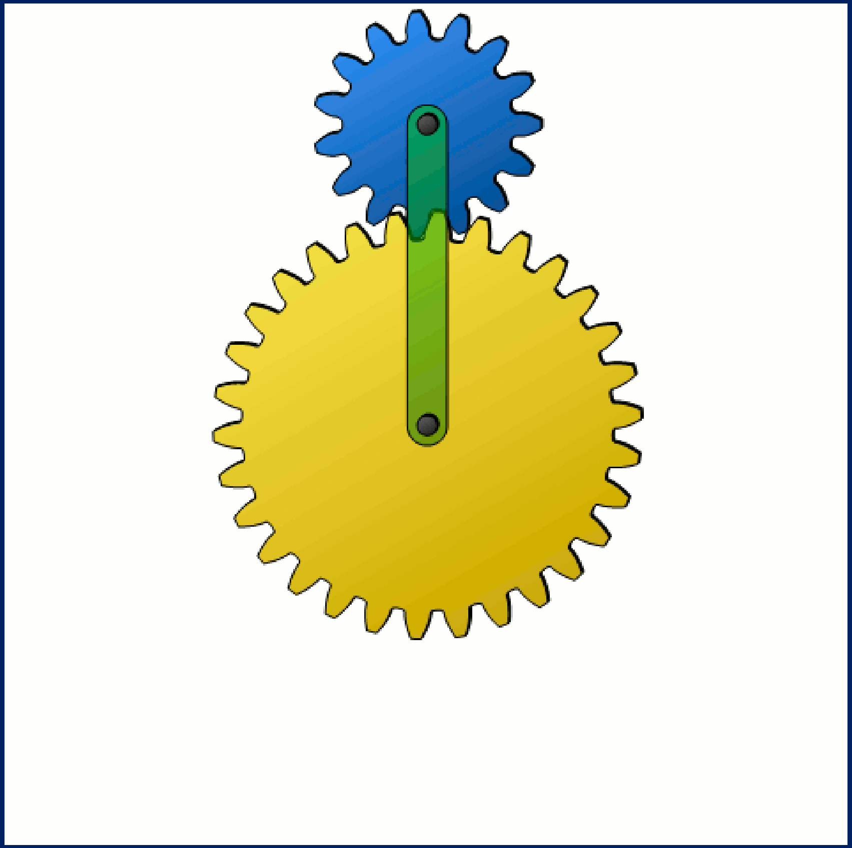
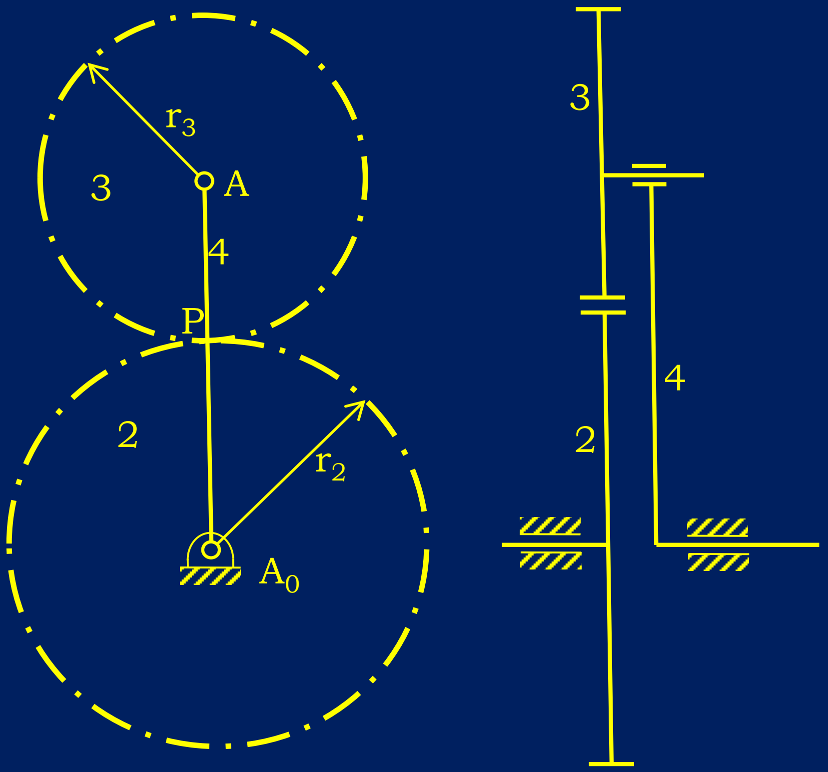
3. Gears

Planetary Gear Trains:

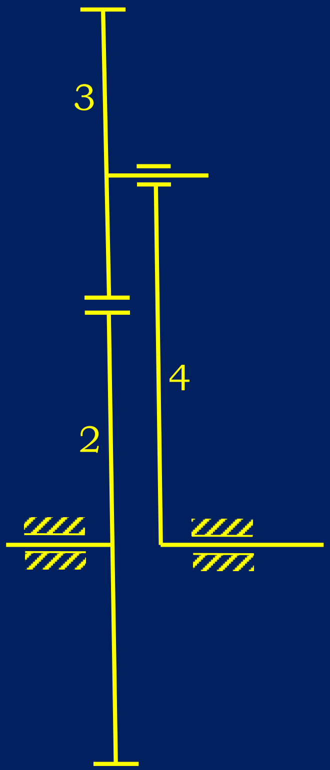
At least one gear is *not* directly connected to the fixed link by a revolute joint, the gear train is planetary gear train.





https://www.tec-science.com/mechanical-power-transmission/planetary-gear/fundamental-equation-of-planetary-gears-willis-equation/

3. Gears



Planetary Gear Trains:

$$F = \lambda(\ell - j - 1) + \sum_{i=1}^{j} f_i$$

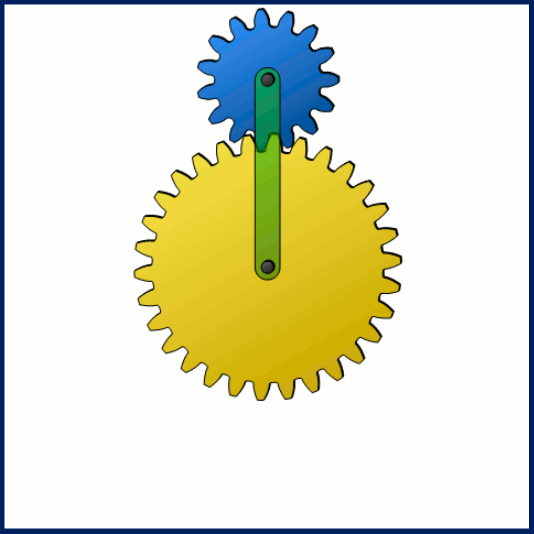
$\lambda = 3$

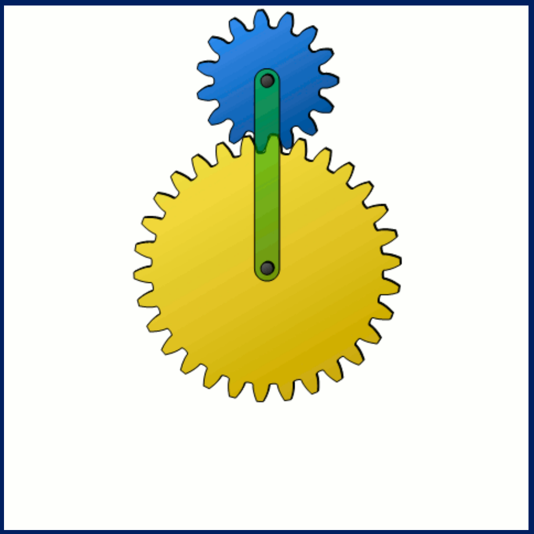
$\ell = 4$

$j = 4\ (3R + GP^*)$

$$\sum_{i=1}^{4} f_i = 5$$

$F = 3(4 - 4 - 1) + 5 = 2$





https://www.tec-science.com/mechanical-power-transmission/planetary-gear/fundamental-equation-of-planetary-gears-willis-equation/

3. Gears

Planetary Gear Trains:

Assume all angular velocities counter clockwise positive:

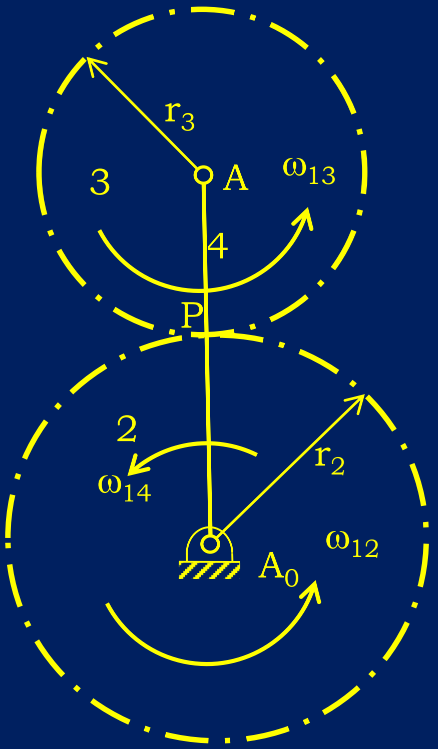
$$V_{P_2} = V_{P_3}$$

$$V_{P_2} = \omega_{12} r_2 \ (\leftarrow)$$

$$V_{P_3} = V_A + V_{P_3/A} = \omega_{14}(r_2 + r_3) - \omega_{13} r_3 (\leftarrow)$$

$$\omega_{12} r_2 = \omega_{14}(r_2 + r_3) - \omega_{13} r_3$$

$$R_{23} = -\frac{r_2}{r_3} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}}$$



3. Gears

Planetary Gear Trains:

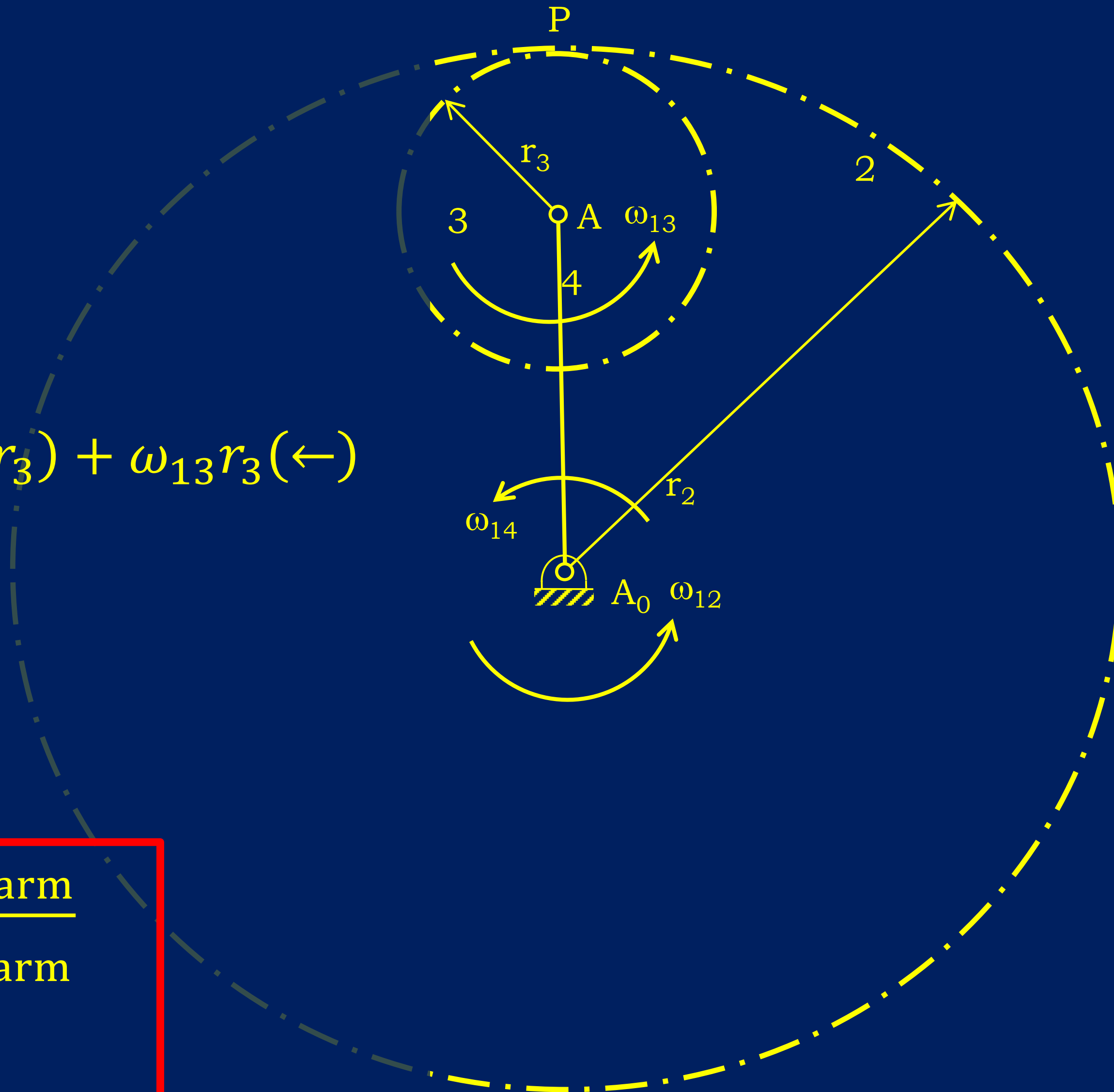
$$V_{P_2} = V_{P_3}$$

$$V_{P_2} = \omega_{12} r_2 \ (\leftarrow)$$

$$V_{P_3} = V_A + V_{P_3/A} = \omega_{14}(r_2 - r_3) + \omega_{13} r_3 (\leftarrow)$$

$$\omega_{12} r_2 = \omega_{14}(r_2 - r_3) + \omega_{13} r_3$$

$$R_{23} = \frac{r_2}{r_3} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}}$$



$$R_{ij} = \mp \frac{r_i}{r_j} = \mp \frac{T_i}{T_j} = \frac{\omega_{1j} - \omega_{1\text{arm}}}{\omega_{1i} - \omega_{1\text{arm}}}$$

- for sun, + for ring gear

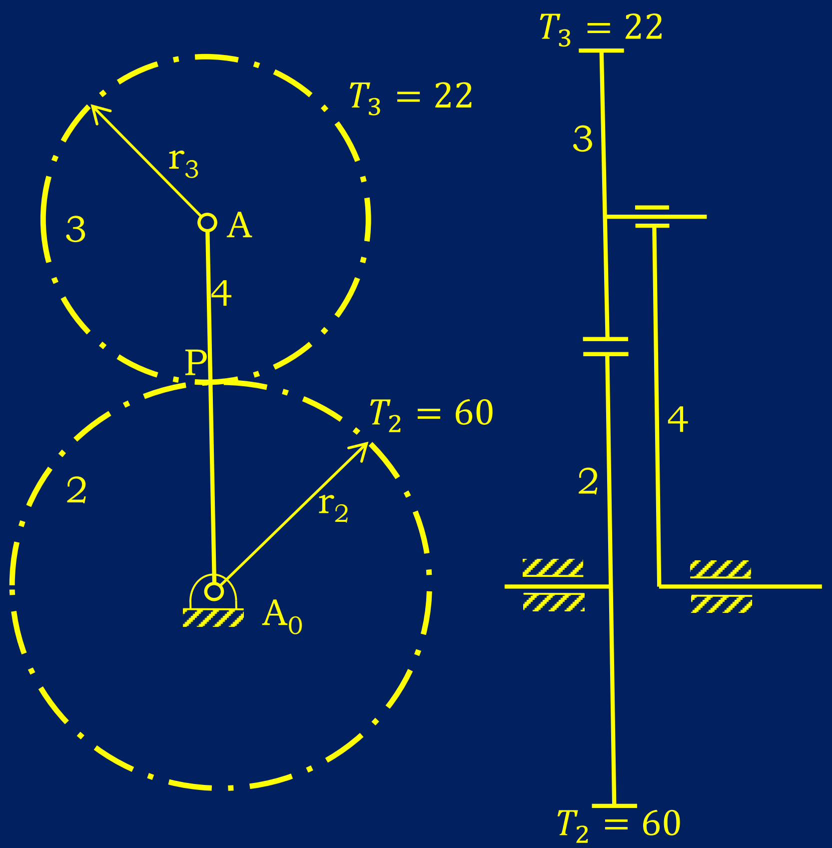
3. Gears

Example:

Given $\omega_{12} = 150\, rpm\ (CW)$, $\omega_{14} = 100\, rpm\ (CCW)$, determine ω_{13} for $T_2 = 60$ and $T_3 = 22$.

$$R_{23} = -\frac{T_2}{T_3} = -\frac{60}{22} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}} = \frac{\omega_{13} - 100}{-150 - 100}$$

$$\omega_{13} = 782\, rpm\ (CCW)$$



3. Gears

Example:

Determine the output speed and direction of rotation for $\omega_{12} = 3000\, rpm$

Mesh 1, planetary, arm 2, external

$$R_{34} = -\frac{T_3}{T_4} = -\frac{40}{38} = \frac{\omega_{14} - \omega_{12}}{\omega_{13} - \omega_{12}}$$

Mesh 2, planetary, arm 2, external

$$R_{35} = -\frac{T_{3'}}{T_5} = -\frac{42}{36} = \frac{\omega_{15} - \omega_{12}}{\omega_{13} - \omega_{12}}$$

Mesh 3, simple, internal

$$R_{56} = \frac{T_5}{T_6} = \frac{120}{54} = \frac{\omega_{16}}{\omega_{15}}$$

Mesh 4, simple, external

$$R_{46} = -\frac{T_{4'}}{T_6} = -\frac{12}{54} = \frac{\omega_{16}}{\omega_{14}}$$

Four equations–four unknowns, $\omega_{16} = -\frac{26}{1305}\omega_{12}$