PHYS 430 - Second Midterm
5 May 2007

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.
Total points is 110, with 10 bonus points.
YOU HAVE 4 HOURS WITH A POSSIBILITY OF EXTENSION

1. We have discussed that both the Bose-Einstein distribution and the Fermi-Dirac Distribution reduces to the Boltzmann distribution in a certain limit. Give a physical interpretation for this limit. (10 points) As one gets away from this limit, the ideal gas equation of state gets modified. Considering a boson gas, a fermion gas and a Boltzmannian gas with the same temperature and the same volume, compare their pressures and explain.(10 points) (Total 20 points. Do not use equations, you will loose extra 2 points for each equation you write)

2. Starting from the definitions

$$\begin{aligned} \Omega &= -T \ln \sum_n \sum_{states\ k} e^{-\beta(\epsilon_{nk} - \mu n)} \\ S &= \frac{E - \Omega - \mu N}{T} \end{aligned}$$

   where $E = \langle \epsilon_{nk} \rangle$, $N = \langle n \rangle$ and without using any of the equations given at the end of the exam, show that

$$d\Omega = -SdT - PdV - Nd\mu \tag{1}$$

   (*Hint:* Start from the first equation, i.e. the definition of Ω, and consider infinitesimal changes in V, T and μ, and write down the change in Ω) (20 points)

3. (a) According to the equipartition theorem, what should be the specific heat per molecule for an ideal gas made up of diatomic molecules? Explain each contribution to the total result. (10 points)

   (b) In the figure, the dependence of the specific heats per molecule of some real diatomic molecules are shown. It is observed that in general, the specific heat depends on the temperature. It s also observed the for some molecules there are temperature ranges where the temperature dependence is minimum, e.g. between $300K$ and $600K$, the specific heats of HCl and H_2 have a constant value of $\frac{5}{2}$, above $800K$ the others have a specific heat which is almost constant at $\frac{7}{2}$. Explain these constant values using the equipartition theorem. (15 points)

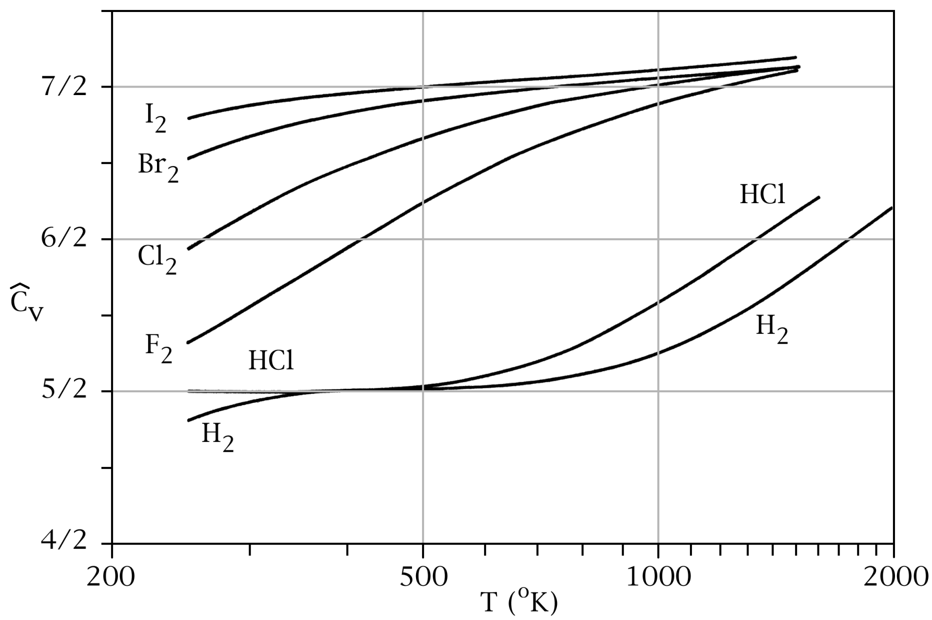

Figure 1: From Lewis, G.N.; Randall, M. (1961). Thermodynamics, 2nd Edition, New York: McGraw-Hill Book Company.

4. Consider a system consisting of only two particles. The particles can only be in states 1 and 2 and the energies of these states are given by ϵ_1 and ϵ_2 respectively. If the system has a temperature T, write down the partition function if

   (a) The particles are distinguishable

   (b) The particles are identical particles that satisfy the Boltzmannian distribution

   (c) The particles are identical Fermions

   (d) The particles are identical Bosons

   (*WARNING:* The number of particles is fixed) (5 points each, 20 points total)

5. Consider a gas of noninteracting point like particles forced to move only in two dimension and enclosed in a surface area A. The temperature of the gas is given as T. Suppose the energy of each of these particles is related to its momenta through $\epsilon = cp$ where c is some constant and p is the magnitude of their momenta. Calculate the specific heat per particle for this system. (25 points)

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy &= \int_0^{2\pi} d\phi \int_0^\infty d\rho\rho, \qquad \rho^2 = x^2 + y^2, \qquad \tan\phi = \frac{x}{y} \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For anything else, you need to derive it.