3. Gears

Planetary Gear Trains:
At least one gear is *not* directly connected to the fixed link by a revolute joint, the gear train is planetary gear train.
3. Gears

Planetary Gear Trains:

\[ F = \lambda (\ell - j - 1) + \sum_{i=1}^{j} f_i \]

\( \lambda = 3 \)
\( \ell = 4 \)
\( j = 4 \left( 3R + GP^* \right) \)
\( \sum_{i=1}^{4} f_i = 5 \)

\[ F = 3(4 - 4 - 1) + 5 = 2 \]
3. Gears

Planetary Gear Trains:
Assume all angular velocities counter clockwise positive:

\[ V_{P_2} = V_{P_3} \]
\[ V_{P_2} = \omega_{12} r_2 \quad (\leftarrow) \]
\[ V_{P_3} = V_A + V_{P_3/A} = \omega_{14} (r_2 + r_3) - \omega_{13} r_3 \quad (\leftarrow) \]
\[ \omega_{12} r_2 = \omega_{14} (r_2 + r_3) - \omega_{13} r_3 \]
\[ R_{23} = -\frac{r_2}{r_3} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}} \]
3. Gears

Planetary Gear Trains:

\[ V_{P_2} = V_{P_3} \]

\[ V_{P_2} = \omega_{12} r_2 \quad (\leftarrow) \]

\[ V_{P_3} = V_A + V_{P_3/A} = \omega_{14}(r_2 - r_3) + \omega_{13}r_3 \quad (\leftarrow) \]

\[ \omega_{12} r_2 = \omega_{14}(r_2 - r_3) + \omega_{13}r_3 \]

\[ R_{23} = \frac{r_2}{r_3} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}} \]

\[ R_{ij} = \frac{\bar{r}_i}{r_j} = \frac{T_i}{T_j} = \frac{\omega_{1j} - \omega_{1\text{arm}}}{\omega_{1i} - \omega_{1\text{arm}}} \]

- for sun, + for ring gear
3. Gears

Example:
Given \( \omega_{12} = 150 \text{ rpm (CW)} \), \( \omega_{14} = 100 \text{ rpm (CCW)} \), determine \( \omega_{13} \) for \( T_2 = 60 \) and \( T_3 = 22 \).

\[
R_{23} = -\frac{T_2}{T_3} = -\frac{60}{22} = \frac{\omega_{13} - \omega_{14}}{\omega_{12} - \omega_{14}} = \frac{\omega_{13} - 100}{-150 - 100}
\]

\( \omega_{13} = 782 \text{ rpm (CCW)} \)
3. Gears

Example:

Determine the output speed and direction of rotation for $\omega_{12} = 3000 \, \text{rpm}$

Mesh 1, planetary, arm 2, external

\[ R_{34} = -\frac{T_3}{T_4} = -\frac{40}{38} = \frac{\omega_{14} - \omega_{12}}{\omega_{13} - \omega_{12}} \]

Mesh 2, planetary, arm 2, external

\[ R_{35} = -\frac{T_{3'}}{T_5} = -\frac{42}{36} = \frac{\omega_{15} - \omega_{12}}{\omega_{13} - \omega_{12}} \]

Mesh 3, simple, internal

\[ R_{56} = \frac{T_5}{T_6} = \frac{120}{54} = \frac{\omega_{16}}{\omega_{15}} \]

Mesh 4, simple, external

\[ R_{46} = -\frac{T_{4'}}{T_6} = -\frac{12}{54} = \frac{\omega_{16}}{\omega_{14}} \]

Four equations–four unknowns, $\omega_{16} = -\frac{26}{1305} \omega_{12}$