2.3.2 Examples related to the Poisson Process

Ex: Starting at $t = 0$, buses depart from the METU campus according to a Poisson process at rate 0.2 buses per minute. Each bus is independently assigned to go to Kizilay, or to Ulus, with probabilities 5/6 and 1/6, respectively. Starting at $t = 0$, people arrive at the bus stop according to a Poisson process of rate 1.8 people per minute, independently of the bus arrival process. People get on the first bus that comes (they do not distinguish between the destinations.)

(a) I arrive at the bus stop and a bus just left as I was arriving. How long do I expect to wait?

(b) I arrive at random. How much do I expect to wait? What is the probability that the first bus I see is to Ulus?

(c) Compute the probability that exactly 5 buses depart in the first hour. Compute the probability that at least 5 buses depart in a one-hour interval.

(d) What is the probability that among the first 5 bus departures, 3 go to Ulus? Also compute the probability that the fifth bus departure is the 3rd bus departure to Ulus.

(e) Find the PMF and the expectation of N, the number of people getting on a bus.

(f) I show up at the bus stop at a random time. What is the expected number of people on my bus?

(g) I arrive at the bus stop and learn that within the half hour previous to my arrival, exactly 6 buses departed. What is the probability that all 6 actually departed in the 10 minutes prior to my arrival? (The interested student can show a generalization of this: Given the number of arrivals occurring in a given interval, the arrival times are uniformly distributed in the interval. We will use this result in the analysis of "shot noise" in the next lecture.)

Ex: "The Random Telegraph Signal" Consider a process $X(t), t \geq 0$, that takes values ± 1 and changes polarity at time instants that arrive according to a Poisson process. Show that, if $X(0)$ takes the values 1 or -1 equiprobably, then $X(t) = 1$ with probability $1/2$ at an t. Compute the covariance of $X(t_1)$ and $X(t_2)$ for any t_1 and t_2.

Ex: "Shot Noise" Consider an impulse train $Z(t) = \sum_{i=1}^{\infty} \delta(t - S_i)$, where $\{S_i, i \geq 1\}$ are the arrival times in a Poisson process of rate λ. Let's pass $Z(t)$ through an LTI system with impulse response $h(t)$. The resulting filtered process, $X(t) = \sum_{i=1}^{\infty} h(t - S_i)$ is known as "shot noise". For example, suppose $h(t)$ is the current pulse resulting from a photon hitting a detector. Then, $X(t)$ would be value of the current at time t. Compute $E[X(t)]$.

This completes our introduction to discrete stochastic processes. The interested student can follow EE531, available on METU OpenCourseWare:
https://ocw.metu.edu.tr/course/view.php?id=323