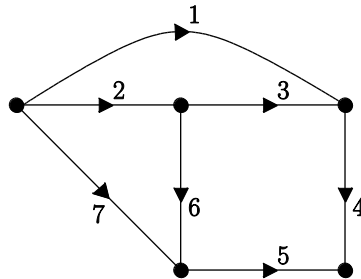


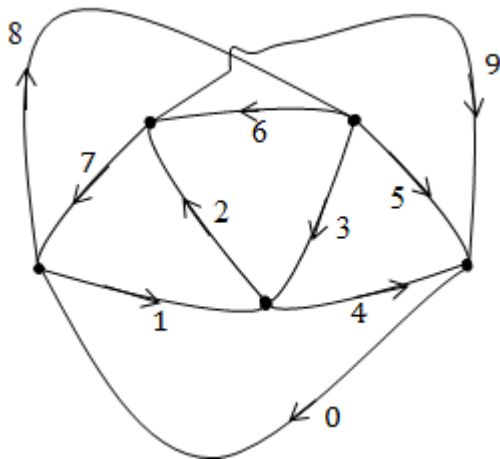
### HOMEWORK I

**Question 1** Consider the circuit graph below.



- Pick a tree. Determine the fundamental cutsets and the fundamental loops.
- Write the current equations on the fundamental cutsets and the voltage equations on the fundamental loops.
- Assign arbitrary nonzero branch voltages to the tree branches. Then compute the cotree-branch voltages.
- Assign arbitrary nonzero branch currents to the cotree branches. Then compute the tree-branch currents.
- Verify Tellegen's Theorem using the numerical values in Part (b) and Part (c).

**Question 2**



The figure on the left is the oriented graph of a lumped circuit.

Some of the branch currents and voltages are given below.

Find the missing currents and voltages.

$i_0 = 5A$	$i_1 = 3A$	$i_2 = ?$	$i_3 = 4A$	$i_4 = ?$	$i_5 = -7A$	$i_6 = ?$	$i_7 = ?$	$i_8 = 2A$	$i_9 = -4A$
$v_0 = 4V$	$v_1 = 3V$	$v_2 = 6V$	$v_3 = ?$	$v_4 = ?$	$v_5 = -2V$	$v_6 = ?$	$v_7 = ?$	$v_8 = ?$	$v_9 = ?$

**Answer:**  $i_2 = -9A$ ,  $i_4 = 16A$ ,  $i_6 = 5A$ ,  $i_7 = 0A$ ,  $v_3 = 5V$ ,  $v_4 = -7V$ ,  $v_6 = 11V$ ,  $v_7 = -9V$ ,  $v_8 = -2V$ ,  $v_9 = -13V$

**Question 3** For a lumped circuit  $N$  made up of single-branch elements, the reduced incidence matrix  $A$  is as given below.

$$A = \begin{bmatrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

- a) Obtain the circuit graph.  
 b) Let the circuit  $\hat{N}$  be the dual of  $N$ . Some of the branch currents and voltages for  $\hat{N}$  are given below. Find the missing currents and voltages.

$\hat{i}_1 = ?$	$\hat{i}_2 = 3 \text{ A}$	$\hat{i}_3 = ?$	$\hat{i}_4 = 1 \text{ A}$	$\hat{i}_5 = 1 \text{ A}$	$\hat{i}_6 = 6 \text{ A}$	$\hat{i}_7 = ?$
$\hat{v}_1 = ?$	$\hat{v}_2 = -5 \text{ V}$	$\hat{v}_3 = -1 \text{ V}$	$\hat{v}_4 = ?$	$\hat{v}_5 = ?$	$\hat{v}_6 = ?$	$\hat{v}_7 = 3 \text{ V}$

**Question 4** Certain measurements are performed on four different lumped circuits. All four circuits have the same graph  $G$  with six branches. For the first three circuits, the branch voltage readings (in Volts) are recorded in the following table.

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$
Circuit #1	2	5	-3	-1	3	2
Circuit #2	1	-5	6	2	-1	-4
Circuit #3	9	8	1	?	5	3

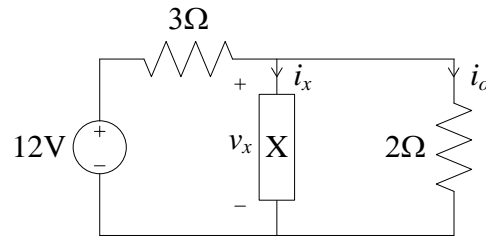
For the fourth circuit, the branch current readings (in Amps) are provided below.

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$
Circuit #4	?	?	2	1	-3	-4

- a) Find the missing current values,  $i_1$  and  $i_2$ , for the Circuit #4.  
 b) Find the missing voltage value  $v_4$  for the Circuit #3.

Answer: a)  $i_1 = -3 \text{ A}$ ,  $i_2 = 6 \text{ A}$  b)  $4 \text{ V}$

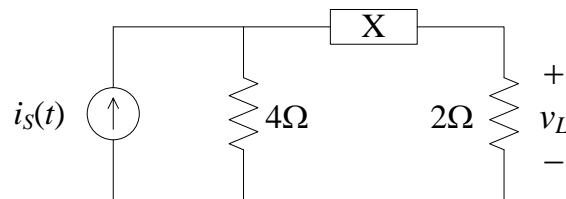
**Question 5** Consider the circuit shown below where “X” is a resistive element. The current through the  $2\ \Omega$  resistor is measured to be  $i_o(t) = 1.5\cos(t)$  A.



- Find the current  $i_x(t)$ .
- Find the instantaneous power delivered to X.
- Is X a time-invariant element? Explain.
- Is X a passive element? Explain.

Answer:  $i_x(t) = 4 - 2.5\cos(t)$  A

**Question 6** Consider the circuit shown below where “X” is a time-invariant resistive element.



When  $i_s(t) = 6 + \cos(\omega t)$  A, the voltage across the  $2\ \Omega$  resistor is measured to be  $v_L(t) = 6 - 2\cos(\omega t)$  V.

- Find the instantaneous power delivered to X.
- Is X a linear element? Explain.
- Is X a passive element? Explain.
- Find a possible branch relation for X.

Answer:  $P_x(t) = 18 + 24\cos(\omega t) - 10\cos^2(\omega t)$  W

$$V_x = -10 i_x + 36$$

### Question 7

In the circuit shown below, "X" is a resistive element. The results of two different experiments are shown in the table where  $P_s$  is the power supplied by the voltage source.

- Determine the value of R.
- Find  $i_x$  when  $V_s$  is 13 V.
- Is the element X linear or non-linear? Explain.
- Is the element X active or passive? Explain.

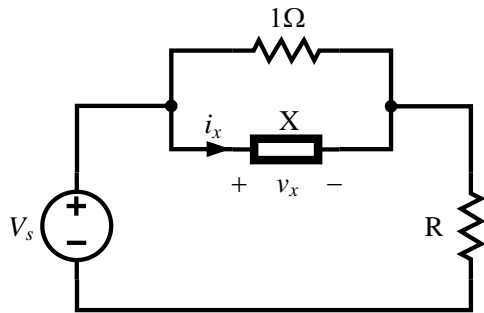


Table: Experiment results

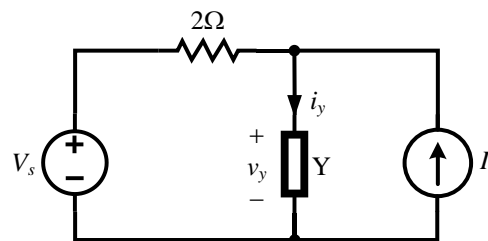
$V_s$	$P_s$	$i_x$
5 V	5 W	2A
13 V	26 W	?

Answer: a)  $6\Omega$  b) 1 A

### Question 8

In the circuit shown below, "Y" is a non-linear element whose branch relation is  $v_y = 3i_y^3$ . The following are given:

- $V_s > 0$ .
- The power absorbed by the  $2\Omega$  resistor is 18 W.
- The power absorbed by Y is 48 W.
- The voltage source is absorbing power.



Find the power delivered by the current source.

Answer: 120 W