Fall 2020-21

EE 503

Homework #1

Due: Nov. 18, 2020

The following assignment is to clarify the concept of correlation and the decorrelating transformations.

Part 1: 2-D Gaussian Distribution

a) Using meshgrid, mesh and contour commands of Matlab, generate curves of 2-D zero-mean Gaussian distributions using the analytical definition. Experiment with the following parameters:

i. $\sigma_x^2 = 1$; $\sigma_y^2 = 1$; $\rho_{xy} = 0$

ii. $\sigma_x^2 = 1$; $\sigma_y^2 = 10$; $\rho_{xy} = 0$

iii. $\sigma_x^2 = 1$; $\sigma_y^2 = 1$; $\rho_{xy} = 0.25$

iv. $\sigma_x^2 = 1$; $\sigma_y^2 = 10$; $\rho_{xy} = 0.25$

v. $\sigma_x^2 = 1$; $\sigma_y^2 = 1$; $\rho_{xy} = 0.50$

vi. $\sigma_x^2 = 1$; $\sigma_y^2 = 1$; $\rho_{xy} = 0.99$

Only submit level curve plot for part vi and 3-D mesh plot for part ii.

b) Write a Matlab function with the inputs σ_x^2, σ_y^2 and ρ_{xy} (correlation coefficient) and the output r where r is the variable on the right hand side of the following relation

$$[x \quad y]\begin{bmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}^{-1}\begin{bmatrix} x \\ y \end{bmatrix} \le r^2$$

defining an ellipse. The output r should be the value for which the points generated by 2-D zero-mean Gaussian distribution (with the parameters of σ_x^2, σ_y^2 and ρ_{xy}) lies in the ellipse with 50% probability. Submit your Matlab function.

Part 2: Decorrelation

Step1: Use randn command to generate a 2x1000 matrix whose columns are 2-D Gaussian random vectors with zero mean and $\sigma_{x1}^2 = 1$; $\sigma_{x2}^2 = 1$ (x_1 and x_2 are the first and second entries of the random vector $\boldsymbol{x}$). Each matrix column is independent and identically distributed. Use scatter command (or plain plot command) to see the distribution of 1000 randomly generated vectors.

Step 2: Transform 1000 random vectors generated in Step 1 with the matrix $\mathbf{A}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, that is multiply each vector by matrix $\boldsymbol{A}$, $\mathbf{y}=\mathbf{Ax}$ Use scatter command to see the distribution after the transformation.
Estimate the correlation coefficient using

$$\hat{r}_{xy}=\frac{\sum_{k=1}^{1000} y_1[k]y_2[k]}{\sqrt{\sum_{k=1}^{1000}(y_1[k])^2\sum_{k=1}^{1000}(y_2[k])^2}}$$

where $y_1[k]$ and $y_2[k]$ are the first and second entries of the k^{th} random vector $\boldsymbol{y}$. Compare your estimate with the true correlation coefficient induced by the transformation.

Using the function written in part 1b) draw an ellipse on the scatter chart for which the probability of points in the scatter chart has $\frac{1}{2}$ probability of being inside of the ellipse. Roughly compare the number of points lying inside and outside of the ellipse.

Step 3: Find an orthogonal matrix that decorrelates 2x1000 random vector generated in Step 2. Apply the decorrelating transform and show its effect on the random vectors using scatter plot. Calculate the variance of first and second element of the random after decorrelation. To verify the calculated variance, plot the histogram of first entry of decorrelated vectors and place a Gaussian pdf curve with zero mean and calculated variance.

Step 4: Find a matrix that decorrelates 2x1000 random vector set in Step 2 such that the variances of first and second elements of the vector after decorrelation is unity. Show the scatter plot after the decorrelation resulting in unity variance.

Step 5: Use Cholesky transformation to find a lower triangular matrix that decorrelates 2x1000 vector set in Step 2. What are the variances after decorrelation? Can you find another lower triangular matrix resulting in unit variance distribution for the first and second entries of the vector as in Step 4. Show the scatter plot after unit variance decorrelation.

Step 6: Random vectors generated in Step 1 is processed by a second matrix $\mathbf{B}=\begin{bmatrix}1 & -2\\ -3 & 4\end{bmatrix}$. If we call a random vector in Step 1 as **x;** in this step we have two sets of random vectors **y=Ax** (as in Step 2)**;** and $\mathbf{z=Bx+[4\ 4]^T}$ where $\mathbf{[4\ 4]^T}$ is the mean of random vector **z**.

Find a decorrelating transform that decorrelates both **y** and **z** vectors. Show the scatter plot of **y** and **z** before and after decorrelation (please use different colors for the **y** and **z** vectors).