2^{nd} Homework
Due: 25 October 2007

1. Show explicitly that the force created by a spherical shell of radius R and mass M on a test mass m is given by

$$\vec{F} = -G_N \frac{Mm}{r^2} \hat{r} \tag{1}$$

   where r is the distance between the test mass and the center of the shell, $\hat{r}$ is a unit vector along the direction of the position vector of m with respect to the center of M. (*Do not just copy the derivation in the book. Make you own derivation explaining why you do each step*)

2. Consider a sphere of radius R that has a spherical whole inside of radius $R' < R$ the center of which is located at a distance r from the center of the larger sphere. Calculate the gravitational potential energy created at any point in space (both inside and outside the large sphere) if this object has a mass M. (*Hint:* Mathematically, a hole in an object of mass density ρ can be treated as the object without a hole plus, another object with mass density $-\rho$ placed at the position of the hole)

3. A binary star system consists of two stars of masses m_1 and m_2 orbiting about each other. Suppose that the orbits of the stars are circles of radii r_1 and r_2 centered on the center of mass. Show that the period of the orbital is given by:

$$T^2 = \frac{4\pi^2}{G(m_1 + m_2)} (r_1 + r_2)^3 \tag{2}$$

4. The Sun is moving in a circular orbit around the center of our galaxy. The radius of this orbit is 3×10^4 light years. Calculate the period of the orbital motion and calculate the orbital speed of the Sun. The mass of our Galaxy is 4×10^{41} kg and all this mass can be regarded as concentrated at the center of the Galaxy.

5. On of the problems in modern physics is the problem of dark matter. Dark matter is believed to make about 24% of all the energy of the universe. (Only 4% is believed to be in the form of ordinary matter). In the following step, we will try to understand how do we now that there is dark matter.

(a) Let $v(r)$ denote the speed that a mass should have in order for it to have a circular orbit of radius r around a mass M. Obtain an expression for $v(r)$.

(b) Consider a spherical galaxy of radius R_0. Assume that the mass of the galaxy is distributed uniformly until a radius R. Let $v(r)$ be the speed of a star that orbits the center of the galaxy at a distance r. Plot $v(r)$ as a function of r between $r = 0$ and $r = 2R_0$. This plot is called the rotation curve of the galaxy.

(c) Visit the web page http://burro.cwru.edu/JavaLab/RotcurveWeb/main.html. In the applet, there are experimental observations of rotation curves of three galaxies. Compare the general features of your plot and the data. Identify R_0 for each of the galaxies such that for $r < R_0$, you plot and the experimental data have same general features. How does the experimental data compare with your plot with $r > R_0$?

(d) What should be the mass profile of this dark matter, such that outside the galaxy, where there is only dark matter, the rotation curve is flat?