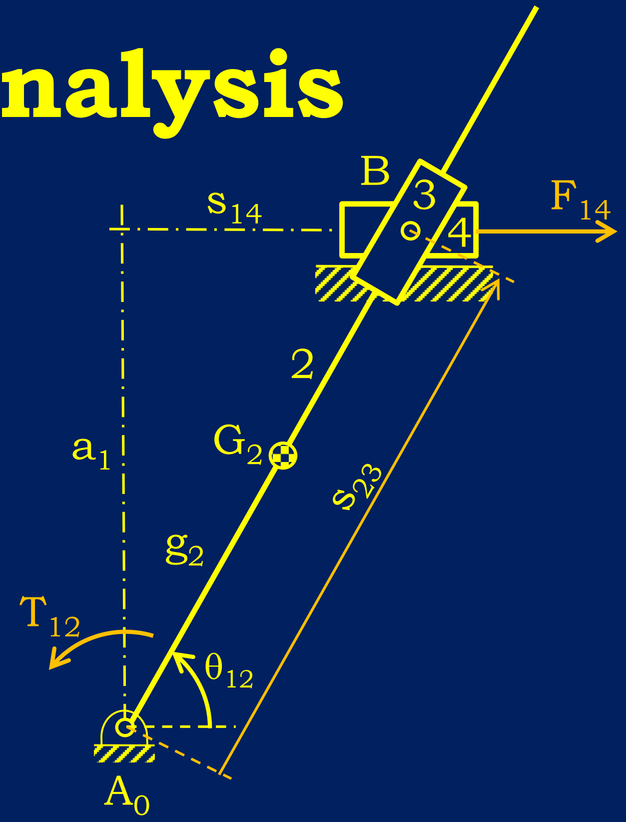
Dynamic Force Analysis

Example:

Determine the torque T_{12} to be applied on link 2 for a given motion (i.e. θ_{12}, $\dot{\theta}_{12}$ and $\ddot{\theta}_{12}$) and F_{14}.

Solution Procedure:

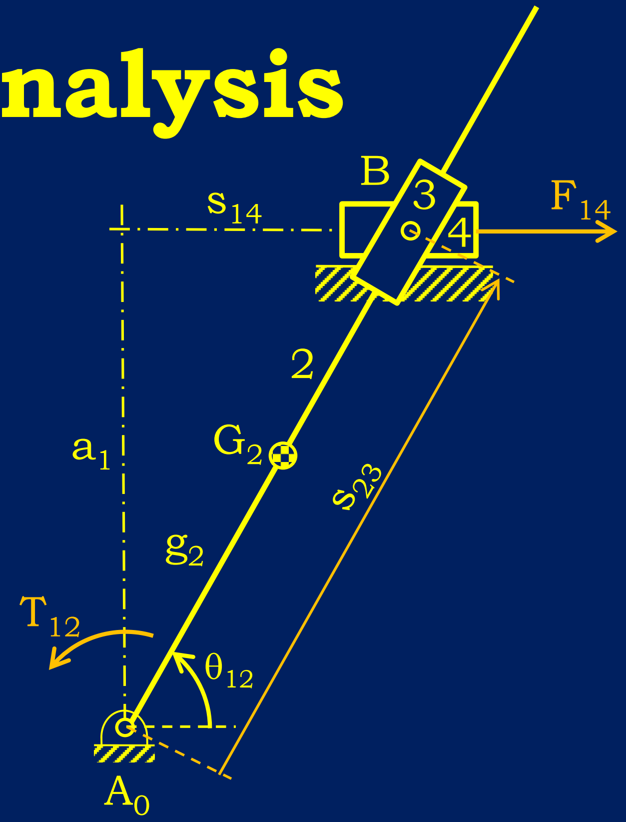
- Perform kinematic analysis.
- Draw free body diagrams with inertia forces and write the equations of equilibrium.
- Solve the equations for unknown force.



Dynamic Force Analysis

Example:

- Perform kinematic analysis.



Disconnect and reconnect B:

$$s_{23}e^{i\theta_{12}} = ia_1 + s_{14}$$

$$Re: s_{23}cos\theta_{12} = s_{14}$$

$$Im: s_{23}sin\theta_{12} = a_1$$

$$s_{23} = \frac{a_1}{sin\theta_{12}}$$

$$s_{14} = s_{23}cos\theta_{12} = \frac{a_1}{tan\theta_{12}}$$

$$\frac{d}{dt}Re: \dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}s_{23}sin\theta_{12} = \dot{s}_{14}$$

$$\frac{d}{dt}Im: \dot{s}_{23}sin\theta_{12} + \dot{\theta}_{12}s_{23}cos\theta_{12} = 0$$

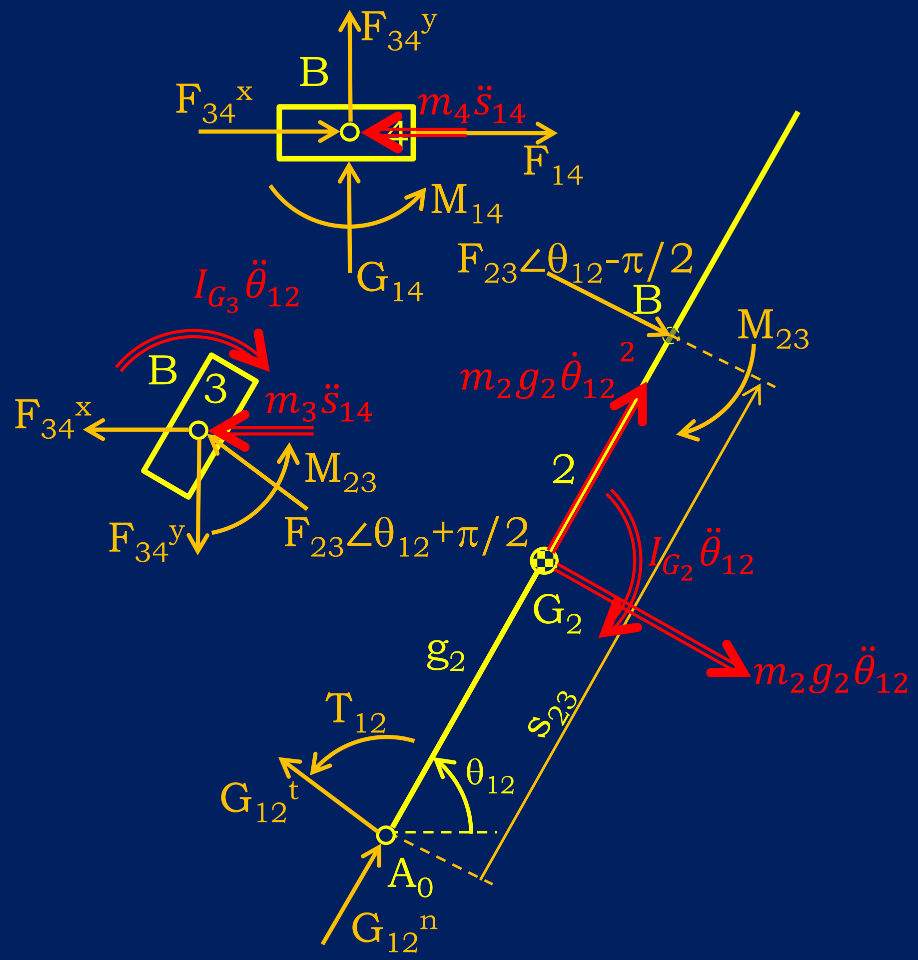
$$\frac{d^2}{dt^2}Re: \ddot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \ddot{\theta}_{12}s_{23}sin\theta_{12} - \dot{\theta}_{12}\dot{s}_{23}sin\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}cos\theta_{12} = \ddot{s}_{14}$$

$$\frac{d^2}{dt^2}Im: \ddot{s}_{23}sin\theta_{12} + \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} + \ddot{\theta}_{12}s_{23}cos\theta_{12} + \dot{\theta}_{12}\dot{s}_{23}cos\theta_{12} - \dot{\theta}_{12}{}^2 s_{23}sin\theta_{12} = 0$$

Dynamic Force Analysis

Example:

- Draw free body diagrams with inertia forces and write the equations of equilibrium.



Dynamic Force Analysis

Example:

Equations of Equilibrium

Link 4:

$$\sum M_B = 0$$

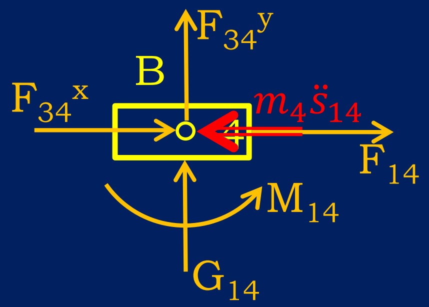
$$M_{14} = 0$$

$$\sum F_x = 0$$

$$F_{14} + F_{34}{}^{x} - m_4\ddot{s}_{14} = 0 \rightarrow F_{34}{}^{x}$$

$$\sum F_y = 0$$

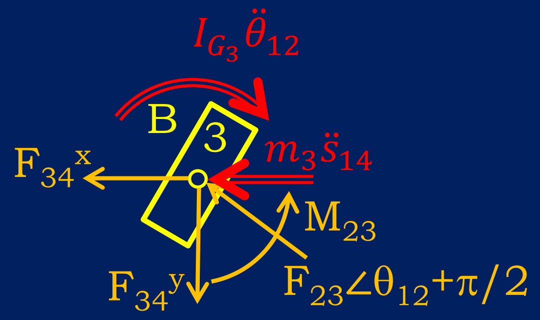
$$G_{14} + F_{34}{}^{y} = 0 \rightarrow ?$$



Dynamic Force Analysis

Example:

Link 3:



$$\sum M_B = 0$$

$$M_{23} - I_{G_3}\ddot{\theta}_{12} = 0 \rightarrow M_{23} = I_{G_3}\ddot{\theta}_{12}$$

$$\sum F_x = 0$$

$$F_{23}cos\left(\theta_{12} + \frac{\pi}{2}\right) - F_{34}{}^{x} - m_3\ddot{s}_{14} = 0 \rightarrow F_{23}$$

$$\sum F_y = 0$$

$$F_{23}sin\left(\theta_{12} + \frac{\pi}{2}\right) - F_{34}{}^{y} = 0 \rightarrow F_{34}{}^{y}$$

Dynamic Force Analysis

Example:

Link 2:

$$\sum M_{A_0} = 0$$

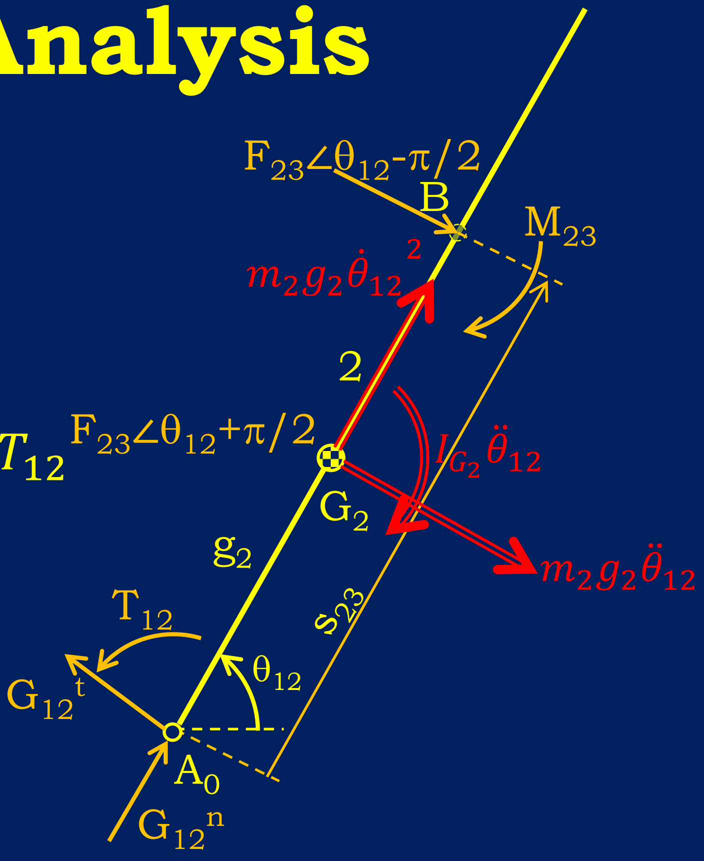
$$-s_{23}F_{23} - M_{23} - g_2 m_2 g_2 \ddot{\theta}_{12} - I_{G_2}\ddot{\theta}_{12} + T_{12} = 0 \rightarrow T_{12}$$

$$\sum F_n = 0$$

$$G_{12}{}^n + m_2 g_2 \dot{\theta}_{12}{}^2 = 0 \rightarrow G_{12}{}^n$$

$$\sum F_t = 0$$

$$G_{12}{}^t - m_2 g_2 \ddot{\theta}_{12} - F_{23} = 0 \rightarrow G_{12}{}^t$$



Dynamic Force Analysis

Example:

Link 2, alternative FBD:

$$I_{A_{0_2}} = I_{G_2} + m_2 g_2{}^2$$

$$a_{G_2}{}^t = g_2 \ddot{\theta}_{12}$$

$$\sum M_{A_0} = 0$$

$$-s_{23}F_{23} - M_{23} - I_{A_{0_2}}\ddot{\theta}_{12} + T_{12} = 0 \rightarrow T_{12}$$

$$\sum F_n = 0$$

$$G_{12}{}^n + m_2 g_2 \dot{\theta}_{12}{}^2 = 0 \rightarrow G_{12}{}^n$$

$$\sum F_t = 0$$

$$G_{12}{}^t - m_2 g_2 \ddot{\theta}_{12} - F_{23} = 0 \rightarrow G_{12}{}^t$$