Final Exam - June 1, 2009

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not put the right units and put the vector signs for vectors. For vector quantities, you have to express both the magnitude and the direction. The questions might contain unnecessary information, or insufficient information. In the latter case, make necessary assumptions.
Everybody Should solve the first three questions, questions worth of 50 points. From the remaining questions, select questions to solve such that the total points will be 50 (or 60 if you have not chosen any question worths 10 points). Mark the questions that you have solve below.

1	2	3	4	5	6	7	8	9

1. Express each of the units $Volt$, $Farad$, $Ampere$, Ohm in terms of kg (for mass), m(for length), sec(for time) and C (for coulomb). (10 points)

2. Consider a wire in the shape of a square of side L. What is the self inductance of this wire? (20 points)

3. Show that if there is a lens with focal length f, an object put at a distance s will have its image at a distance s', given by
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}.$$
If $s < f$, s' turns out to be negative. What does negative s' mean? (20 points)

4. Show that if there is a ship that moves with a velocity v relative to the observers reference frame, the time in the ship slows down by the formula $t = t'/\sqrt{1 - v^2}$ (in units where $c = 1$). (10 points)

5. Consider a disc of radius R. The disc has mass m, radius R and rotates around it symmetry axis. If the disc has a total angular momentum L, along the symmetry axis, what is the magnetic dipole moment of the disc in terms of the given quantities? (20 points)

6. Consider a circuit consisting of an inductor with inductance L and a capacitor with capacitance C connected in parallel and then a resistor with resistance R is attached in series. If this circuit is driven by an emf source $\mathcal{E} = \mathcal{E}_0 \sin \omega t$, calculate the currents passing through the capacitor and the inductor separately. Show that the final expression have the correct units. (20 points)

7. Consider a hemispherical shell that has a uniform charge density σ distributed uniformly over its surface. Calculate the electric field at any point on the symmetry access. (the symmetry access is the line that passes through the center of the sphere and is perpendicular to the plane whose boundary is the rim of the shell). Show that the final expression have the correct units. (20 points)

8. Consider a U shaped wire and a conducting rod of mass m. The rod is placed on the wire such that the rod and the wire form a rectangle. Assume that this system is placed in a uniform gravitational field such that the rod is parallel to the ground and the rectangle makes an angle θ with the ground. The rod is free to slide on the wire. Assume that the whole system has an electric resistivity R and it does not change as the rod moves. We have seen in the first term that the rod would slide down with an acceleration whose magnitude is given by $a = g \sin\theta$. Assume that this system is put in a uniform magnetic field that is given by $\vec{B} = B_0 \hat{z}$. ($\hat{z}$ points upwards, in the opposite direction of the gravitational acceleration). If the rod is initially moving with a velocity v, write down an equation that will govern its subsequent motion(15 points). What is the angle θ for which the rod will slide down with constant velocity, v?(10 points) (Assume that the total resistance of the rectangle formed by the U shaped wire and the rod is R and is constant.) Show that the expression for the angle θ has the correct units. (5 points) (30 points total)

9. Consider two dipoles $\vec{p}_1$ and $\vec{p}_2$ that are separated by $\vec{r}$, Calculate the electrostatic energy of this system, and express it in terms of $\vec{p}_1$, $\vec{p}_2$ and $\vec{r}$. (Note that $\vec{p}_i$ and $\vec{r}$ need not have any simple orientations) (20 points)

Some numerical coefficients:

$$\begin{aligned} e &= 1.6 \times 10^{-19}\ C \\ \frac{1}{4\pi\epsilon_0} &= 9.0 \times 10^{9} Nm^2/C^2 \\ \frac{\mu_0}{4\pi} &= 10^{-7}\ N/A^2 \end{aligned} \tag{1}$$

Coulomb Law: $\vec{F}^E = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}\hat{r}$

Magnetic force between two moving particles: $\vec{F}_M = \frac{\mu_0}{4\pi}\frac{qq'}{r^2}\vec{v} \times (\vec{v}' \times \hat{r})$

Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2} d\vec{l} \times \hat{r}$

Gauss' Law: $\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon}$

Faraday's Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$

$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$,

$\oint \vec{B} \cdot d\vec{S} = 0$