1. In spherical coordinates, the position vector of a particle can be written as:

$$\vec{r} = r\hat{r} \tag{1}$$

where

$$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z} \tag{2}$$

For a general motion of the particle, all of r, θ and ϕ change with time. For such a general motion, calculate $\vec{v}$ and $\vec{a}$. If one defines, $\vec{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_\phi\hat{\phi}$ and $\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} + a_\phi\hat{\phi}$,

   (a) Express $v_{r,\theta,\phi}$ and $a_{r,\theta,\phi}$ in terms of r, θ, ϕ and their derivatives.

   (b) Express the kinetic energy of the particle in terms of these variables.

   (c) Consider a particle carrying out uniform circular motion. Assume that the circle lies on the xy-plane. For such a particle r and θ have fixed values, and hence $dr/dt = 0$ and $d\theta/dt = 0$. Show that the acceleration of this particle has a single component and points towards the center.

   (d) Consider a circular loop and a bead of mass m that is allowed to move freely on the loop (*i.e* ignore friction). If the loop is placed vertically and rotates at a constant velocity around the z axis, what is the total mechanical energy of the bead? By taking the derivative of the energy with respect to time, and equating the result to zero, obtain an equation satisfied by θ. (*Hint:* The position of the vector can be parameterized by $r = constant$ and $d\phi/dt = \omega = constant$)

2. A box of mass 40 kg is initially at rest on the floor. The coefficient of kinetic friction between the box and the floor is $\mu_k = 0.60$. A woman pushes horizontally against the box with a force of 250 N until the box attains a speed of 2.0 m/s.

   (a) What is the change of kinetic energy of the box?

(b) What is the work done by the friction force on the box?

(c) What is the work done by the woman on the box?

3. A pendulum consists of a mass tied to a string of length $1.0\ m$. Suppose that this pendulum is initially held at an angle $30°$ with the vertical and then released. What is the speed with which the mass swings through its lowest point? At what angle will the mass have one half of this speed?

4. A particle initially sits on top of a large smooth sphere of radius R. The particle begins to slide down the sphere, without friction. At what angular position will the particle loose contact with the surface of the sphere? where will the particle land on ground? (H. C. Ohanian, "Physics," Q7.55, pg 183) (*Hint:* What is the condition on the normal force so that the particle can be considered in contact with the sphere?).

5. A particle is subjected to a force that depends on position as follows:

$$\vec{F} = (4N)\hat{x} + (2N/m)\hat{y} \tag{3}$$

(a) Calculate the work done by this force as the particle moves from the origin to the points $(x, y) = (1m, 1m)$ along a straight line

(b) Calculate the work down by this force if the particle moves from the point $(x, y) = (1m, 1m)$ first to the point $(x, y) = (0, 1m)$ along a straight line, and then to the origin along the y axis.

(c) Is the force conservative? If it it, what is the corresponding potential energy?

6. Suppose that the potential energy of a particle moving along the x axis is

$$U(x) = \frac{b}{x^2} - 2\frac{c}{x} \tag{4}$$

where b and c are positive constants.

(a) Sketch the general shape of $U(x)$. Where is the equilibrium points?

(b) Suppose the energy of the particle is $E = -\frac{1}{2}c^2/b$. Find the turning points of the motion.

(c) Suppose the energy of the particle is $E = \frac{1}{2}c^2 b$. Find the turning point.

7. Consider a projectile traveling horizontally and slowing down under the influence of air resistance. The mass of this projectile is $43.36\ kg$ and the speed as a function of time is

$$v = (655.9\ m/s) - (61.14\ m/s^2)t + (3.26\ m/s^3)t^2 \tag{5}$$

(a) What is the position as a function of time if the projectile starts at $x = 0$?

(b) What is the acceleration of the projectile as a function of time? Calculate the force exerted by air.

(c) What is the instantaneous power removed from the projectile by the air resistance?

(d) What is the average power for the time interval from 0 to 3.0 s?