PHYS 430 - Make-up Exam
June 12, 2007

**Name and Surname:**
**Student ID:**
**Department:**
**Signature:**
**Reason for taking the Make-Up:**

## INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.
YOU HAVE 4 HOURS

1. Explain the following concepts: (5 points each, total 25 points. Do not use equations, you will loose extra 2 points for each equation you write)
   (a) Fermi Energy
   (b) Bose-Einstein condensation
   (c) Microcanonical distribution
   (d) Ensemble
   (e) Equipartition theorem

2. Consider a gas of non-interacting ideal gas confined in a volume $V$. Use Boltzmannian distribution and the grand canonical ensemble to show that
$$PV = NT \tag{1}$$
where $P$ is the pressure, $V$ is the volume, $N$ is the number of particles and $T$ is the temperature of the system. (25 points)(You will not gain any points if you do not use grand canonical distributions.)

3. In the canonical distribution, the free energy of a single particle in a non-interacting gas can be written as
$$F = -T \ln \sum_{CM\ \ states} \sum_{internal\ \ states} e^{-\beta(\epsilon^{tr}+\epsilon^{int})} = F_{id} + F_{int} \tag{2}$$
where $F_{id}$ is the free energy of a single point-like particle, and
$$F_{int} = -T \ln \sum e^{-\beta\epsilon^{int}} \tag{3}$$

is the internal free energy. If the specific heat of this system is known to have a constant value of $c_V$, what is the most general form of $F_{int}$. (25 points) (*Hint:* Deriving $c_V$ from the free energy would give a differential equation for $F_{int}$. You need to solve the equation)

4. Consider a system of $N$ spin-$\frac{1}{2}$ particles. Each spin can only be in two states with energies $\pm\epsilon$. If the system has a temperature $T$, what is the entropy o the system? (25 points)

5. Consider a system of fermions at a temperature $T$. There are two states that have energy $\epsilon$ and a single state that has energy $2\epsilon$. If the system has the chemical potential $\mu$, what is the average number of particles that have energy $\epsilon$ or $2\epsilon$? (25 points)

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty dx x^n e^{-x} &= n! \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy &= \int_0^{2\pi} d\phi \int_0^\infty d\rho\rho, \qquad \rho^2 = x^2 + y^2, \qquad \tan\phi = \frac{x}{y} \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For anything else, you need to derive it.