HOMEWORK I

Question 1 The terminal equation of a 3-branch LTI coupled inductors (CI) is given below,

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} 10 & 7 & 4 \\ 7 & 5 & 3 \\ 4 & 3 & 2 \end{bmatrix} D \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix}.$$

a) Is this CI passive or active? Why?

b) Compute the stored energy in the CI given

(i) $i_1 = 1$ A, $i_2 = -2$ A, $i_3 = -1$ A. **(ii)** $i_1 = 1$ A, $i_2 = -2$ A, $i_3 = 1$ A.

c) The terminals of the third branch are short circuited, i.e., $v_3 = 0$. Obtain the terminal equation of the 2-branch CI,

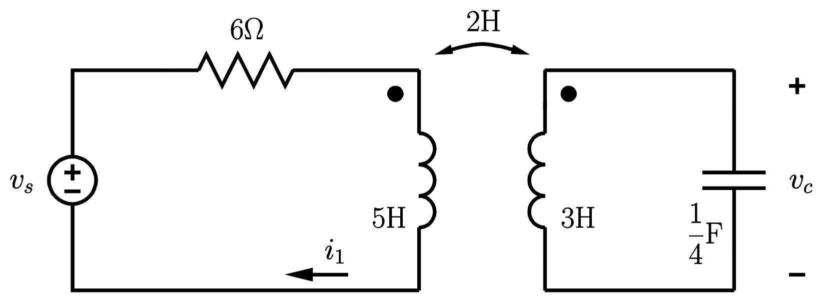
$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} D \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}.$$

Obtain the T and π equivalent models, and the equivalent model involving an ideal transformer. Comment.

Hint: For part (a), see Lec. 2 video 22nd minute and then look up the keyword "quadratic form" from internet sources.

Ans: a) Passive, b) (i) 4 J, c) $v_1(t) = 2\frac{d}{dt}i_1(t) + \frac{d}{dt}i_2(t)$

Question 2 Consider the circuit below.

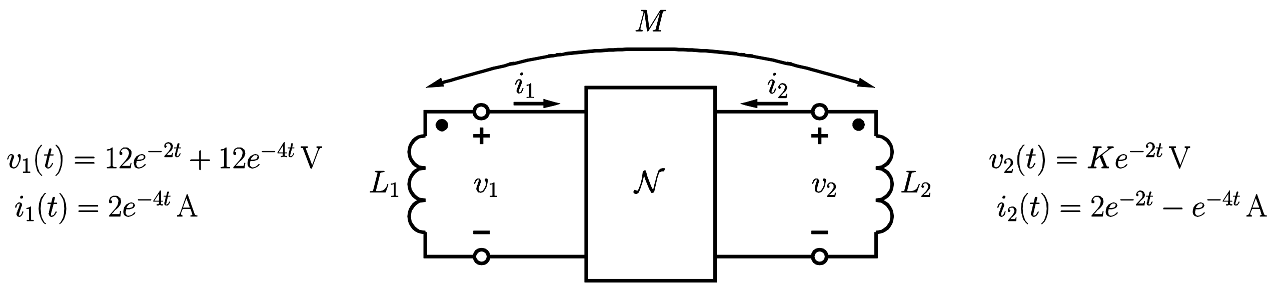


Given $i_1(0) = 2\sqrt{2}$ A and $v_c(t) = 4\cos(2t)$ V.

a) Find $i_1(t)$ and the source voltage $v_s(t)$.

b) Compute the stored energies in the coupled inductors and the capacitor at $t = \frac{\pi}{8}$ sec.

Question 3 Consider the circuit below where the two-port $\mathcal{N}$ is LTI resistive.



a) Find the inductances L_1, L_2, M and the constant K.

b) Find the resistance parameters of the two-port $\mathcal{N}$.

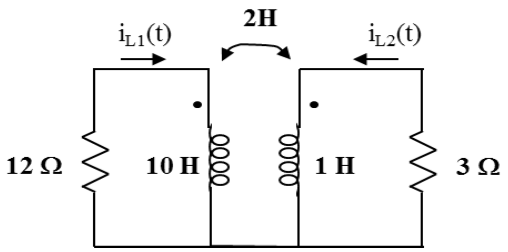
c) Find the instantaneous power input, p(t), to $\mathcal{N}$.

d) Computing the integral of p(t) from zero to infinity, find the energy delivered to $\mathcal{N}$ on the interval $[0, \infty)$.

e) Compute the stored energy in the coupled inductors at t = 0. Comment.

f) Observe that the coefficients a_k and b_k of circuit variables with the form $a_k \exp(\alpha t)$ and $b_k \exp(\beta t)$ for $\alpha \neq \beta$ do not interact or affect each other. Is this a coincidence specific to this problem? Comment.

Question 4 Consider the circuit below.



a) Obtain the state equation.

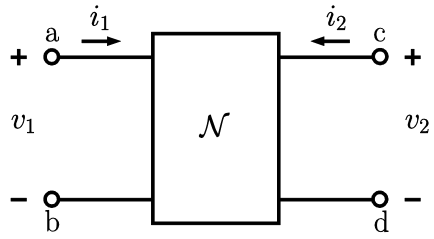
b) Find the natural frequencies.

c) Obtain the state transition matrix.

d) Given $i_{L1}(0) = 2$ A and $i_{L2}(0) = -8$ A, find $i_{L1}(t)$ and $i_{L2}(t)$ for $t \geq 0$, and compute the total energy delivered to the resistors on the interval $[0, \infty)$.

Ans: b) $s_1 = -1, s_2 = -6$ d) $i_{L1}(t) = 2e^{-6t}A, t \geq 0$

Question 5 The following two-port $\mathcal{N}$ is LTI resistive.

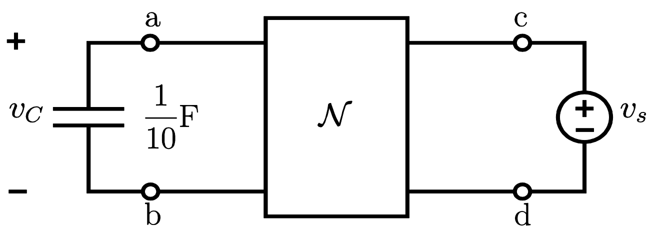


$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

a) Is $\mathcal{N}$ a passive or an active two-port? Why?

<u>Hint:</u> Denote the resistance matrix by R. Write $R = R_s + R_{ss}$ where $R_s = (R + R^T)/2$ is a symmetric matrix and $R_{ss} = (R - R^T)/2$ is a skew-symmetric matrix.

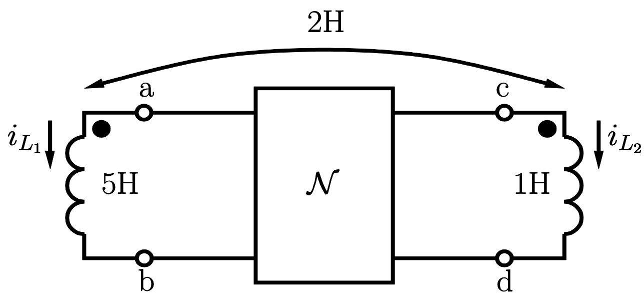
b) Consider the following circuit.



(i) Obtain the state equation.

(ii) For $v_C(0) = 2$ V and $v_s(t) = 3$ V, find $v_C(t)$ for $t \geq 0$.

c) Consider the following circuit.



(i) Obtain the state equation.

(ii) Determine the natural frequencies.

(iii) Obtain the state transition matrix.

(iv) Let $i_{L1}(0) = -3$ A. Find $i_{L2}(0)$ such that $i_{L1}(t) \to 0$ as $t \to \infty$.

Ans: a) Active, b) (ii) $v_c(t) = -10e^{2t} + 12\ V, t \geq 0$ c) (ii) $s_1 = 5, s_2 = -1$, (iv) $i_{L2}(0) = 3\ A$