Ex: 3 buses departed from METU to Kizilay this morning. The first bus had only 10 passengers on board. The second bus had 70, and the third had 40 passengers. EGO, the company who runs the buses, wanted to find out whether the buses are too crowded or not, on average. They decided to sample several passengers at random, and ask them their estimate of how many people there were on the bus. What is the expected result? Compare this with the average number of customers on the three buses.

The example above is to illustrate the random incidence “paradox”. In the Poisson process, when a point in time is selected at random, larger intervals are more likely to get selected.

2.3.1 Splitting and Merging Poisson Processes

Ex: Show that, when we send each arrival of a Poisson process at rate λ to a process A with probability p, and process B with probability $1-p$, the resulting processes A and B are Poisson with rates $p\lambda$ and $(1-p)\lambda$. (Hint: (i) Express the transform of the interarrival time as a geometric sum of Exponentials, and use Definition 1 of the Poisson process, or, (ii) use the Baby Bernoulli definition of the Poisson process.)

Note: It can be shown that the processes resulting from randomly splitting each arrival in a Poisson process as above are independent of each other.

Ex: Show that when we merge two INDEPENDENT Poisson processes at rates λ_a and λ_b, we get a Poisson process at rate $\lambda_a + \lambda_b$. (Hint: Consider racing Exponentials.)

Extending the above by induction, we find that when we merge independent Poisson processes of rates $\lambda_i, i = 1, 2, \ldots, k$, we obtain a Poisson process of rate $\sum_{i=1}^{k} \lambda_i$.

Ex: Starting at $t = 0$, buses depart from the METU campus according to a Poisson process at rate 0.2 buses per minute. Each bus is independently assigned to go to Kizilay, or to Ulus, with probabilities 5/6 and 1/6, respectively. Starting at $t = 0$, people arrive at the bus stop according to a Poisson process of rate 1.8 people per minute, independently of the bus arrival process. People get on the first bus that comes (they do not distinguish between the destinations.)

(a) I arrive at the bus stop and a bus just left as I was arriving. I intend to go to Ulus. How long do I expect to wait?

(b) I arrive at random. What is the probability that the first bus departure I see is to Ulus?

(c) Compute the probability that exactly 5 buses depart in the first hour.

(d) What is the probability that among the first 5 bus departures, 3 go to Ulus? Also compute the probability that the fifth bus departure is the 3rd bus departure to Ulus.

(e) Find the probability that k people arrive, between two bus departures. Also compute the expected number of people getting on a bus.

(f) I show up at the bus stop at a random time. What is the expected number of people getting on the bus with me?