Final - January 20, 2009

Name and Surname:
Student ID:
Department:
Signature:

Instructions: In each of the question, explain why you do each step. The questions might contain unnecessary information. If the question does not contain sufficient information, make any necessary assumptions. If you make unnecessary assumptions, you will lose points.

1. Consider a particle that is free to move in one dimension only. Assume that, initially the particle is stationary at the point $x = 1m$. Suppose that, after $t = 0$, the particle is subject to the time dependent force $F = (2\ N/s)t$. What is the position of the particle as a function of time? (10points) (10 points)

2. Consider two masses, each 1 kg, attached at the end of a rod of length 2 m. What is the moment of inertia of this system for rotations around an axis passing through one end of the rod and perpendicular to the rod? (Ignore the mass of the rod) (10 points)

3. Consider a loop made of a string of length L and has a linear mass density ρ. If the loop is rotating with an angular frequency w around an axis passing through the center and perpendicular to the plane of the loop, what is the tension in the string? (20 points)

4. Ask yourself a question and answer it. The question should be based on some phenomenon that you observe in everyday life. (Not something that you see on TV or you think exists somewhere on earth. I will not believe if you claim that your family is a member of the Valley of the Wolves, etc.) The question should involve doing some explicit calculation. First explain the phenomenon. Then ask the question. Then solve your own question. (The solution should be based on thing that you learned this term) (20 points)

5. Consider a straight tunnel that goes through Earth. Assume that the closest approach of the tunnel to the center of the Earth is d. Denote the mass and radius of Earth by m_E and R_E. If you put a mass, m, in the tunnel, write down the equation of motion for the position of the mass in the tunnel ignoring any friction effects (In calculating the gravitational pull, assume that the mass distribution of Earth is uniform and ignore the mass excavated for the tunnel) (10 points). What kind of motion the mass will execute? Assuming that initially, the mass is released with zero velocity from one end of the tunnel, write down its position as a function of time. (10 points) (Your answer should not involve anything other than m, m_E, R_E, d and G_N.)

6. The father finally traps the unlucky lover in a dead end. The boy, still trying to get away from the father, plans to run fast and pass from the side of the father. His speed relative to the father is v_b. As the boy tries to pass by the father, the father tries to grab him. The boy, as he is running,

to get away, rotates around his own axis with angular velocity ω. But the father manages to grab the boy, and they are locked together. What is their angular velocity after the father grabs the boy. For this problem, model the boy and the father both as filled cylinders with masses $m_b = m$ and $m_{father} = 2m$, assume the radius of the boy is R. They are both of height L. Moreover, at the instant of collision, the distance between the centers of the boy and the father is $R_b + R_{father}$, i.e., they barely touch each other. (20 points)

Useful formulae:

You can use the following formula's without deriving them. For anything else, you need to derive it:

$$\begin{aligned} \vec{F} &= m\vec{a}\ , \quad \vec{a} = \frac{d\vec{v}}{dt}\ , \quad \vec{v} = \frac{d\vec{x}}{dt} \\ a_r &= \frac{v^2}{r} \end{aligned} \tag{1}$$

where a_r is the radial acceleration of an object making circular motion on a circle of radius r

$$I = \int dM d^2 \tag{2}$$

where dM is the mass of an infinitesimal volumes and d is the distance from the rotation access.

$$\begin{aligned} dV &= r^2 dr \sin\theta d\theta d\phi \\ &= \rho d\rho d\phi dz \\ &= dxdydz \end{aligned} \tag{3}$$

where $r^2 = x^2 + y^2 + z^2 = \rho^2 + z^2$, $\cos\theta = z/r$, $\tan\phi = y/x$