Phys 430 - 2nd HOMEWORK

1. Consider a system of N non-interacting spins. Suppose that each spin
   can have the values $s_i = \pm 1$ and the total energy is given as

$$E = \alpha \sum_{i=1}^{N} s_i \tag{1}$$

   where α is some positive constant. Using the microcanonical distribu-
   tion, if the system has total energy $E_T = \alpha M$, where M is an integer
   between $-N$ and N, calculate the entropy and the temperature T of
   the system. Sketch S as a function of E_T, and T as a function of E_T.

2. Study the previous problem in the canonical distribution. Assume that
   the system has a temperature T. Calculate the average energy of the
   system and show that the relation between the energy and temperature
   is the same as the one obtained in the previous problem. Calculate $\langle s_1 \rangle$.

3. Show that for a particle in a 3D box,

$$\sum_{states} \simeq \int \frac{d^3 p \, d^3 q}{(2\pi\hbar)^3} \tag{2}$$

   To show this, first write the sum in terms of summation over the sin-
   gle particl quantum numbers. Then convert the summation over the
   integers to an integral. And then make a change of variables from
   integration over quantum numbers to integration over the momenta.

4. Consider a non-interacting ideal gas at a temparature T in two dimen-
   sions. Calculate the surface tension. (*Hint:* The analog of pressure in
   two dimensions is called the surface tension.)

5. Consider a cylinder of radius R and height h filled with N non-interacting
   particles of mass m at a temperature T. If the cylinder is under uniform
   gravitational field, calculate the density distribution.