6^{th} HOMEWORK

Due May 20, 2009

1. If the preferential direction of two adjacent sheets of Polaroid are at right angles, no light will pass through. However, if you now slip a third sheet of Polaroid between the other two and orient its preferential direction so that it lies between the other two, then some light will pass through the three sheets. Explain.

2. Suppose there is a small sphere inside water and it lies d meters deep. Suppose you look at the sphere from outside the water. What would be the apparent depth of the sphere if you look at it at angle close to the normal? close to $90°$ to the normal?

3. Show that if you have two thin lenses of focal length f_1 and f_2 placed close to each other, than the equivalent focal length, f_{eq} of the combined system will be given by $f_{eq}^{-1} = f_1^{-1} + f_2^{-1}$

4. Suppose that you have a curved mirror such that any ray passing parallel to a given axis passes through the same point after reflection, independent of the distance of the ray from the axis. Show that the shape of the mirror has to be a paraboloid, i.e. the shape can be obtained by revolving a parabola around its symmetry axis.

5. Consider two points P and Q that are in mediums of refractive index n_1 and n_2 respectively. The two mediums are separated by the xy plane. Consider a light ray that goes from point P to a point R on the interface between the mediums and than from point R to point Q. Calculate how long it takes for the light ray to go from point P to point Q. Show that this time is a minimum if the direction of the light ray in two media satisfy the Snell's Law.

6. Rainbows are produced by the refraction of sunlight by drops of water. Consider a spherical drop of water. The light ray is refracted at the point A, which then undergoes a total internal reflection at the point B and then refracted again as it leaves the drop at the point C. The angle of incidence at the point A is θ and the angle of refraction is θ'.

   (a) By geometry, show that the angles of incidence and reflection at B, coincide with θ' and that the angles of incidence and refraction at C coincide with θ and θ', respectively.

(b) Show that the angular deflection of the ray from its path is $\theta - \theta'$ at A, $\pi - 2\theta'$ at B, and $\theta - \theta'$ at C. These angles are all in radians, and they are measured clockwise from the incident path at each point.

(c) The total angular deflection of the ray by the raindrop is $\Delta = 2(\theta - \theta') + \pi - 2\theta'$. A rainbow will form when all the rays within and infinitesimal angle $d\theta$ of angles of incidence suffer the same angular deflection, i.e., when the derivative $d\Delta/d\theta = 2 - 4d\theta'/d\theta$ is zero. If this condition is satisfied, the rays sent back by the raindrops are concentrated, producing a bright zone in the sky. Show that the critical angle θ_c, at which $d\Delta/d\theta = 0$, is given by

$$\cos^2 \theta_c = \frac{1}{3}(n^2 - 1)$$

where n is the index of the refraction of water.

(d) The index of refraction for red light in water is 1.330. Find θ_c and find Δ in degrees and minutes of arc. Draw a diagram showing a red ray coming from the Sun, hitting the drop, and reaching the eye of a rainbow watcher.

(e) The index of refraction for violet light in water is 1.342. Find θ_c and find Δ. On top of the preceding diagram, draw a violet ray coming from the sun, hitting the drop (at a different point), and reaching the eye of the rainbow watcher. Will the watcher see red color above or below violet?