HOMEWORK III

Question 1 A third order linear time-invariant circuit includes a single independent source. The natural responses for two sets of initial conditions are given below.

The responses to a set of initial conditions:	All voltages and currents in the circuit are in the form Ae^{-4t}.
The responses to another set of initial conditions:	All voltages and currents in the circuit are in the form $B\cos(3t+\theta)$.

Let x(t) be a voltage or a current in the circuit.

(a) Write the general form of x(t) when the input is $5\sin(2t+60°)$ and the initial conditions are arbitrary.

(b) Can you find a bounded input so that x(t) is unbounded?

Question 2 The natural frequencies of and the inputs to some linear time-invariant dynamic circuits are given below. Write the general form of complete response for any voltage or current for each case. Discuss the boundedness of the responses. Show the transient and steady-state parts of the responses provided that they are well defined.

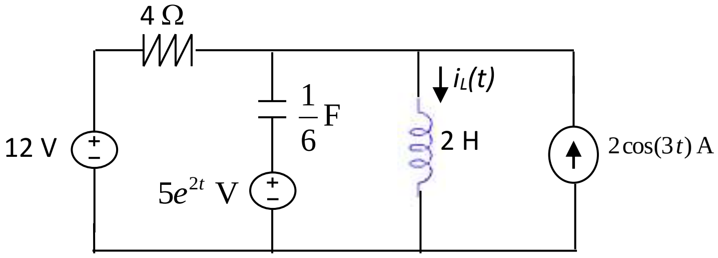
a) Natural frequencies: $\{-2, -3\}$, input: u(t).

b) Natural frequencies: $\{1, -1\pm j\}$, input: cos(2t)u(t).

c) Natural frequencies: $\{-2\pm j3\}$, input: cos(4t)u(t).

d) Natural frequencies: $\{\pm j2\}$, input: cos(2t)u(t).

Question 3 Consider the circuit below. Find the particular solution to the differential equation satisfied by $i_L(t)$.



Question 4 Refer to the circuit of HW-1/Q4. Let $i_{L1}(0) = 2$ A and $i_{L2}(0) = -8$ A.

a) Obtain the mesh equation. Determine the natural frequencies of the circuit.
b) Solve the mesh equation by Laplace transformation, and find $i_{L1}(t)$ and $i_{L2}(t)$ for $t \geq 0$.

Question 5 Refer to the circuit of HW-2/Q2. Let $\gamma = 17/12$ mho, $i_L(0) = I_o$, and $v_C(0) = V_o$.

a) Obtain the state equation.
b) Solve the state equation by Laplace transformation, and express $i_L(t)$ and $v_C(t)$ in terms of I_o and V_o.
c) Obtain the node equation.
d) Obtain the modified node equation.
e) Solve the node equation by Laplace transformation, and express the Laplace transforms of node voltages in terms of I_o and V_o.
f) Using the results of Part (e), find $I_L(s)$ and $V_C(s)$.
g) By inverse Laplace transformation, express $i_L(t)$ and $v_C(t)$ in terms of I_o and V_o.
h) Using the results of Part (g), find the initial condition sets for single mode excitation.

Question 6 Refer to the circuit of HW-2/Q3. Let $v_C(0) = V_o$ and $i_L(0) = I_o$.

a) Let $\alpha = 2$ mho.
  (i) Obtain the state equation.
  (ii) Solve the state equation by Laplace transformation, and express $v_C(t)$ and $i_L(t)$ in terms of V_o and I_o.
  (iii) Obtain the node equation.
  (iv) Solve the node equation by Laplace transformation, and express $v_C(t)$ and $i_L(t)$ in terms of V_o and I_o.
b) Let $\alpha = 3$ mho. Repeat Part (a).

Question 7 Refer to the circuit of HW-2/Q4. Let $\beta = 2$ mho, $v_C(0) = V_o$, and $i_L(0) = I_o$.

a) Obtain the state equation. Determine the natural frequencies of the circuit.
b) Solve the state equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $V_s(s)$, V_o, and I_o.
c) Obtain the node equation. Determine the natural frequencies of the circuit.
d) Solve the node equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $V_s(s)$, V_o, and I_o.
e) Obtain the mesh equation. Determine the natural frequencies of the circuit.
f) Solve the mesh equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $V_s(s)$, V_o, and I_o.

Question 8 Refer to the circuit of HW-2/Q5. Let $v_C(0) = V_o$ and $i_L(0) = I_o$.

a) Obtain the state equation. Determine the natural frequencies of the circuit.
b) Solve the state equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $I_s(s)$, V_o, and I_o.
c) Using the results of Part (b), obtain the state transition matrix.
d) Obtain the node equation. Determine the natural frequencies of the circuit.

e) Solve the node equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $I_s(s)$, V_o, and I_o.
f) Obtain the mesh equation. Determine the natural frequencies of the circuit.
g) Solve the mesh equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $I_s(s)$, V_o, and I_o.
h) Let $i_s(t) = 10e^{-t}u(t)$ A.
    (i) Find the zero-state solutions for $v_C(t)$ and $i_L(t)$.
    (ii) Transform the circuit to the phasor domain. Find the particular solutions for $v_C(t)$ and $i_L(t)$.

Question 9 Refer to the circuit of HW-2/Q6. Let $v_{C1}(0) = V_o$ and $i_L(0) = I_o$.

a) Obtain the state equation. Determine the natural frequencies of the circuit.
b) Solve the state equation by Laplace transformation, and express $V_{C1}(s)$ and $I_L(s)$ in terms of $V_s(s)$, V_o, and I_o.
c) Obtain the modified node equation. Determine the natural frequencies of the circuit.
d) Solve the modified node equation by Laplace transformation, and express $V_{C1}(s)$ and $I_L(s)$ in terms of $V_s(s)$, V_o, and I_o.
e) Let $v_s(t) = 18u(t)$ V.
    (i) Find the zero-state solutions for $v_{C1}(t)$ and $i_L(t)$.
    (ii) Transform the circuit to the phasor domain. Find the particular solutions for $v_{C1}(t)$ and $i_L(t)$.

Question 10 Refer to the circuit of HW-2/Q8. Let $v_C(0) = V_o$, $i_{L1}(0) = I_{o1}$, and $i_{L2}(0) = I_{o2}$.

a) Obtain the node equation.
b) Obtain the determinant of the node admittance matrix $Y_n(s)$.
c) Determine the natural frequencies of the circuit, and the natural frequencies of each branch current and each branch voltage.
d) Solve the node equation by Laplace transformation, and express $V_C(s)$, $I_{L1}(s)$, $V_{L1}(s)$, and $I_x(s)$ in terms of $V_s(s)$, V_o, I_{o1}, and I_{o2}.
e) Given $V_o = 3$ V, $I_{o1} = 4$ A, and $I_{o2} = -2$ A, find the zero-input solution for $i_x(t)$.
f) Let $i_s(t) = 9e^{-t}u(t)$ A.
    (i) Find the zero-state solutions for $v_C(t)$, $i_{L1}(t)$, $v_{L1}(t)$, and $i_x(t)$.
    (ii) Transform the circuit to the phasor domain. Find the particular solutions for $v_C(t)$, $i_{L1}(t)$, $v_{L1}(t)$, and $i_x(t)$.
g) Let $v_s(t) = 10\cos(4t - 20^o)u(t)$ V.
    (i) Find the zero-state solution for ix(t).
    (ii) Transform the circuit to the phasor domain. Find the particular solution for ix(t).

Question 11 Refer to the circuit of HW-2/Q7. Let $v_C(0) = V_o$, $i_{L1}(0) = I_{o1}$, and $i_{L2}(0) = I_{o2}$.

Obtain the mesh equation.