HOMEWORK II

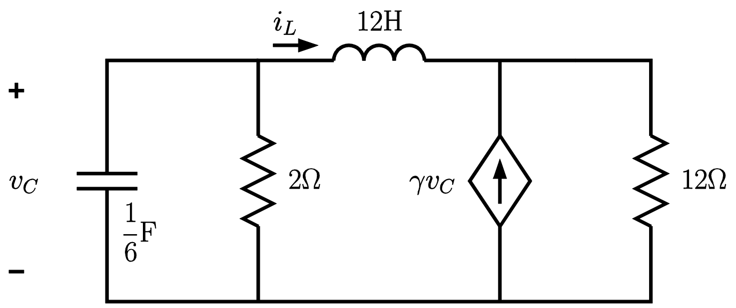
Question 1. Given the state equation of an LTI dynamic circuit,

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix} = \begin{bmatrix}1 & -1 & 0\\ 2 & 4 & 5\\ -2 & -2 & -3\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix}$$

a) Find the natural frequencies of the circuit.
b) Determine real initial values $x_1(0), x_2(0)$, and $x_3(0)$ so that $x_1(t), x_2(t)$, and $x_3(t)$ are bounded for $t \geq 0$.
c) For the initial values determined above, find $x_1(t), x_2(t)$, and $x_3(t)$ for $t \geq 0$.

<u>Ans</u>: (a) $s_1 = 2,\ \ s_2 = j,\ \ s_3 = -j.$
(b) (i) $x_1(0) = a/2,\quad x_2(0) = a,\quad x_3(0) = -a;\quad a \in R;$
(ii) $x_1(0) = b/2,\quad x_2(0) = 0,\quad x_3(0) = 0;\quad b \in R;$
The answer is any vector that can be expressed as a linear combination vectors shown in parts (i) and (ii), that is $x_1(0) = (a+b)/2,\quad x_2(0) = a, x_3(0) = -a;$ $a \in R, b \in R.$

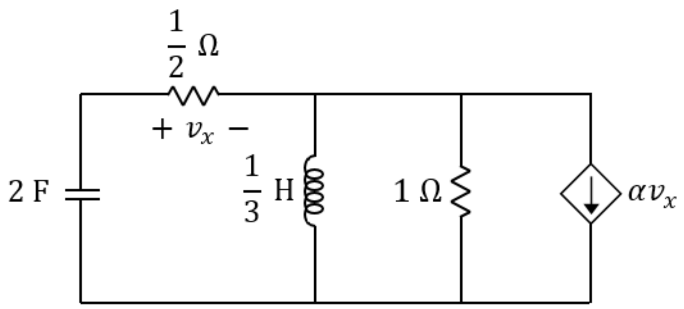
Question 2. Consider the circuit below.



a) Obtain the state equation.
b) Given $v_c(t) = (K_1 + K_2 t)e^{\lambda t}$, find λ and γ.
c) Let $\gamma = \frac{17}{12}$ mho.
i. Find the natural frequencies of the circuit.
ii. Find suitable initial conditions $i_L(0)$ and $v_C(0)$ such that the energy delivered to the 2 Ω resistor over the time interval $[0, \infty)$ is 5 J.

<u>Ans</u>: (b) $\lambda = -2,\quad \gamma = -1/12\ mhos.$
(c) (i) $s_1 = 1,\ \ s_2 = -5.$
(ii) $v_C(0) = 10\,V,\ \ i_L(0) = 10/3\,A;$ or $v_C(0) = -10\,V,\ \ i_L(0) = -10/3\,A.$

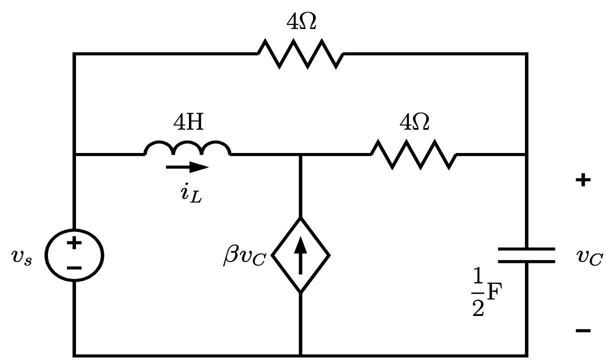
Question 3. Consider the circuit below.



a) Let $\alpha \neq 3$. Obtain the state equation.
b) Find α if one of the natural frequencies is at -3. Find the other natural frequency.
c) Find the range of α values to guarantee the stability of the circuit.
d) What happens if $\alpha = 3$?

Ans: (b) $\alpha = 2\ mhos,\ \ s_2 = -1.$
(c) $\alpha < 3\ mhos.$
(d) Single state variable, $v_C = -i_L.$

Question 4. Consider the circuit below.



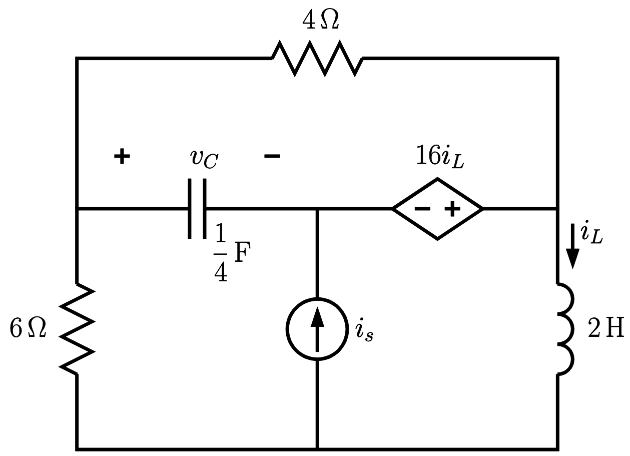
a) Obtain the state equation.

For the following parts, take $v_s(t) = 0$.

b) Let $\beta = 2$ mho and $v_C(0) = 4$ V. Find $i_L(0)$ so that only a single mode is excited.
c) Given $v_C(t) = \cos(\omega t)$ V, determine β and ω and find $i_L(t)$.

Ans: (b) $i_L(0) = -6\ A$ or $i_L(0) = -3\ A.$
(c) $\beta = 0.75\ mhos,\ \ \omega = 1\ rad/sec.$

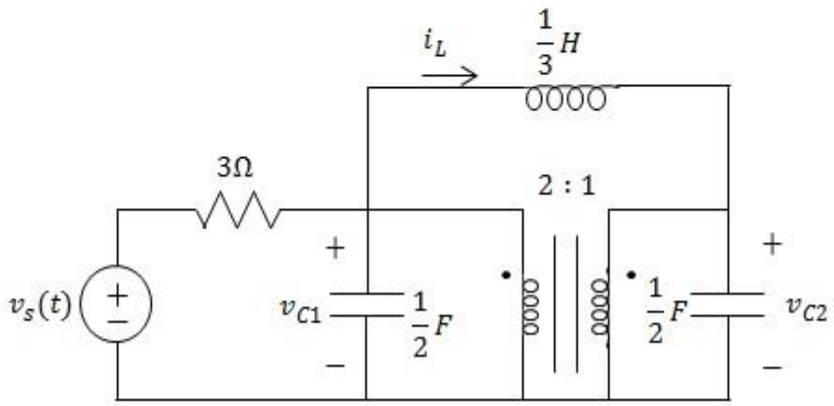
Question 5. Consider the circuit below.



a) Obtain the state equation.
b) Find the natural frequencies of the circuit.
c) Obtain the state transition matrix.
d) Given $i_s(t) = 10e^{-t}$ A, find the particular solutions for $v_C(t)$ and $i_L(t)$.

Ans: (b) $s_{1,2} = 2 \pm j$.
(d) $i_{L_p}(t) = 2e^{-t}\ A$.

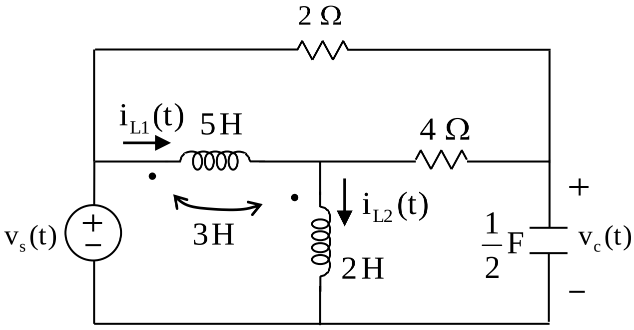
Question 6. Consider the circuit below.



a) Obtain the state equation.
b) Find the natural frequencies of the circuit.
c) Given $v_s(t) = 18$ V, find the particular solutions for $v_{C1}(t), v_{C2}(t)$, and $i_L(t)$.
d) Solve part (c) by studying the DC steady-state of the system. Does steady-state exist?

Ans: (b) $s_{1,2} = -4/15 \pm j\sqrt{254}/15$.
(c) $v_{C_{1p}}(t) = 0,\quad i_{L_p}(t) = 12\ A$.

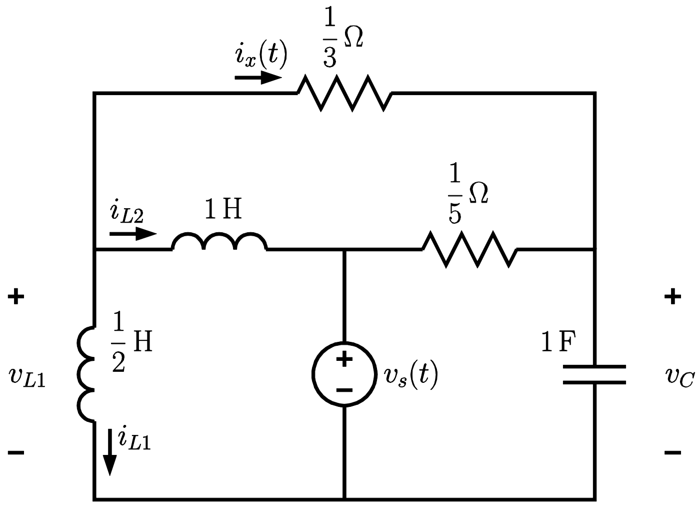
Question 7. Consider the circuit below.



a) Obtain the state equation.
b) Find the natural frequencies of the circuit.
c) What are the observed natural frequencies of the circuit for the branch variables $v_C(t), i_{L1}(t)$, and $i_{L2}(t)$?

Ans: (b) $s_1 = 0,\ \ s_{2,3} = -53/2 \pm \sqrt{2497}/2.$
(c) v_C: s_2, s_3; i_{L_1}, i_{L_2}: s_1, s_2, s_3.

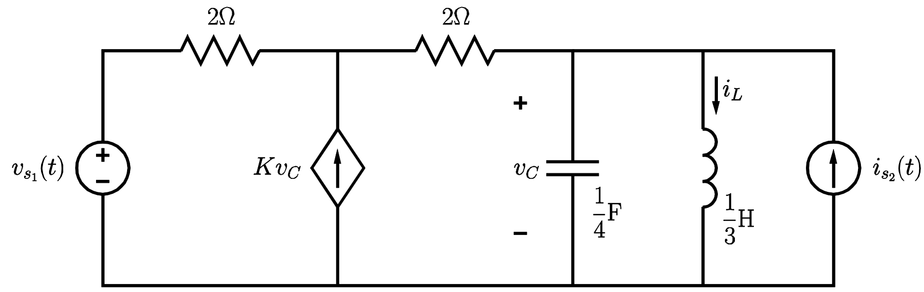
Question 8. Consider the circuit below.



a) Obtain the state equation.
b) Find the natural frequencies of the circuit.
c) Let the initial values of dynamic elements be specified. Write the forms of the zero-input solutions (do not determine the coefficients) for $v_C(t), i_{L2}(t), v_{L1}(t)$, and $i_x(t)$.
d) Given $v_s(t) = 9e^{-t}$ V, find the particular solutions for $v_C(t), i_{L1}(t), i_{L2}(t), v_{L1}(t)$, and $i_x(t)$.

Ans: (b) $s_1 = 0,\ \ s_2 = -2,\ \ s_3 = -4.$
(d) $i_{x_p}(t) = -33e^{-t}$ A

Question 9. Consider the circuit below.



a) Obtain the state equation.
b) Express the natural frequencies in terms of K.
c) Determine the value of K so that the natural frequencies are purely imaginary. For this value of K, obtain the state transition matrix.
For the following parts, take $K = 4$.
d) Find the natural frequencies of the circuit.
e) Obtain the state transition matrix.
f) Find the zero-input solution given $v_C(0) = 7$ V, $i_L(0) = 4$ A.
g) Given $v_{s1}(t) = 8$ V and $i_{s2}(t) = 3e^{6t}$ A,
- **i.** Find the particular solution, $\underline{x}_p(t)$.
- **ii.** Find the zero-state solution.
- **iii.** Determine the initial state $\underline{x}(0)$ so that $\underline{x}(t) = \underline{x}_p(t)$ for $t \geq 0$.

h) Given $v_{s1}(t) = 0$ and $i_{s2}(t) = e^{4t}$ A, find the zero-state solution.

Ans: (c) $K = 0.5\ mhos.$
(d) $s_1 = 4,\ \ s_2 = 3.$
(f) $v_{C_{zi}}(t) = 12e^{4t} - 5e^{3t}$ V, $t \geq 0.$
(g) (i) $i_{L_p}(t) = 2 + 6e^{6t}\ A.$
(ii) $i_{L_{zs}}(t) = -12e^{4t} + 4e^{3t} + 2 + 6e^{6t}\ A.$
(iii) $v_C(0) = 12\ V,\ \ i_L(0) = 8\ A.$
(h) $v_{C_{zs}}(t) = -12e^{4t} + 16te^{4t} + 12e^{3t}\ V, t \geq 0.$