PHYS 430 - Final Exam
June 8, 2007

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.
Answer ONLY 5 of the questions.
Which Questions Do You Want Graded: | | | | | |
YOU HAVE 2 HOURS 30 MINUTES WITH NO EXTENSION

1. During the term, we have discussed three different ensembles: micro canonical, canonical, and grand canonical. Compare and discuss the three distributions corresponding to these three ensembles. In each case, do not forget to mention what is conserved and what is allowed to vary in each ensemble. (Total 20 points. Do not use equations, you will loose extra 2 points for each equation you write)

2. Calculate the Fermi energy of a two dimensional electron gas that is made of N electrons confined to a surface area A. (20 points)

3. For a system of non-interacting identical point like fermions or bosons whose energy depends on momentum through $\epsilon = \alpha p^n$, calculate $\frac{\Omega}{E}$ where Ω is the free energy in grand canonical distribution and E is the average energy. (20 points)

4. Consider the subsystem of a boson system consisting of all the particles in the single particle state that have the energy ϵ. If the system has a temperature T and chemical potential μ (25 points total)

   (a) What is that average of the square of the number of particles? (10 points)

   (b) Show that the fluctuation in the number of particles is given by (10 points)

$$\Delta n^2 = \langle n^2 \rangle - \langle n \rangle^2 = T \left(\frac{\partial \langle n \rangle}{\partial \mu} \right) \Bigg|_{T,V} \tag{1}$$

(c) Calculate the relative fluctuation in the number of particles. (5 points)

5. In Fig. 5, the temperature-entropy diagram of a heat engine undergoing a reversible cyclic process is shown. Calculate the efficiency of this heat engine. (20 points)

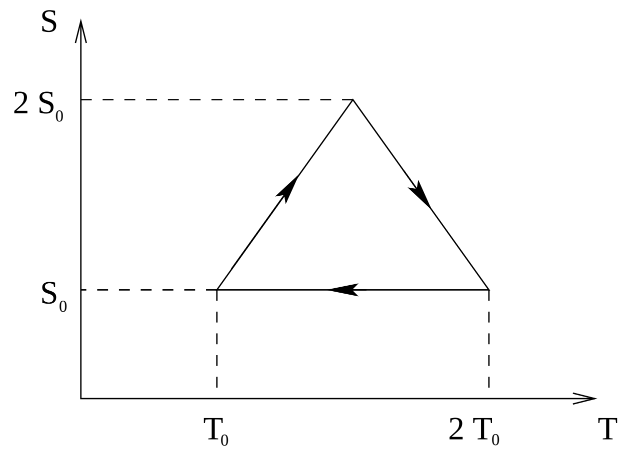

Figure 1: TS diagram of a reversible heat engine

6. Consider a system of identical bosons. The single particle states can be labeled by an integer n and the energy of the n^{th} state is given by $E_n = n\epsilon$, $n = 0, 1, 2, \cdots$. (25 points)

   (a) If the system has only two bosons and the total energy of the system is 4ϵ, what is the entropy of the system? (10 points)

   (b) If one more boson is added to the system, what should be the new energy of the system so that the entropy of the system remains the same. (10 points)

   (c) What is the chemical potential of this system? (5 points)

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty dx x^n e^{-x} &= n! \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy &= \int_0^{2\pi} d\phi \int_0^\infty d\rho\rho, \qquad \rho^2 = x^2 + y^2, \qquad \tan\phi = \frac{x}{y} \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}$$

For anything else, you need to derive it.