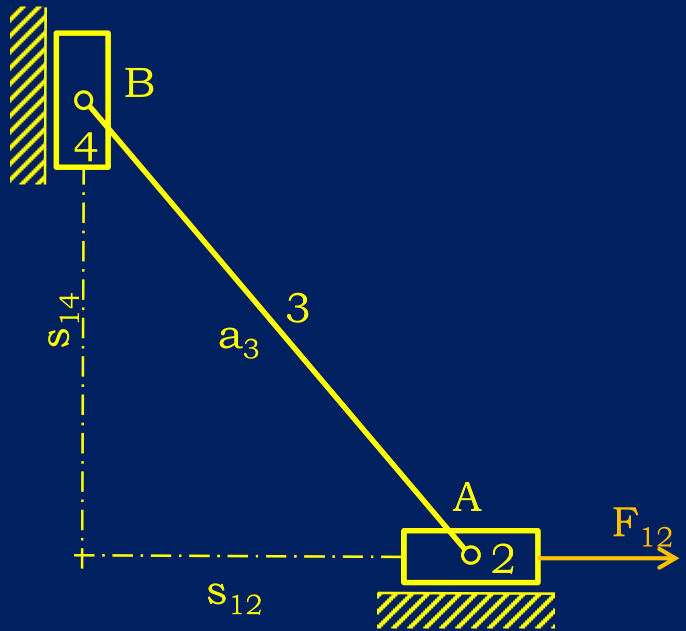
Dynamic Force Analysis

Example:

Determine the force F_{12} to be applied on link 2 so that it has a constant velocity to the right for any position of the mechanism (i.e. s_{12}). The mechanism is in horizontal plane and masses of links 2 and 4 are negligible compared to link 3 which is a uniform slender rod of mass m.



Solution Procedure:

- Perform kinematic analysis.
- Draw free body diagrams with inertia forces and write the equations of ***dynamic*** equilibrium.
- Solve the equations for unknown force.

Dynamic Force Analysis

Example:

Perform kinematic analysis with s_{12} input.

Disconnect and reconnect B:

$$s_{12} + a_3 e^{i\theta_{13}} = is_{14}$$

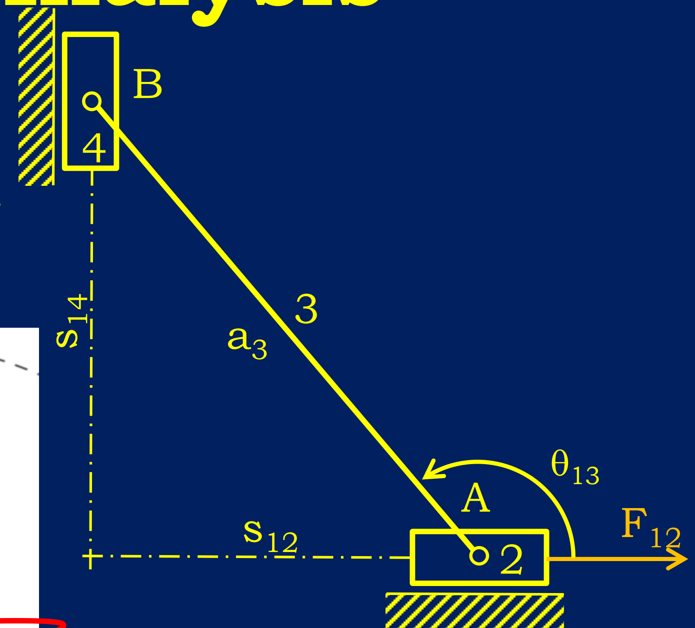
$$Re:\ s_{12} + a_3 cos\theta_{13} = 0$$

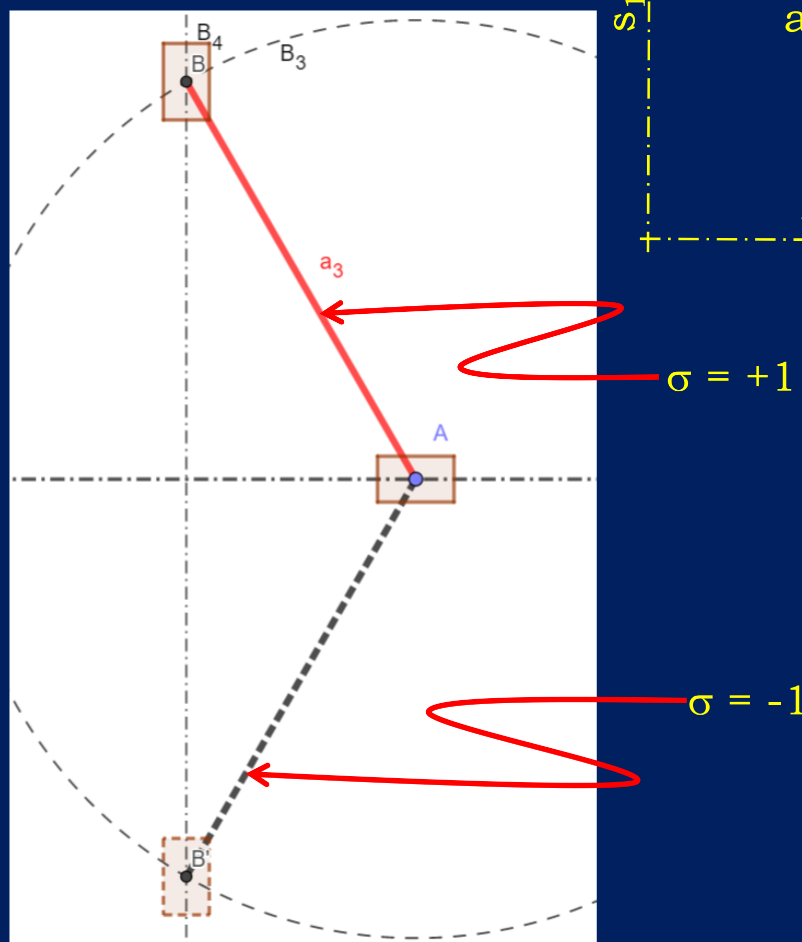
$$Im:\ a_3 sin\theta_{13} = s_{14}$$

$$cos\theta_{13} = -\frac{s_{12}}{a_3}$$

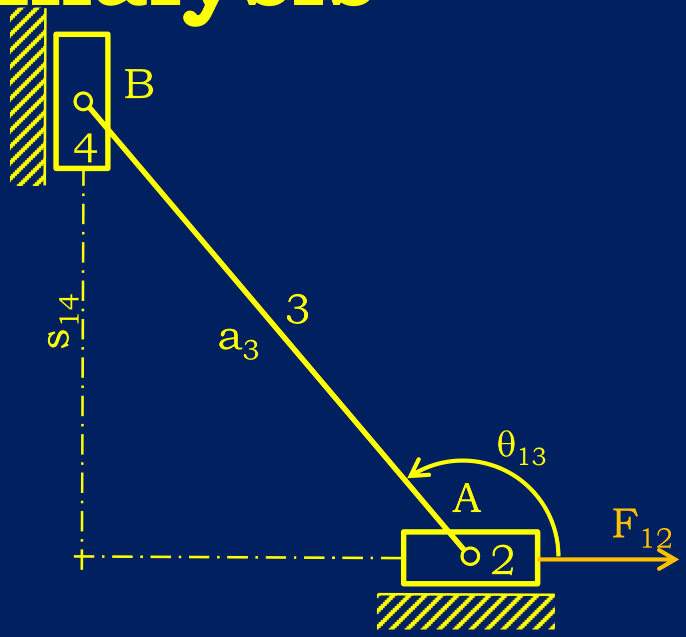
$$\theta_{13} = \sigma cos^{-1}\left(-\frac{s_{12}}{a_3}\right)$$

$$s_{14} = a_3 sin\theta_{13}$$





Dynamic Force Analysis



Example:

Perform kinematic analysis.

Re: $s_{12} + a_3 cos\theta_{13} = 0$

Im: $a_3 sin\theta_{13} = s_{14}$

$$\frac{d}{dt}(Re): \dot{s}_{12} - \dot{\theta}_{13} a_3 sin\theta_{13} = 0$$

$$\frac{d}{dt}(Im): \dot{\theta}_{13} a_3 cos\theta_{13} = \dot{s}_{14}$$

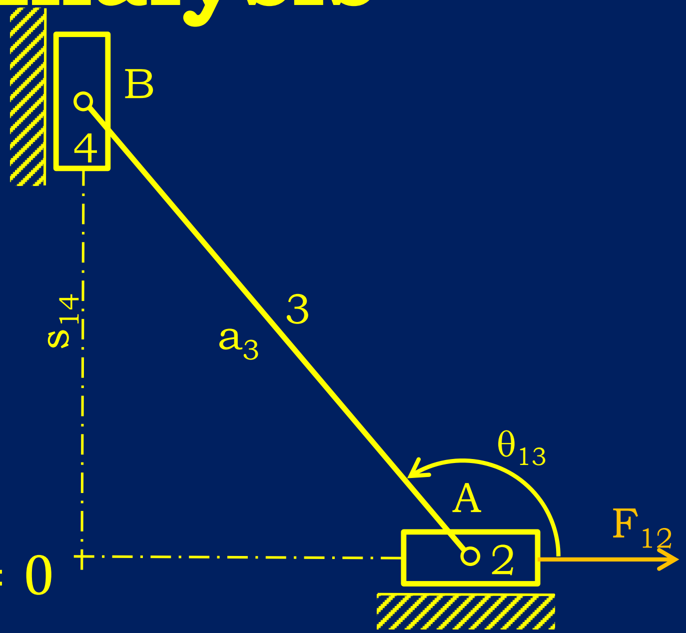
$$\dot{\theta}_{13} = \frac{\dot{s}_{12}}{a_3 sin\theta_{13}}$$

$$\dot{s}_{14} = \frac{\dot{s}_{12}}{tan\theta_{13}}$$

Dynamic Force Analysis

Example:

Perform kinematic analysis.



$$\frac{d}{dt}(Re):\ \dot{s}_{12} - \dot{\theta}_{13} a_3 sin\theta_{13} = 0$$

$$\frac{d}{dt}(Im):\dot{\theta}_{13} a_3 cos\theta_{13} = \dot{s}_{14}$$

$$\frac{d^2}{dt^2}(Re):\ \ddot{s}_{12} - \ddot{\theta}_{13} a_3 sin\theta_{13} - \dot{\theta}_{13}{}^2 a_3 cos\theta_{13} = 0$$

$$\frac{d^2}{dt^2}(Im):\ddot{\theta}_{13} a_3 cos\theta_{13} - \dot{\theta}_{13}{}^2 a_3 sin\theta_{13} = \dot{s}_{14}$$

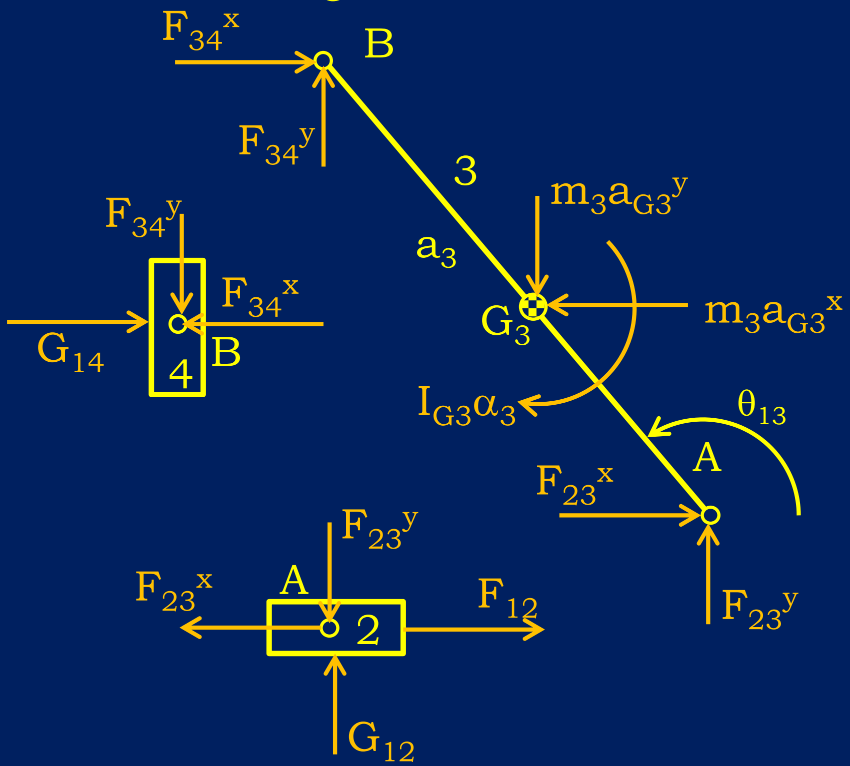
$$\ddot{\theta}_{13} = \frac{\ddot{s}_{12} - \dot{\theta}_{13}{}^2 a_3 cos\theta_{13}}{a_3 sin\theta_{13}}$$

$$\ddot{s}_{14} = \ddot{\theta}_{13} a_3 cos\theta_{13} - \dot{\theta}_{13}{}^2 a_3 sin\theta_{13}$$

Dynamic Force Analysis

Example:

Draw free body diagrams with inertia forces and write the equations of equilibrium.



Link 4:

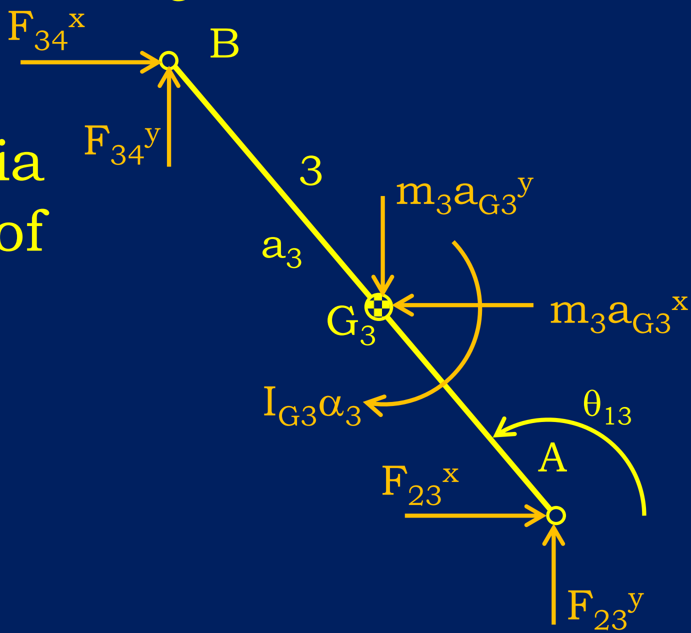
$$\sum F_x = 0$$

$$G_{14} - F_{34}{}^{x} = 0 \rightarrow ?$$

$$\sum F_y = 0$$

$$F_{34}{}^{y} = 0$$

Dynamic Force Analysis



Example:

Draw free body diagrams with inertia forces and write the equations of equilibrium.

Link 3:

$$\sum F_x = 0$$

$$F_{23}{}^{x} + F_{34}{}^{x} - m_3 a_{G_3}{}^{x} = 0 \rightarrow ?$$

$$\sum F_y = 0$$

$$F_{23}{}^{y} + F_{34}{}^{y} - m_3 a_{G_3}{}^{y} = 0 \rightarrow F_{23}{}^{y} = m_3 a_{G_3}{}^{y}$$

$$\sum M_A = 0$$

$$a_3 F_{34}{}^{x} sin(0 - \theta_{13}) + a_3 F_{34}{}^{y} sin\left(\frac{\pi}{2} - \theta_{13}\right)$$

$$+ \frac{a_3}{2} m_3 a_{G_3}{}^{x} sin(\pi - \theta_{13}) + \frac{a_3}{2} m_3 a_{G_3}{}^{y} sin\left(\frac{3\pi}{2} - \theta_{13}\right)$$

$$- I_{G_3}\alpha_3 = 0 \rightarrow F_{34}{}^{x}$$

$$a_{G_3} = a_{G_3}{}^{x} + i a_{G_3}{}^{y} = \ddot{r}_{G_3}$$

$$r_{G_3} = s_{12} + \frac{a_3}{2} e^{i\theta_{13}}$$

$$\dot{r}_{G_3} = \dot{s}_{12} + i\dot{\theta}_{13}\frac{a_3}{2} e^{i\theta_{13}}$$

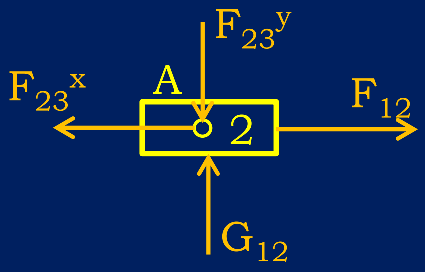
$$\ddot{r}_{G_3} = \ddot{s}_{12} + i\ddot{\theta}_{13}\frac{a_3}{2} e^{i\theta_{13}} - \dot{\theta}_{13}{}^{2}\frac{a_3}{2} e^{i\theta_{13}}$$

Dynamic Force Analysis

Example:

Draw free body diagrams with inertia forces and write the equations of equilibrium.

Link 2:



$$\sum F_x = 0$$

$$F_{12} - {F_{23}}^x = 0 \rightarrow F_{12}$$

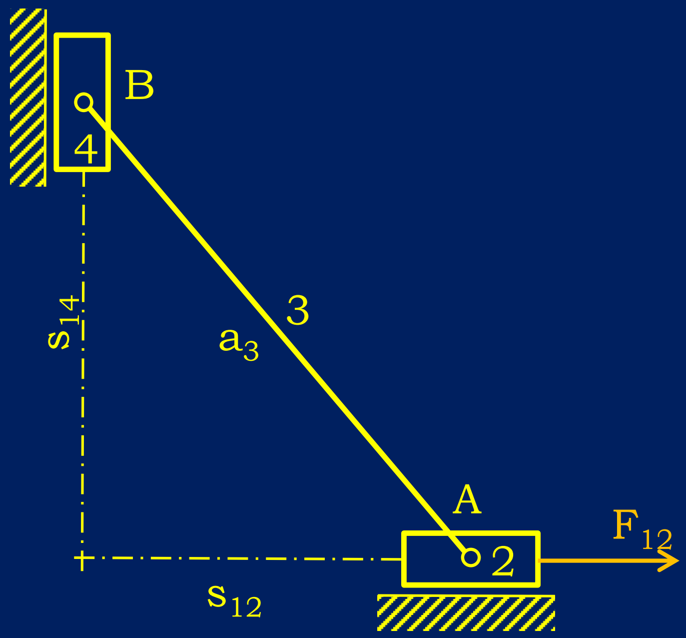
$$\sum F_y = 0$$

$$G_{12} - {F_{23}}^x = 0 \rightarrow G_{12}$$

Dynamic Force Analysis

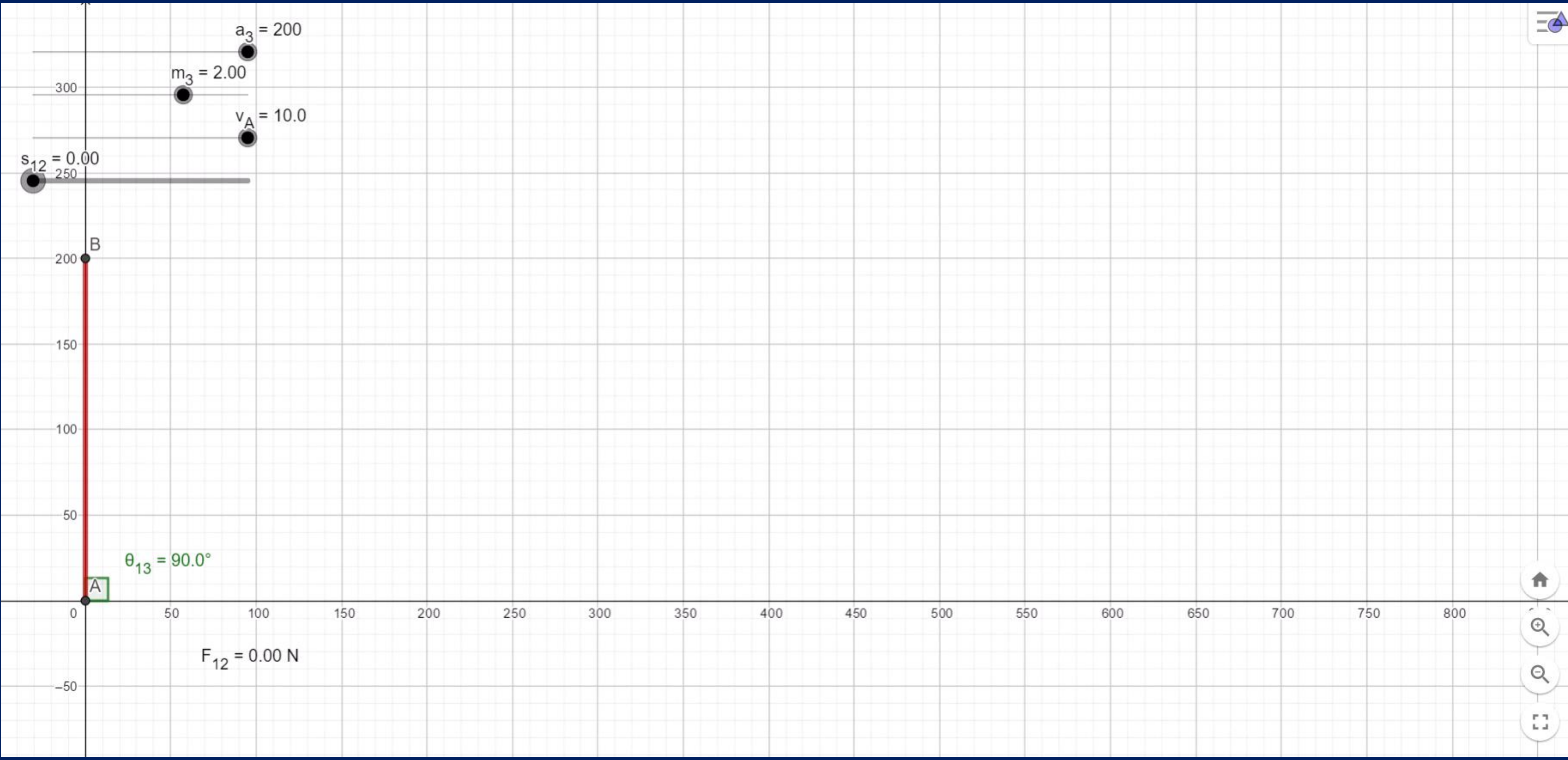
Example:

Determine the force F_{12} to be applied on link 2 so that it has a constant velocity of 10 mm/s to the right for any position of the mechanism (i.e. $0 \leq s_{12} \leq 200\,mm$). The mechanism is in horizontal plane and masses of links 2 and 4 are negligible compared to link 3 which is a uniform slender rod of length a_3 = 200 mm and of mass m_3 = 2 kg.



Dynamic Force Analysis

Solution with Geogebra



Dynamic Force Analysis

Solution with Excel

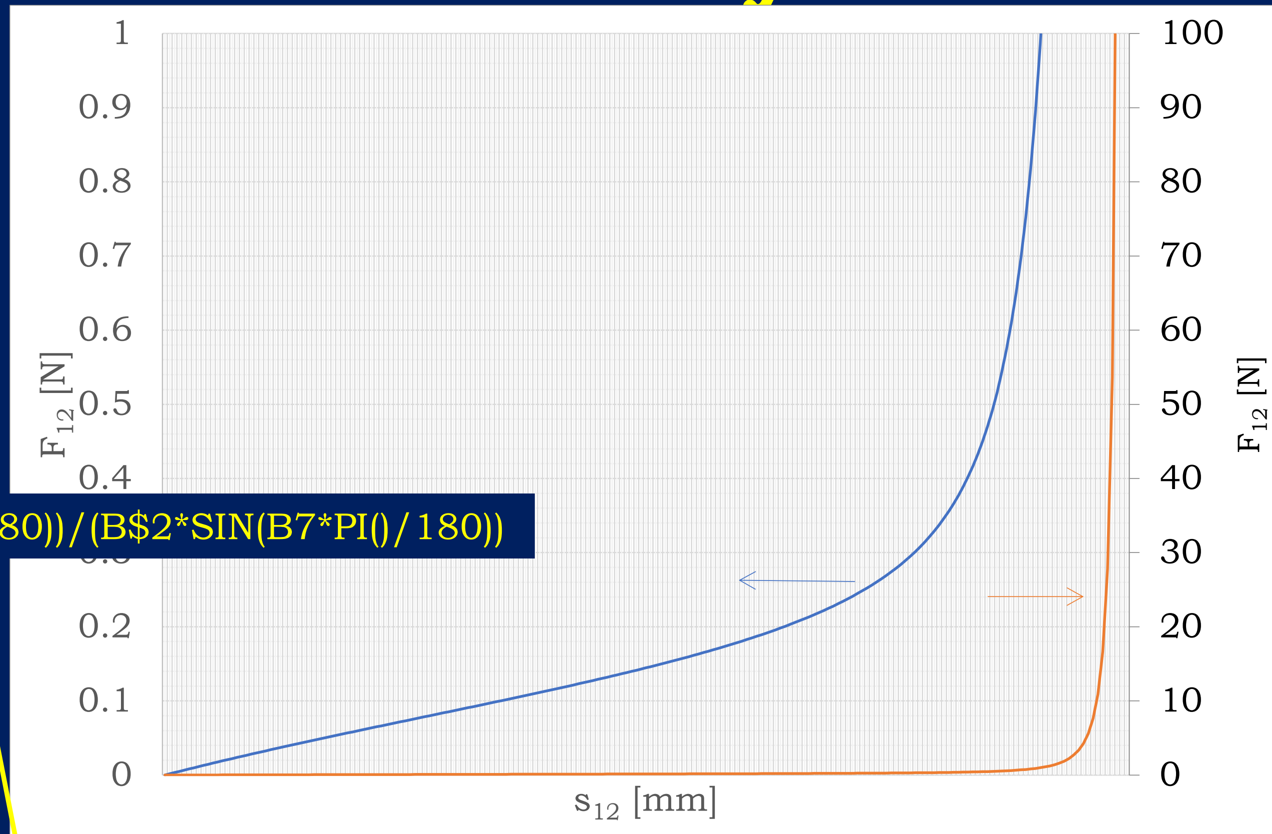
=ACOS(-A7/B$2)*180/PI()

=B$2*SIN(B7*PI()/180)

=B$4/(B$2*SIN(B7*PI()/180))

=B$4/TAN(B7*PI()/180)

=(B$5-D7^2*B$2*COS(B7*PI()/180))/(B$2*SIN(B7*PI()/180))



v_A	10		I_{G3}	6666.7						
a_3	200									
m_3	2									
$s_{12}d$	10									
$s_{12}dd$	0									
s_{12}	θ_{13}	s_{14}	$\theta_{13}d$	$s_{14}d$	$\theta_{13}dd$	$s_{14}dd$	a_{G3x}	a_{G3y}	F_{34x}	F_{23x}
0	90	200	0.05	6E-16	-2E-19	-0.5	0	-0.25	-3E-17	3E-17
1	90.286	200	0.05	-0.05	1E-05	-0.5	0	-0.2488	-0.0017	0.0017
2	90.573	199.99	0.05	-0.1	3E-05	-0.5001	0	-0.2475	-0.0033	0.0033
3	90.859	199.98	0.05	-0.15	4E-05	-0.5002	0	-0.2463	-0.0049	0.0049
4	91.146	199.96	0.05	-0.2	5E-05	-0.5003	0	-0.245	-0.0066	0.0066