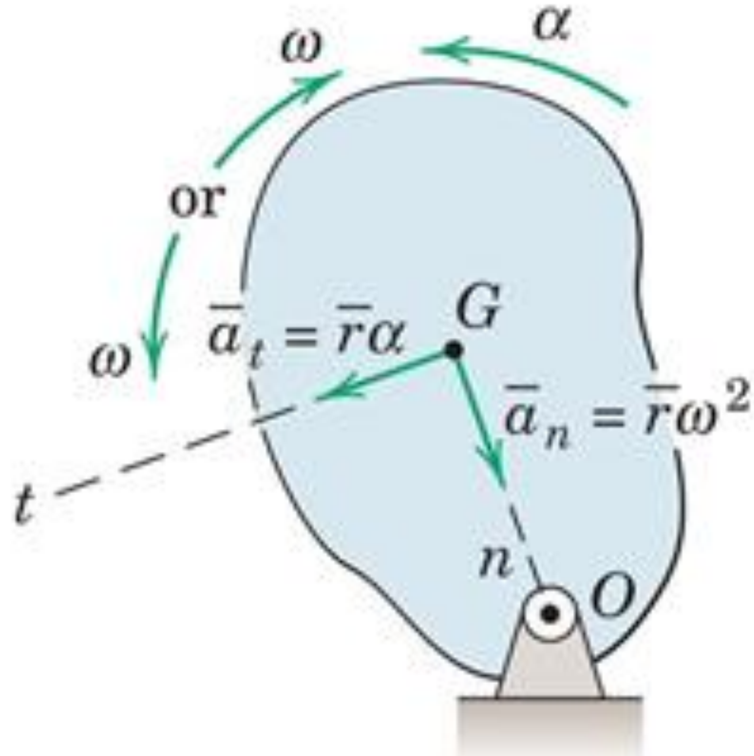
$$1.8B_y + 20 * 9.81 * 1 = 20 * 15.4, B_y = 62.1\ N, B = 188.3\ N$$

6/4 Fixed Axis Rotation

All points on the rigid body follow circular paths centered at the axis of rotation.

$$\sum \vec{F} = m\vec{a}_G \begin{cases} \sum F_n = ma_n \\ \sum F_t = ma_t \end{cases}$$

$$\sum M_G = I_G\alpha$$



OR

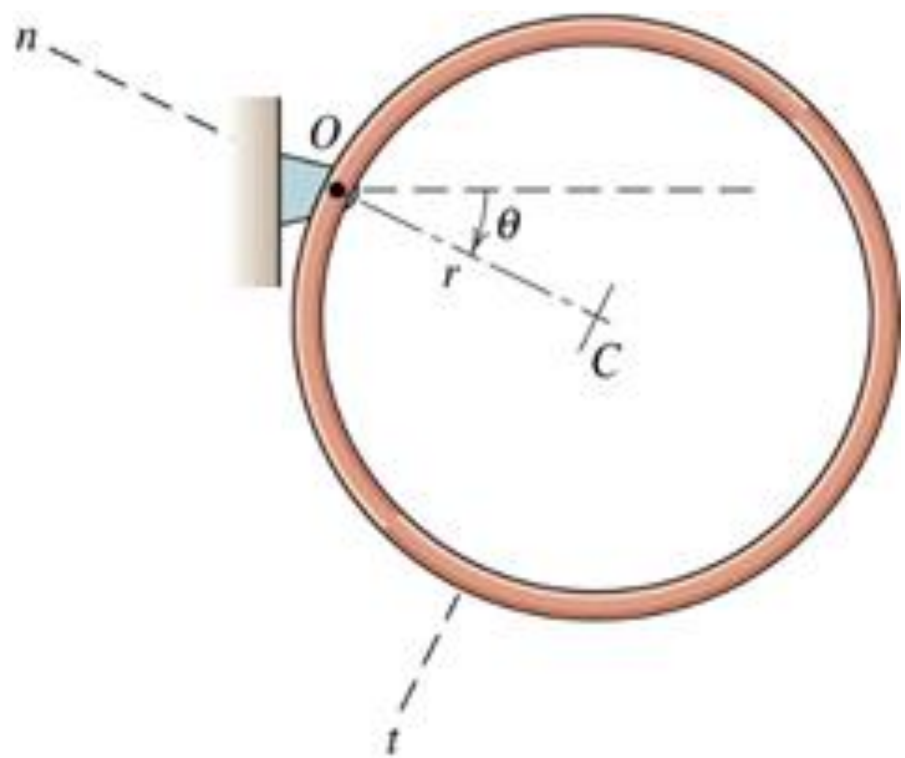
$$\sum M_O = I_O\alpha = I_G\alpha + ma_t r = I_G\alpha + m\alpha r^2 = (I_G + mr^2)\alpha$$

For bodies rotating about G

$$\sum \vec{F} = \vec{0}$$

6/54 (4th), 6/55 (5th), None (6th), 6/47 (7th), None (8th)

The narrow ring of mass m is free to rotate in the vertical plane about O. If the ring is released from rest at $\theta = 0°$, determine the expressions for the n- and t-components of the force at O in terms of θ.



$$\sum F_n = ma_n = m\omega^2 r$$

$$\sum F_t = ma_t = m\alpha r$$

$$\sum M_O = I_O\alpha$$

$$I_O = mr^2 + mr^2$$

$$-mgrcos\theta = 2mr^2\alpha, \alpha = -\frac{g}{2r}cos\theta$$

$$F_t(\theta) + mgcos\theta = m\left(-\frac{g}{2r}cos\theta\right)r$$

$$F_t(\theta) = -\frac{m}{2}gcos\theta$$

$$F_n(\theta) - mgsin\theta = m\omega^2 r$$

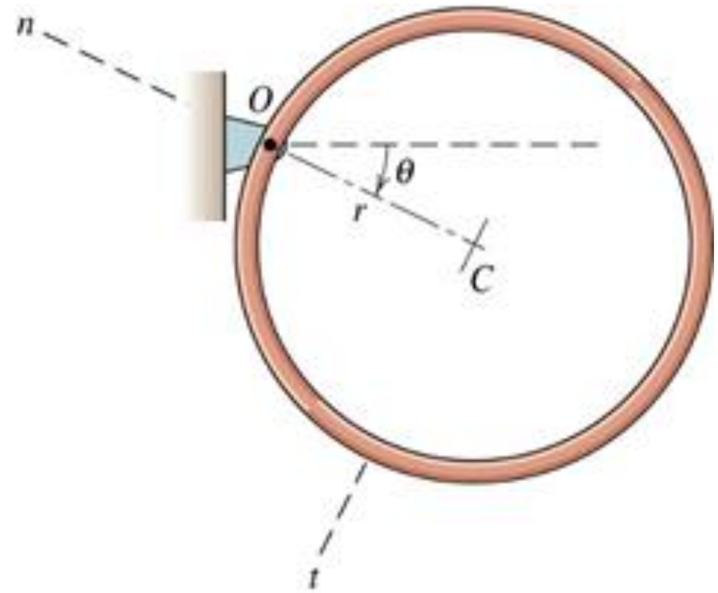
$$\omega d\omega = \alpha d\theta$$

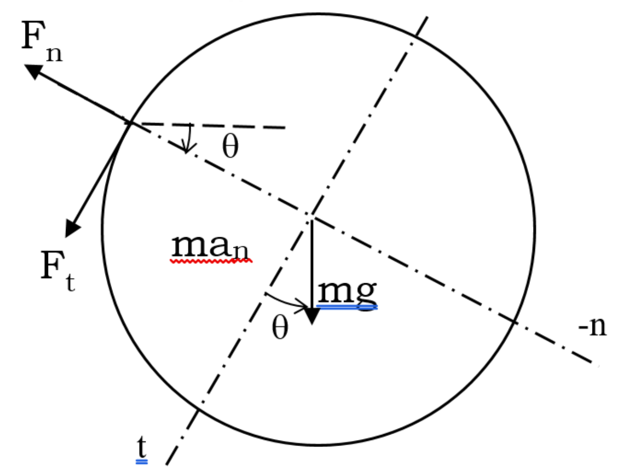
$$\alpha = -\frac{g}{2r}cos\theta$$

$$\int_0^{\omega}\omega d\omega = \int_0^{\theta}\alpha d\theta = \int_0^{\theta} -\frac{g}{2r}cos\theta d\theta$$

$$\omega^2 = \frac{g}{r}sin\theta$$

$$F_n(\theta) = 2mgsin\theta$$





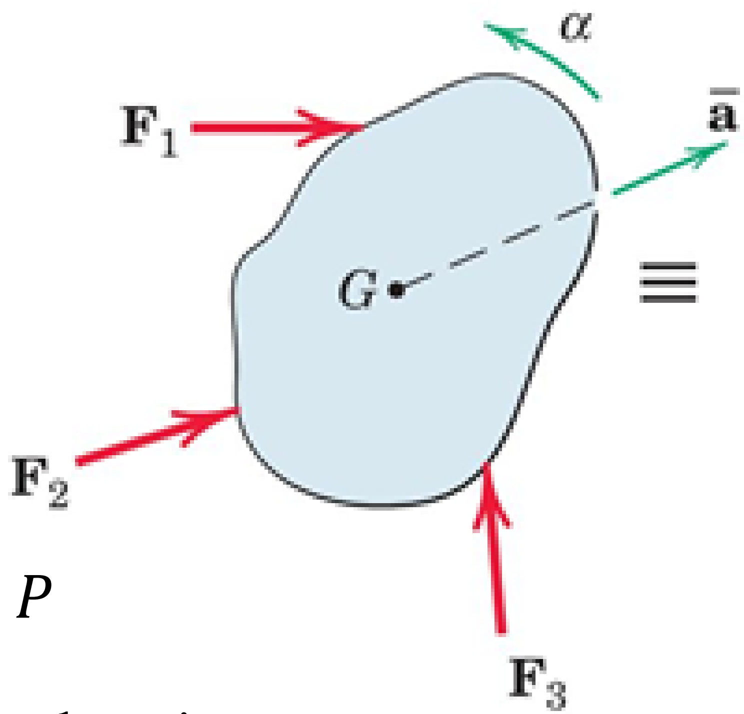
6/5 General Plane Motion

$$\sum \vec{F} = m\vec{a}_G \begin{cases} x - y \\ n - t \\ r - \theta \end{cases}$$

$$\sum \vec{M}_G = I_G \vec{\alpha}$$

OR

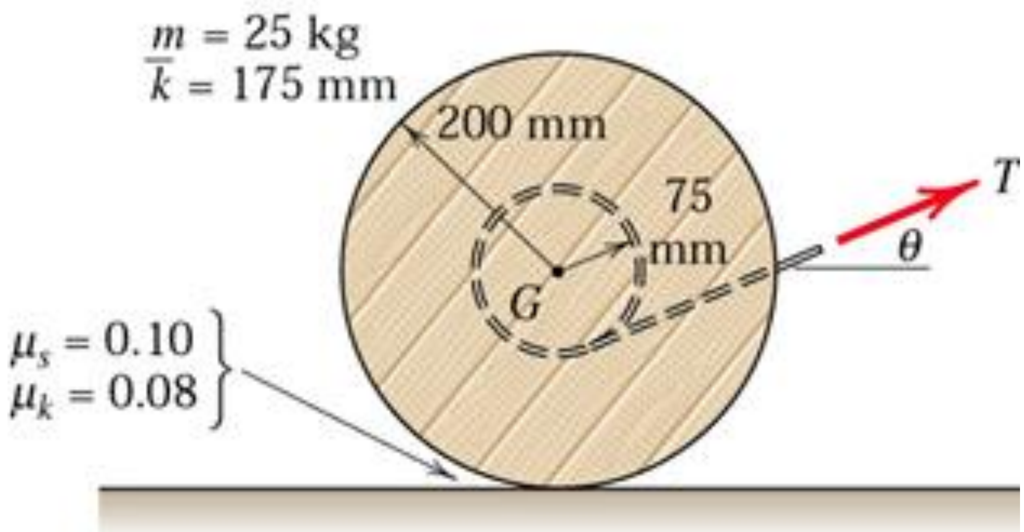
$$\sum M_P = I_G \vec{\alpha} + Moment\ of\ m\vec{a}_G\ about\ P$$



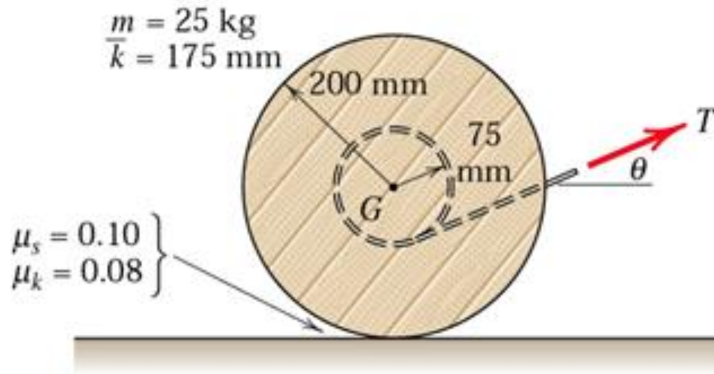
For unconstrained motion the acceleration components are independent (i.e there is no kinematic relation among accelerations), for constrained motion there are constraint equations between acceleration components and reaction forces to impose the constraints.
Use consistent directions (assume unknown acceleration components in positive coordinate directions).
In writing equations any force in positive coordinate direction is positive.
Use consistent action-reaction force pairs.

6/89 (4th), 6/83 (5th), None (6th), 6/88 (7th), None (8th)

The circular disk of 200 mm radius has a mass of 25 kg with centroidal radius of gyration 175 mm and has a concentric circular groove of 75 mm radius cut into it. A steady force T is applied at an angle θ to a cord wrapped around the groove as shown. If T = 30 N, θ = 0, μ_s = 0.10, and μ_k = 0.08, determine the angular acceleration α of the disk, the acceleration, a_G, of its mass center G, and the friction force which the surface exerts on the disk.



$$\sum F_y = 0, N = mg = 25 * 9.81 = 245\, N$$



$$\sum F_x = ma_{G_x}, 30 - F_f = 25a_G$$

$$\sum M_G = I_G\alpha$$

$$0.075 * 30 - 0.2F_f = 25 * 0.175^2\alpha$$

Assuming disk rolls without slipping (i.e. $F_f \leq \mu_s N$)

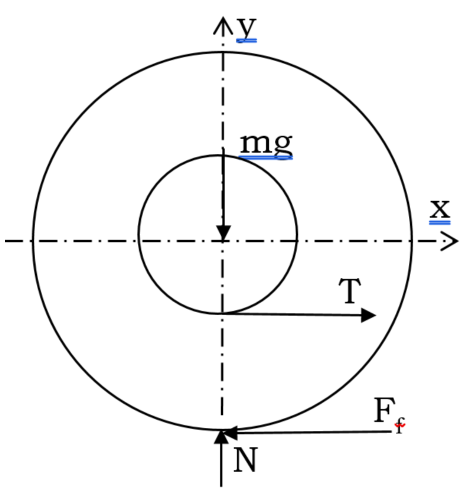
$$a_G = -r\alpha$$

Solution yields

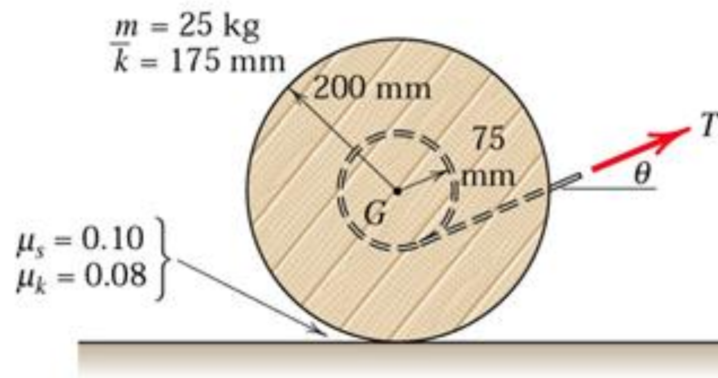
$$a_G = 0.425\, m/s^2, \alpha = -2.12\, rad/s^2$$

$$F_f = 19.38\, N$$

$$\begin{cases} F_{f_{smax}} = 0.1 * 245 = 24.5\, N \\ F_{f_{kmax}} = 0.08 * 245 = 19.62\, N \end{cases}$$



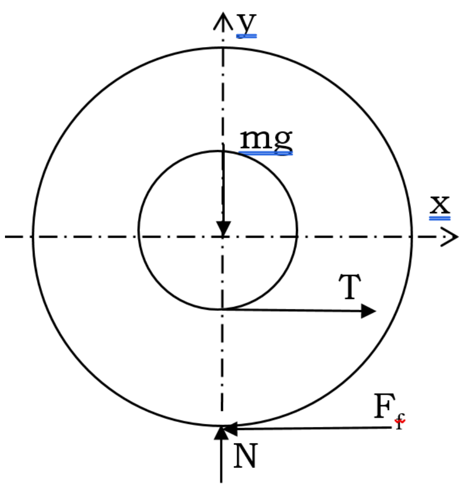
Required friction force is less than the maximum static and kinetic friction forces therefore rolls without slipping.



Let $\mu_s = 0.05$, and $\mu_k = 0.04$

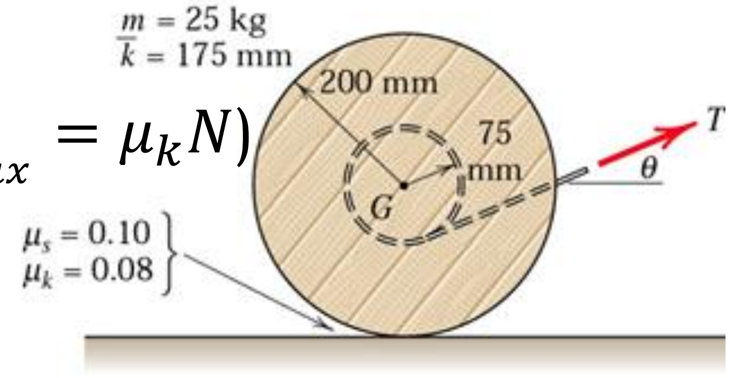
$$\begin{cases} F_{f_{s_{max}}} = 0.05 * 245 = 12.25\,N \\ F_{f_{k_{max}}} = 0.04 * 245 = \ 9.80\,N \end{cases}$$

Required friction force ($F_f = 19.38\,N$) is more than the maximum static and kinetic friction forces therefore disk rolls with slipping!



$$\sum F_y = 0, N = mg = 25 * 9.81 = 245\ N$$

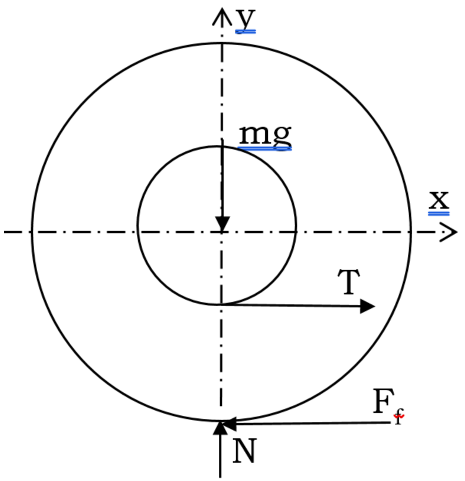
Disk rolls with slipping (i.e. $F_f = F_{f_{kmax}} = \mu_k N$)



$$\sum F_x = ma_{G_x}, 30 - F_{f_{kmax}} = 25a_G\ (1)$$

$$\sum M_G = I_G \alpha$$

$$0.075 * 30 - 0.2F_{f_{kmax}} = 25 * 0.175^2 \alpha\ \ (2)$$



$a_G \neq -r\alpha$ (Slipping!)

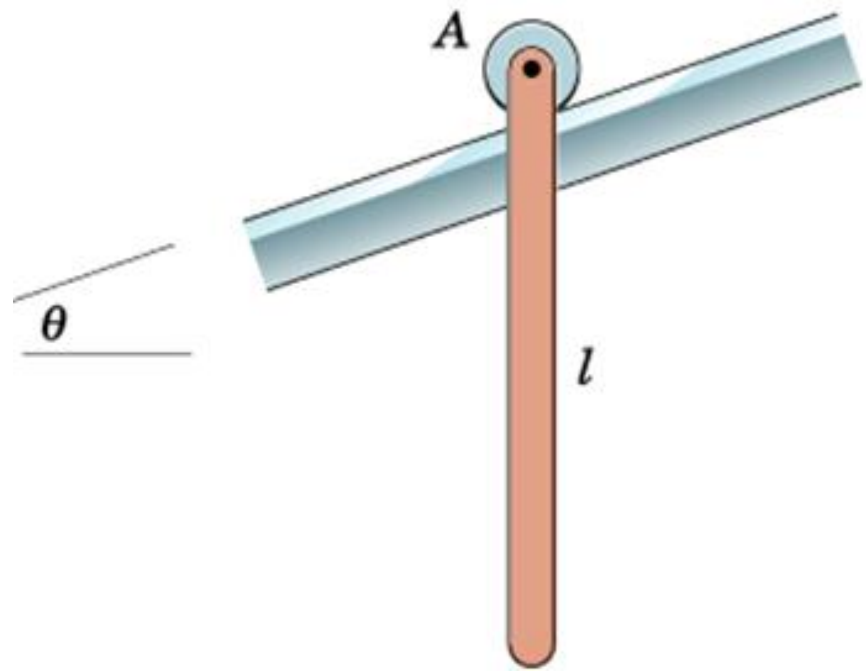
Solution yields

$(1) \rightarrow a_G = 0.808 m/s^2$

$(2) \rightarrow \alpha = 0.379\ rad/s^2$

6/97 (4th), 6/94 (5th), 6/98 (6th), None (7th), None (8th)

The slender rod of mass m and length ℓ is released from rest in the vertical position with the small roller at end A resting on the incline. Determine the initial acceleration of point A.



$$\sum M_A = I_G\alpha + ma_G d$$

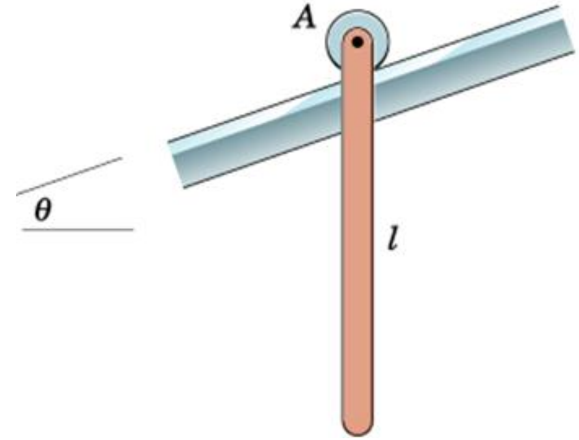
$$a_A = a_G + \frac{\ell}{2}cos\theta\alpha, a_{A/G_n} = 0\ \omega = 0$$

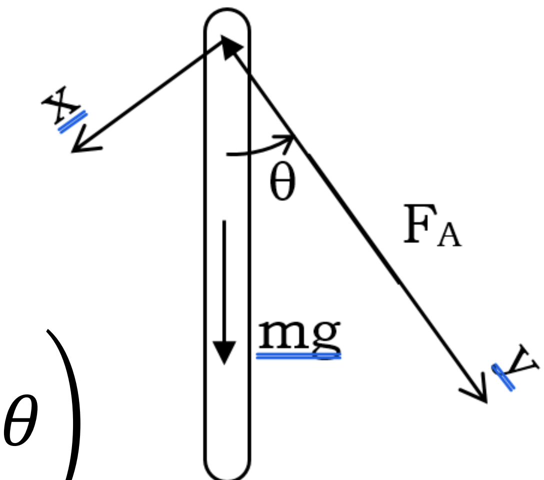
$$0 = \frac{1}{12}m\ell^2\alpha - m\frac{\ell}{2}\alpha\frac{\ell}{2} + ma_A\frac{\ell}{2}cos\theta$$

$$\sum F_y = 0\,, F_A - mgcos\theta = 0$$

$$\sum F_x = ma_{G_x}\,, mgsin\theta = m\left(a_A - \alpha\frac{\ell}{2}cos\theta\right)$$

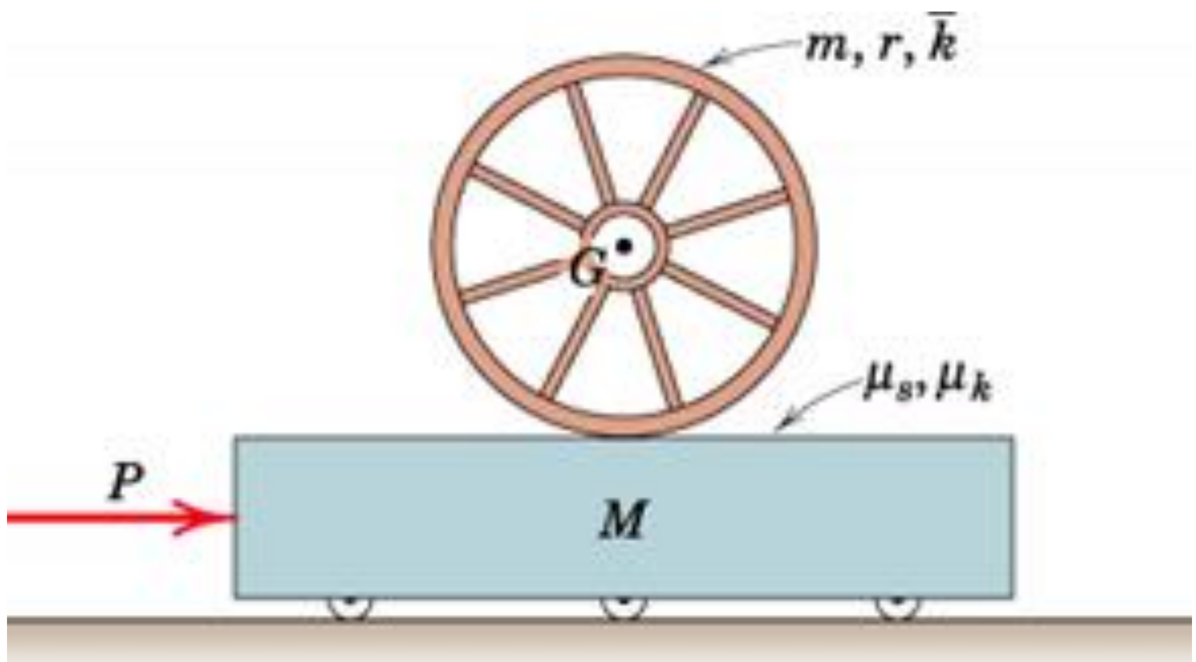
$$a_A = \frac{gsin\theta}{1 - \frac{3}{4}cos^2\theta}$$





6/100 (4th), None (5th), 6/103 (6th), None (7th), None (8th)

Determine the maximum horizontal force P which may be applied to the cart of mass M for which the wheel will not slip as it begins to roll on the cart. The wheel has mass m, rolling radius r, and radius of gyration k. The coefficients of static and kinetic friction between the wheel and the cart are μ_s and μ_k respectively.



No-slip condition at the contact point P is: $a_{P_t} = a_C$

$a_{P_t} = a_C = a_G + r\alpha, \omega = 0$

Also for $P_{max}, F_f = \mu_s N$

For the disk:

$$\sum F_y = 0, N = mg$$

$$\sum F_x = ma_{G_x}, \mu_s mg = ma_{G_x}, a_{G_x} = \mu_s g$$

$$\sum M_G = I_G \alpha$$

$$\mu_s mgr = m{k_G}^2 \alpha, \alpha = \frac{\mu_s gr}{{k_G}^2}$$

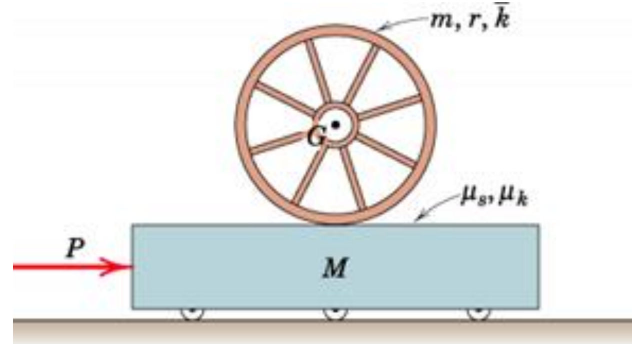
$$a_{P_t} = a_C = \mu_s g + r\frac{\mu_s gr}{{k_G}^2} = \mu_s g\left(1 + \frac{r^2}{{k_G}^2}\right)$$

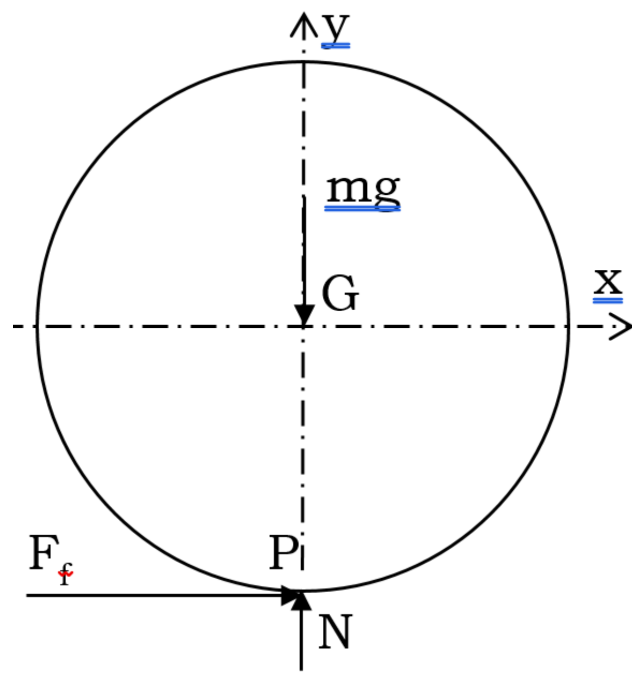
For the cart:

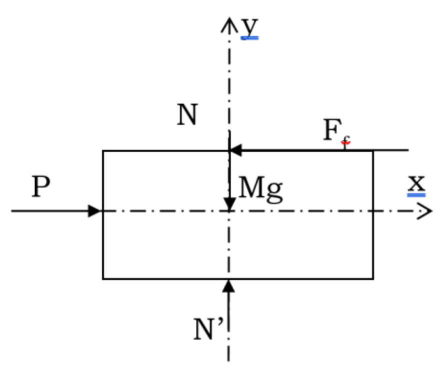
$$\sum F_x = Ma_C$$

$$P - \mu_s mg = M\mu_s g\left(1 + \frac{r^2}{{k_G}^2}\right)$$

$$P = \mu_s g\left[m + M\left(1 + \frac{r^2}{{k_G}^2}\right)\right]$$

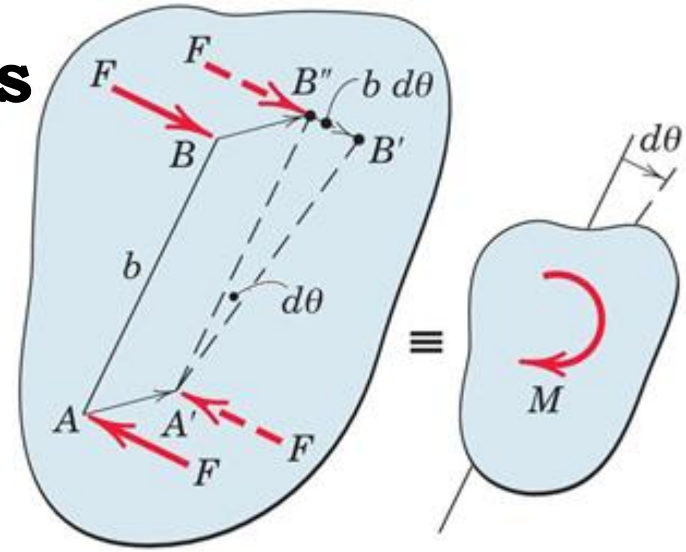






B. Work and Energy

6/6 Work and Energy Relations



Work done by a couple moment is:

$$dU = \vec{F} \cdot d\vec{r} = Fbd\theta = Md\theta$$

$$U = \int_{\theta_1}^{\theta_2} Md\theta$$

Kinetic Energy

- Translation: Since all points on the rigid body have the same velocity

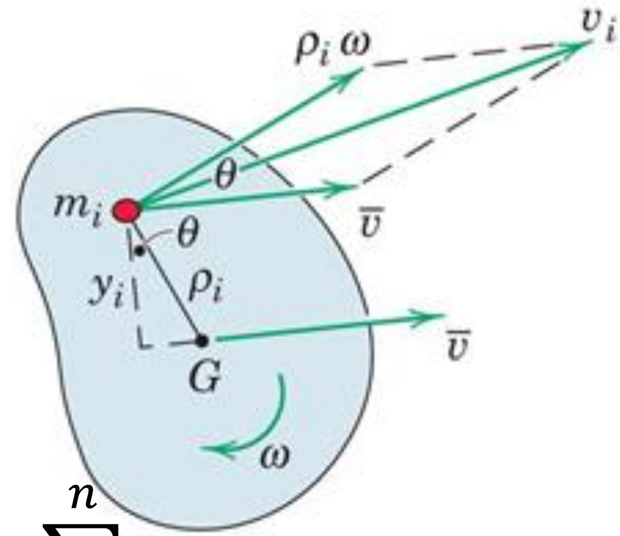
$$T = \frac{1}{2}mv^2$$

- Fixed Axis Rotation:

$$T_i = \frac{1}{2}m_i {v_i}^2 = \frac{1}{2}m_i(r_i\omega)^2$$

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2}m_i(r_i\omega)^2 = \frac{\omega^2}{2}\sum_{i=1}^{n} m_i {r_i}^2 = \frac{1}{2}I_O\omega^2$$

- <u>General Plane Motion</u>:



$$T_i = \frac{1}{2} m_i {v_i}^2$$

$${v_i}^2 = {v_G}^2 + {\rho_i}^2 \omega^2 + 2 v_G \rho_i \omega cos\theta_i$$

$$\sum_{i=1}^{n} m_i v_G \rho_i \omega cos\theta_i = v_G \omega \sum_{i=1}^{n} m_i \rho_i cos\theta_i = v_G \omega \sum_{i=1}^{n} m_i y_i = 0$$

$$T = \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \frac{1}{2} m_i {v_G}^2 + \sum_{i=1}^{n} \frac{1}{2} m_i {\rho_i}^2 \omega^2$$

$$= \frac{1}{2} {v_G}^2 \sum_{i=1}^{n} m_i + \frac{1}{2} \omega^2 \sum_{i=1}^{n} m_i {\rho_i}^2 = \frac{1}{2} m {v_G}^2 + \frac{1}{2} I_G \omega^2$$

Work-Energy Equation:

$$U_{1\rightarrow 2} = \Delta T + \Delta V_g + \Delta V_e$$

Power

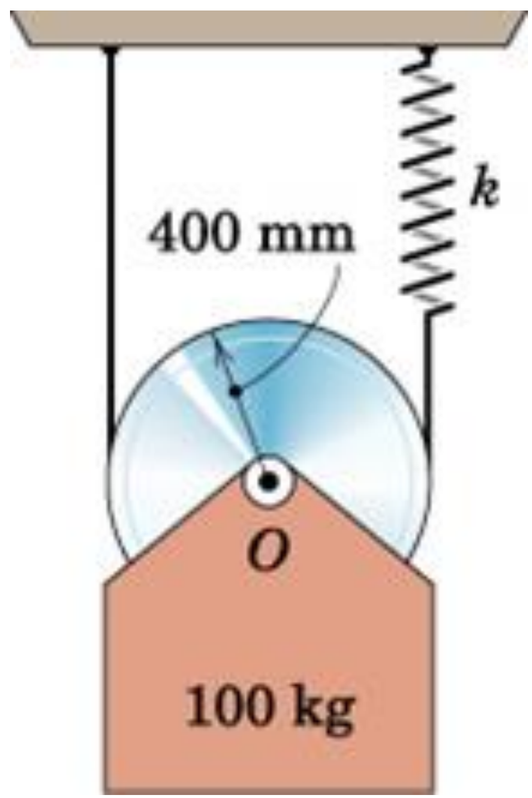
$$\mathbb{P} = \frac{dU}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$\mathbb{P} = \frac{dU}{dt} = M\frac{d\theta}{dt} = M \cdot \omega$$

$$\mathbb{P} = \frac{dU}{dt} = \frac{d}{dt}(T + V_G + V_E)$$

6/143 (4th), 6/134 (5th), 6/140 (6th), None (7th), 6/141 (8th)

The sheave of 400 mm radius has a mass of 50 kg and a radius of gyration of 300 mm. The sheave and its 100 kg load are suspended by the cable and the spring, which has a stiffness of 1.5 kN/m. If the system is released from rest with the spring initially stretched 100 mm, determine the velocity of O after it has dropped 50 mm.



$$U_{1\to2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to2} = 0$$

$$\Delta T = \sum \frac{1}{2} m {v_G}^2 + \frac{1}{2} I_G \omega^2 - 0$$

$$\Delta T = \frac{1}{2}(100 + 50){v_G}^2 + \frac{1}{2} 50 * 0.3^2 \omega^2$$

From kinematics, $v_G = 0.4\omega$

$$\Delta T = 89.1{v_G}^2$$

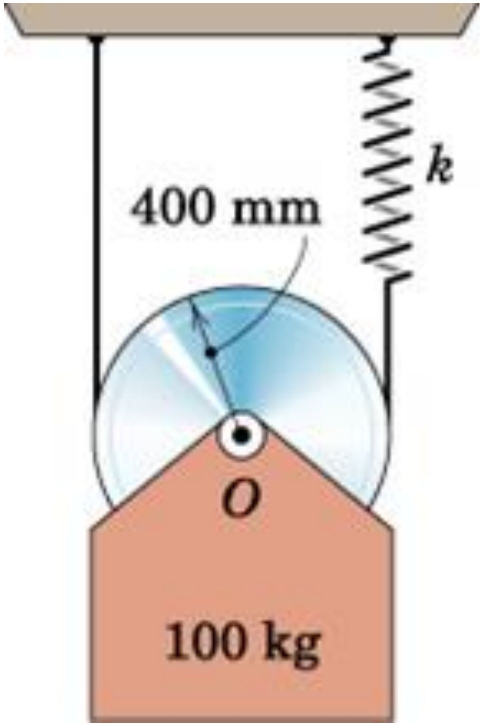
$$\Delta V_g = mg\Delta h = (100 + 50)9{,}81 * (-0.05) = -73.6\,J$$

$$\Delta V_e = \frac{1}{2} k({x_2}^2 - {x_1}^2) = \frac{1}{2} 1500[(0.1 + 2 * 0.05)^2 - 0.1^2]$$

$$\Delta V_e = 22.5\,J$$

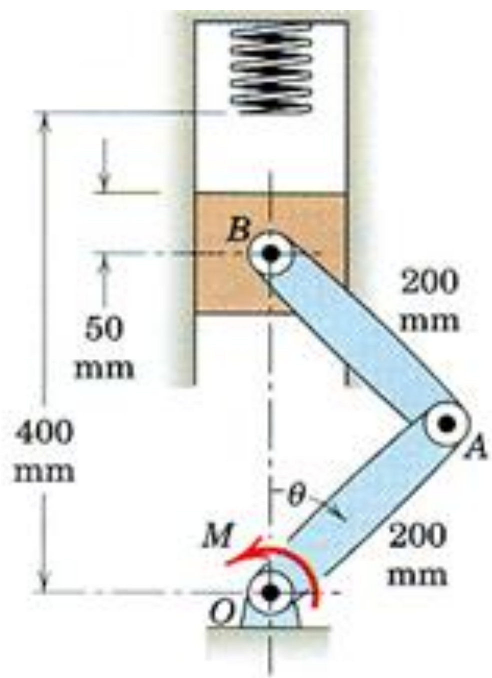
Substitution into work-energy equation yields:

$$v_G = v_O = 0.757\,m/s$$



6/143 (4th), None (5th), None (6th), 6/135 (7th)

Each of the two links has a mass of 2 kg and a centroidal radius of gyration of 60 mm. The slider B has a mass of 3 kg and moves freely in the vertical guide. The spring has a stiffness of 6 kN/m. If a constant torque, M = 20 N.m is applied to link OA through its shaft at O starting from the rest position at $\theta = 45°$, determine the angular velocity ω of OA when $\theta = 0$.



$$U_{1\to 2} = \Delta T + \Delta V_g + \Delta V_e$$

$$U_{1\to 2} = M\Delta\theta = 20 * 45\frac{\pi}{180} = 5\pi\ J$$

$$\Delta V_g = \sum mg\Delta h$$

$$= 2 * 9.81 * 0.1(1 - cos45°) + 2 * 9.81 * 0.3(1 - cos45°) + 3 * 9.81 * 0.4(1 - cos45°)$$
$$= 5.75\ J$$

$$\Delta V_e = \frac{1}{2}kx^2 = \frac{1}{2}6000(0.05)^2 = 7.5\ J$$

For final kinetic energy please recognize that the piston is momentarily at rest in position 2 therefore

$$\omega_{OA} = \omega_{AB} = \frac{v_A}{0.2}$$

$$\Delta T = \frac{1}{2}2I_O\omega^2 = \frac{1}{2}m\left({k_G}^2 + 0.1^2\right)\omega^2 + \frac{1}{2}50 * 0.3^2\omega^2$$
$$= 0.0136\omega^2$$
$$\omega = 9.51\ rad/s$$