PHYS 430 - First Midterm
7 April 2007

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.

YOU HAVE 4 HOURS WITH A POSSIBILITY OF EXTENSION

1. The second law of thermodynamics states that the entropy of an isolated system either increases or remains constant. How does the concept of probability in statistical mechanics lead to the same conclusion? Explain. (20 points)

2. Answer the following questions briefly: (5 points each)(10 points)

   (a) Which of the following can be the entropy of a system. Give a brief reason.

      i. $S = N$

      ii. $S = \ln \frac{EV^{\frac{3}{2}}}{N^{\frac{5}{2}}}$

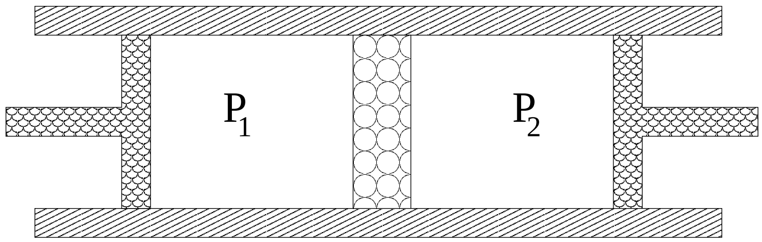
      iii. $S = E\frac{V^{\frac{3}{2}}}{N^{\frac{3}{2}}}$

   (b) If the entropy of a system is given by $S = \ln \left[E\left(\frac{V}{N}\right)^n\right]$, calculate the temperature T, the free energy F, the heat function S for this system

3. Consider the system shown in the figure. The left piston compresses the gas at a constant pressure P_1. The gas passes through the porous membrane. The membrane is such that when the gas passes through it has the lower pressure $P_2 < P_1$. As the gas passes through the membrane, the piston on the right expands so as to keep the pressure constant.

   Suppose that initially all the gas is on the right, and finally all is on the left. Calculate the change in the temperature. (25 points)

   For this purpose:



(a) Consider the passage of a small fixed amount of gas. By equating the change in the energy of this gas as it passes through the medium to the work done on the gas, show that the heat function/enthalpy ($W = E + PV$) is constant. (10 points)

(b) Consider, the rate of change of temperature as pressure changes at fixed enthalpy:

$$\left(\frac{\partial T}{\partial P}\right)_W \tag{1}$$

and express it in terms of the equation of state. (10 points)

(c) Integrate the previous result from P_1 to P_2 to find the change in temperature. (5 points)

4. The car engine can be idealized as follows: First the piston compresses that gas. As the process is very fast, during this stage, there is negligible heat exchange with the environment. Then the gas is ignited, and pressure increases before the piston can move and hence change volume. The the piston starts moving and the gas expands, again almost adiabatically. The gas expands until the gas reaches exhaust when the pressure suddenly drops and we are back to the initial stage and the cycle repeats. Assume that the maximum volume is twice the initial volume. (20 points)

   (a) Draw the PV diagram of this idealized process. Define that labels that you will use in the next part. (10 points)

   (b) Calculate its efficiency. (15 points)

5. Consider a chain of 6 pieces of length a. If each piece can be either vertical or lateral, what is the entropy of the chain, if the coordinates of the end of the chain is $(2a, 2a)$ relative to the beginning?(*Hint:* If you consider the i^{th} piece, even if it is vertical(or lateral) it has

two possible orientations: it can point up or down. Hence you can consider the chain as a chain made of N arrows which can point in four directions only) (20 points)

6. Consider a system of two spins s_i, $i = 1,\ 2$ at temperature T. Each spin can take only two different values $s_i = \pm\frac{1}{2}$. The system has the energy $E = -\epsilon s_1 s_2 - B(s_1 + s_2)$ where ϵ and B are some positive constants. What is the average of the total spin of the system, $S_t = s_1 + s_2$ as a function of temperature T and the coefficient B? Take the limit as $T \to 0$ and then $B \to 0$. What is the average $\langle S_t \rangle$? What happens if you change the order if the limits, i.e. first take the limit $B \to 0$ and then $T \to 0$? (*Hint:* First consider all the states of the system and the probability that the system will be found in that state) (20 points)

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST\ ;\quad W = E + PV\ ;\quad \Phi = E - ST + PV\ ;\quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E)\ ;\quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For anything else, you need to derive it.