2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

θ_{12} is the input and s_{16} is the output.

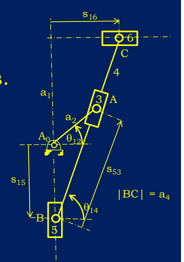
Disconnect A and C to eliminate loops.

Reconnecting A (C disconnected):

$\overrightarrow{A_0A_2} = \overrightarrow{A_0B} + \overrightarrow{BA_3}$

Reconnecting C (A disconnected):

$\overrightarrow{A_0B} + \overrightarrow{BC_4} = \overrightarrow{A_0C_6}$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

$\overrightarrow{A_0A_2} = \overrightarrow{A_0B} + \overrightarrow{BA_3}$

$a_2e^{i\theta_{12}} = -is_{15} + s_{53}e^{i\theta_{14}}$

Re: $a_2cos\theta_{12} = s_{53}cos\theta_{14}$

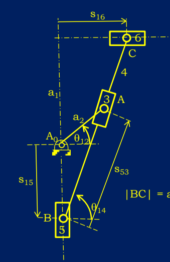
Im: $a_2sin\theta_{12} = -s_{15} + s_{53}sin\theta_{14}$

$\overrightarrow{A_0B} + \overrightarrow{BC_4} = \overrightarrow{A_0C_6}$

$-is_{15} + a_4e^{i\theta_{14}} = ia_1 + s_{16}$

Re: $a_4cos\theta_{14} = s_{16}$

Im: $-s_{15} + a_4sin\theta_{14} = a_1$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

θ_{12} is the input and s_{16} is the output.

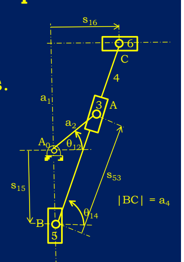
Disconnect A and B to eliminate loops.

Reconnecting A (B disconnected):

$\overrightarrow{A_0A_2} = \overrightarrow{A_0C} + \overrightarrow{CA_3}$

Reconnecting B (A disconnected):

$\overrightarrow{A_0B_5} = \overrightarrow{A_0C} + \overrightarrow{CB_4}$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

$\overrightarrow{A_0A_2} = \overrightarrow{A_0C} + \overrightarrow{CA_3}$

$a_2e^{i\theta_{12}} = ia_1 + s_{16} + (a_4 - s_{53})e^{i(\theta_{14}-\pi)}$

Re: $a_2cos\theta_{12} = s_{16} + (a_4 - s_{53})cos(\theta_{14} - \pi)$

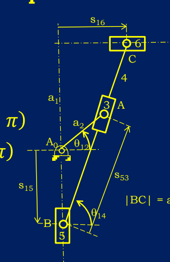
Im: $a_2sin\theta_{12} = a_1 + (a_4 - s_{53})sin(\theta_{14} - \pi)$

$\overrightarrow{A_0B_5} = \overrightarrow{A_0C} + \overrightarrow{CB_4}$

$-is_{15} = ia_1 + s_{16} + a_4e^{i(\theta_{14}-\pi)}$

Re: $0 = s_{16} + a_4cos(\theta_{14} - \pi)$

Im: $-s_{15} = a_1 + a_4sin(\theta_{14} - \pi)$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

θ_{12} is the input and s_{16} is the output.

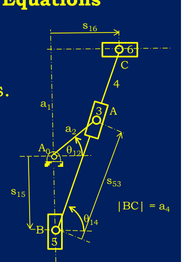
Disconnect B and C to eliminate loops.

Reconnecting B (C disconnected):

$\overrightarrow{A_0B_5} = \overrightarrow{A_0A} + \overrightarrow{AB_4}$

Reconnecting C (B disconnected):

$\overrightarrow{A_0A} + \overrightarrow{AC_4} = \overrightarrow{A_0C_6}$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism

$\overrightarrow{A_0B_5} = \overrightarrow{A_0A} + \overrightarrow{AB_4}$

$-is_{15} = a_2e^{i\theta_{12}} + s_{53}e^{i(\theta_{14}-\pi)}$

Re: $0 = a_2cos\theta_{12} + s_{53}cos(\theta_{14} - \pi)$

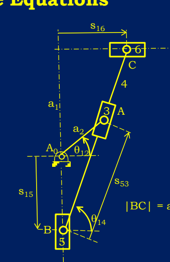
Im: $s_{15} = a_2sin\theta_{12} + s_{53}sin(\theta_{14} - \pi)$

$\overrightarrow{A_0A} + \overrightarrow{AC_4} = \overrightarrow{A_0C_6}$

$a_2e^{i\theta_{12}} + (a_4 - s_{53})e^{i\theta_{14}} = ia_1 + s_{16}$

Re: $a_2cos\theta_{12} + (a_4 - s_{53})cos\theta_{14} = s_{16}$

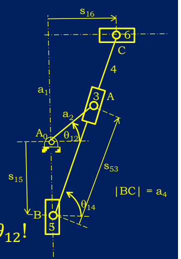
Im: $a_2sin\theta_{12} + (a_4 - s_{53})sin\theta_{14} = a_1$



2. Kinematic Analysis

4. Numerical Solution of Loop Closure Equations

Quick Return Mechanism



Whatever we try:

In total four equations with four unknowns however coupled for *input* θ_{12}!

a. Try numerical solution.

b. Assume s_{16} is known, solve backwards in closed form.

2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

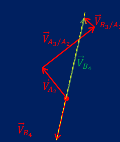
Inverted Slider-Crank Mechanism

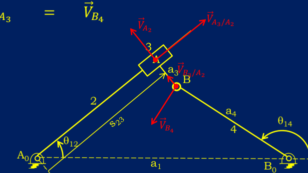
$s_{23}e^{i\theta_{12}} + a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = a_1 + a_4 e^{i\theta_{14}}$

Since loop closure equation is a constrain equation its time derivatives relate velocity and accelerations!

$i\dot{\theta}_{12}s_{23}e^{i\theta_{12}} + \dot{s}_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = i\dot{\theta}_{14}a_4 e^{i\theta_{14}}$

$\vec{V}_{A_2} + \vec{V}_{A_3/A_2} + \vec{V}_{B_3/A_3} = \vec{V}_{B_4}$





2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

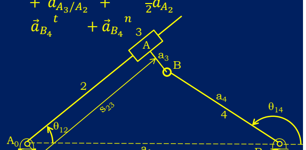
Inverted Slider-Crank Mechanism

$i\dot{\theta}_{12}s_{23}e^{i\theta_{12}} + \dot{s}_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = i\dot{\theta}_{14}a_4 e^{i\theta_{14}}$

$\vec{V}_{A_2} + \vec{V}_{A_3/A_2} + \vec{V}_{B_3/A_3} = \vec{V}_{B_4}$

$i\ddot{\theta}_{12}s_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}\dot{s}_{23}e^{i\theta_{12}} - \dot{\theta}_{12}{}^2 s_{23}e^{i\theta_{12}} + \ddot{s}_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}\dot{s}_{23}e^{i\theta_{12}}$
$+ i\ddot{\theta}_{12}a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} - \dot{\theta}_{12}{}^2 a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = i\ddot{\theta}_{14}a_4 e^{i\theta_{14}} - \dot{\theta}_{14}{}^2 a_4 e^{i\theta_{14}}$

$\vec{a}_{A_2}{}^t + \frac{1}{2}\vec{a}_{A_2}{}^C + \vec{a}_{A_2}{}^n + \vec{a}_{A_3/A_2} + \frac{1}{2}\vec{a}_{A_2}{}^C$
$+ \vec{a}_{B_3/A_3}{}^t + \vec{a}_{B_3/A_3}{}^n = \vec{a}_{B_4}{}^t + \vec{a}_{B_4}{}^n$



2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

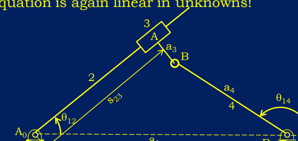
Inverted Slider-Crank Mechanism

$i\dot{\theta}_{12}s_{23}e^{i\theta_{12}} + \dot{s}_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = i\dot{\theta}_{14}a_4 e^{i\theta_{14}}$

Given one velocity (say $\dot{\theta}_{12}$) the others ($\dot{s}_{23}$ and $\dot{\theta}_{14}$) can be determined (recall F = 1). Please recognize the equation is linear in unknowns!

$i\ddot{\theta}_{12}s_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}\dot{s}_{23}e^{i\theta_{12}} - \dot{\theta}_{12}{}^2 s_{23}e^{i\theta_{12}} + \ddot{s}_{23}e^{i\theta_{12}} + i\dot{\theta}_{12}\dot{s}_{23}e^{i\theta_{12}}$
$+ i\ddot{\theta}_{12}a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} - \dot{\theta}_{12}{}^2 a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = i\ddot{\theta}_{14}a_4 e^{i\theta_{14}} - \dot{\theta}_{14}{}^2 a_4 e^{i\theta_{14}}$

Given one acceleration (say $\ddot{\theta}_{12}$) the others ($\ddot{s}_{23}$ and $\ddot{\theta}_{14}$) can be determined (recall F = 1). Please recognize the equation is again linear in unknowns!



2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

Inverted Slider-Crank Mechanism

$s_{23}e^{i\theta_{12}} + a_3 e^{i\left(\theta_{12}-\frac{\pi}{2}\right)} = a_1 + a_4 e^{i\theta_{14}}$

Re: $s_{23}cos\theta_{12} + a_3 sin\theta_{12} = a_1 + a_4 cos\theta_{14}$ (1)

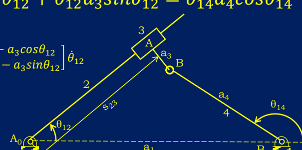
Im: $s_{23}sin\theta_{12} - a_3 cos\theta_{12} = a_4 sin\theta_{14}$ (2)

$(\dot{1})$: $\dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}s_{23}sin\theta_{12} + \dot{\theta}_{12}a_3 cos\theta_{12} = -\dot{\theta}_{14}a_4 sin\theta_{14}$

$(\dot{2})$: $\dot{s}_{23}sin\theta_{12} + \dot{\theta}_{12}s_{23}cos\theta_{12} + \dot{\theta}_{12}a_3 sin\theta_{12} = \dot{\theta}_{14}a_4 cos\theta_{14}$

Let $\dot{\theta}_{12}$ be the input:

$\begin{bmatrix} cos\theta_{12} & a_4 sin\theta_{14} \\ sin\theta_{12} & -a_4 cos\theta_{14} \end{bmatrix} \begin{bmatrix} \dot{s}_{23} \\ \dot{\theta}_{14} \end{bmatrix} = \begin{bmatrix} s_{23}sin\theta_{12} - a_3 cos\theta_{12} \\ -s_{23}cos\theta_{12} - a_3 sin\theta_{12} \end{bmatrix} \dot{\theta}_{12}$



2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

Inverted Slider-Crank Mechanism

$(\dot{1})$: $\dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}s_{23}sin\theta_{12} + \dot{\theta}_{12}a_3 cos\theta_{12} = -\dot{\theta}_{14}a_4 sin\theta_{14}$

$(\dot{2})$: $\dot{s}_{23}sin\theta_{12} + \dot{\theta}_{12}s_{23}cos\theta_{12} + \dot{\theta}_{12}a_3 sin\theta_{12} = \dot{\theta}_{14}a_4 cos\theta_{14}$

$(\ddot{1})$: $\ddot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \ddot{\theta}_{12}s_{23}sin\theta_{12} - \dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}cos\theta_{12} + \ddot{\theta}_{12}a_3 cos\theta_{12} - \dot{\theta}_{12}{}^2 a_3 sin\theta_{12}$
$= -\ddot{\theta}_{14}a_4 sin\theta_{14} - \dot{\theta}_{14}{}^2 a_4 cos\theta_{14}$

$(\ddot{2})$: $\ddot{s}_{23}sin\theta_{12} + \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} + \ddot{\theta}_{12}s_{23}cos\theta_{12} + \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}sin\theta_{12} + \ddot{\theta}_{12}a_3 sin\theta_{12} + \dot{\theta}_{12}{}^2 a_3 cos\theta_{12}$
$= \ddot{\theta}_{14}a_4 cos\theta_{14} + \dot{\theta}_{14}{}^2 a_4 sin\theta_{14}$

Let $\ddot{\theta}_{12}$ be the input:

$\begin{bmatrix} cos\theta_{12} & a_4 sin\theta_{14} \\ sin\theta_{12} & -a_4 cos\theta_{14} \end{bmatrix} \begin{bmatrix} \ddot{s}_{23} \\ \ddot{\theta}_{14} \end{bmatrix} = -\begin{bmatrix} -\dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \ddot{\theta}_{12}s_{23}sin\theta_{12} - \dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}cos\theta_{12} + \ddot{\theta}_{12}a_3 cos\theta_{12} - \dot{\theta}_{12}{}^2 a_3 sin\theta_{12} + \dot{\theta}_{14}{}^2 a_4 cos\theta_{14} \\ \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} + \ddot{\theta}_{12}s_{23}cos\theta_{12} + \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}sin\theta_{12} + \ddot{\theta}_{12}a_3 sin\theta_{12} + \dot{\theta}_{12}{}^2 a_3 cos\theta_{12} - \dot{\theta}_{14}{}^2 a_4 sin\theta_{14} \end{bmatrix}$

$[A]$

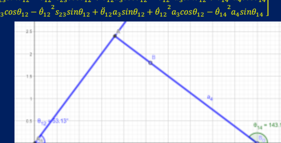
$\begin{bmatrix} cos\theta_{12} & a_4 sin\theta_{14} \\ sin\theta_{12} & -a_4 cos\theta_{14} \end{bmatrix} \begin{bmatrix} \dot{s}_{23} \\ \dot{\theta}_{14} \end{bmatrix} = \begin{bmatrix} s_{23}sin\theta_{12} - a_3 cos\theta_{12} \\ -s_{23}cos\theta_{12} - a_3 sin\theta_{12} \end{bmatrix} \dot{\theta}_{12}$

$det[A] = |A| \neq 0$, there is a solution

$det[A] = -a_4 cos\theta_{12}cos\theta_{14} - a_4 sin\theta_{12}sin\theta_{14}$

$det[A] = -a_4 \cos(\theta_{14} - \theta_{12})$

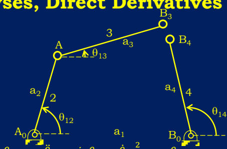
$det[A] = 0$ when $\theta_{14} - \theta_{12} = \frac{\pi}{2}, \frac{3\pi}{2}$



2. Kinematic Analysis

Velocity and Acceleration Analyses, Direct Derivatives

Four-Bar Mechanism



$a_2e^{i\theta_{12}} + a_3e^{i\theta_{13}} = a_1 + a_4e^{i\theta_{14}}$

Re: $a_2cos\theta_{12} + a_3cos\theta_{13} = a_1 + a_4cos\theta_{14}$ (1)

Im: $a_2sin\theta_{12} + a_3sin\theta_{13} = a_4sin\theta_{14}$ (2)

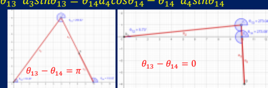
$(\dot{1})$: $-\dot{\theta}_{12}a_2sin\theta_{12} - \dot{\theta}_{13}a_3sin\theta_{13} = -\dot{\theta}_{14}a_4sin\theta_{14}$

$(\dot{2})$: $\dot{\theta}_{12}a_2cos\theta_{12} + \dot{\theta}_{13}a_3cos\theta_{13} = \dot{\theta}_{14}a_4cos\theta_{14}$

$(\ddot{1})$: $-\ddot{\theta}_{12}a_2sin\theta_{12} - \dot{\theta}_{12}{}^2a_2cos\theta_{12} - \ddot{\theta}_{13}a_3sin\theta_{13} - \dot{\theta}_{13}{}^2a_3cos\theta_{13} = -\ddot{\theta}_{14}a_4sin\theta_{14} - \dot{\theta}_{14}{}^2a_4cos\theta_{14}$

$(\ddot{2})$: $\ddot{\theta}_{12}a_2cos\theta_{12} - \dot{\theta}_{12}{}^2a_2sin\theta_{12} + \ddot{\theta}_{13}a_3cos\theta_{13} - \dot{\theta}_{13}{}^2a_3sin\theta_{13} = \ddot{\theta}_{14}a_4cos\theta_{14} - \dot{\theta}_{14}{}^2a_4sin\theta_{14}$

Let θ_{12}, $\dot{\theta}_{12}$ and $\ddot{\theta}_{12}$ be the input:



$$\begin{bmatrix} -a_3sin\theta_{13} & a_4sin\theta_{14} \\ a_3cos\theta_{13} & -a_4cos\theta_{14} \end{bmatrix}\begin{bmatrix} \dot{\theta}_{13} \\ \dot{\theta}_{14} \end{bmatrix} = \begin{bmatrix} a_2sin\theta_{12} \\ -a_2cos\theta_{12} \end{bmatrix}\dot{\theta}_{12}$$

$[A]$

$$\begin{bmatrix} -a_3sin\theta_{13} & a_4sin\theta_{14} \\ a_3cos\theta_{13} & -a_4cos\theta_{14} \end{bmatrix}\begin{bmatrix} \ddot{\theta}_{13} \\ \ddot{\theta}_{14} \end{bmatrix} = -\begin{bmatrix} -\ddot{\theta}_{12}a_2sin\theta_{12} - \dot{\theta}_{12}{}^2a_2cos\theta_{12} - \dot{\theta}_{13}{}^2a_3cos\theta_{13} + \dot{\theta}_{14}{}^2a_4cos\theta_{14} \\ \ddot{\theta}_{12}a_2cos\theta_{12} - \dot{\theta}_{12}{}^2a_2sin\theta_{12} - \dot{\theta}_{13}{}^2a_3sin\theta_{13} + \dot{\theta}_{14}{}^2a_4sin\theta_{14} \end{bmatrix}$$

$det[A] = |A| \neq 0$, there is a solution

$det[A] = a_3a_4(sin\theta_{13}cos\theta_{14} - cos\theta_{13}sin\theta_{14})$

$det[A] = a_3a_4sin(\theta_{13} - \theta_{14})$

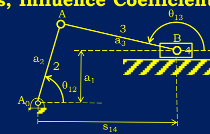
$det[A] = 0$ when $\theta_{13} - \theta_{14} = k\pi$

ME 301 Theory of Machines I

2. Kinematic Analysis

Velocity and Acceleration Analyses, Influence Coefficients

Slider-Crank Mechanism



$a_2e^{i\theta_{12}} = ia_1 + s_{14} + a_3e^{i\theta_{13}}$

Re: $a_2cos\theta_{12} = s_{14} + a_3cos\theta_{13}$ (1)

Im: $a_2sin\theta_{12} = a_1 + a_3sin\theta_{13}$ (2)

$(\dot{1})$: $-\dot{\theta}_{12}a_2sin\theta_{12} = \dot{s}_{14} - \dot{\theta}_{13}a_3sin\theta_{13}$

$(\dot{2})$: $\dot{\theta}_{12}a_2cos\theta_{12} = \dot{\theta}_{13}a_3cos\theta_{13}$

$(\dot{2}) \rightarrow \dot{\theta}_{13} = \frac{a_2cos\theta_{12}}{a_3cos\theta_{13}}\dot{\theta}_{12} = \boldsymbol{g_{23}}\dot{\theta}_{12}$

$\boldsymbol{g_{23}}$ is the velocity influence coefficient of $\dot{\theta}_{13}$ when $\dot{\theta}_{12}$ is the input

$(\dot{1}) \rightarrow \dot{s}_{14} = \boldsymbol{g_{23}}\dot{\theta}_{12}a_3sin\theta_{13} - \dot{\theta}_{12}a_2sin\theta_{12} = \left(\frac{a_2cos\theta_{12}}{a_3cos\theta_{13}}a_3sin\theta_{13} - a_2sin\theta_{12}\right)\dot{\theta}_{12} = a_2\frac{sin(\theta_{13} - \theta_{12})}{cos\theta_{13}}\dot{\theta}_{12} = \boldsymbol{g_{24}}\dot{\theta}_{12}$

$\boldsymbol{g_{24}}$ is the velocity influence coefficient of $\dot{s}_{14}$ when $\dot{\theta}_{12}$ is the input

$\boldsymbol{g_{23}}$ and $\boldsymbol{g_{24}}$ are both undefined when $cos\theta_{13} = 0$ that is $\theta_{13} = \frac{\pi}{2}, \frac{3\pi}{2}$

$\dot{\theta}_{13} = 0$ when $cos\theta_{12} = 0$ that is $\theta_{12} = \frac{\pi}{2}, \frac{3\pi}{2}$

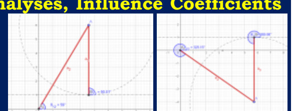
$\dot{s}_{14} = 0$ when $sin(\theta_{13} - \theta_{12}) = 0$ that is $\theta_{13} - \theta_{12} = k\pi$

ME 301 Theory of Machines I

2. Kinematic Analysis

Velocity and Acceleration Analyses, Influence Coefficients

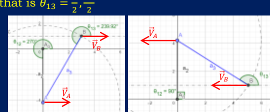
Slider-Crank Mechanism



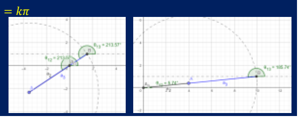
$g_{23} = \frac{a_2cos\theta_{12}}{a_3cos\theta_{13}}$

$g_{24} = a_2\frac{sin(\theta_{13} - \theta_{12})}{cos\theta_{13}}$

g_{23} and g_{24} are both undefined when $cos\theta_{13} = 0$ that is $\theta_{13} = \frac{\pi}{2}, \frac{3\pi}{2}$



$\dot{\theta}_{13} = 0$ when $cos\theta_{12} = 0$ that is $\theta_{12} = \frac{\pi}{2}, \frac{3\pi}{2}$

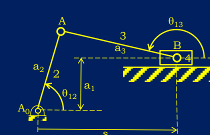


$\dot{s}_{14} = 0$ when $sin(\theta_{13} - \theta_{12}) = 0$ that is $\theta_{13} - \theta_{12} = k\pi$

2. Kinematic Analysis

Velocity and Acceleration Analyses, Influence Coefficients

Slider-Crank Mechanism



$\dot{\theta}_{13} = g_{23}\dot{\theta}_{12}$

$\ddot{\theta}_{13} = \dot{g}_{23}\dot{\theta}_{12} + g_{23}\ddot{\theta}_{12}$

$\dot{s}_{14} = g_{24}\dot{\theta}_{12}$

$\ddot{s}_{14} = \dot{g}_{24}\dot{\theta}_{12} + g_{24}\ddot{\theta}_{12}$

ME 301 Theory of Machines I