1-) Here are some probability warm ups :
a. Suppose you wrote a down a specific sequence of length N of heads and tails, and then flipped a coin N times. What is the probability that the specific sequence you wrote out will correspond to the actual coin tosses?
b. If you throw a coin 100 times, what is the probability that the there will be an equal number of heads and tails?
c. Imagine you are engaging in a little low-stakes coin-toss gambling with a class mate, and you win 1 cent for every head but lose 1 cent for every tail. At some point you find that you are 50 cents ahead.
What is the probability that after 50 more tosses you will still be ahead by at least 5 cents?

2-) The "random walk problem" (1-dimension): A person making steps of length L is just as likely to walk forward as backward. Show that after taking N steps the probability of having gone forward a distance RL (where is R is some integer) is

$$\left(\frac{1}{2}\right)^N \begin{pmatrix} N \\ \frac{1}{2}(N+R) \end{pmatrix}$$

where the notation for the second term on the right is another standard way of writing our binomial multiplicity function:

$$\begin{pmatrix} N \\ M \end{pmatrix} = \frac{N!}{M!(N-M)!}$$

3. In any group of 20 people, what is the probability that at least two will have the same birthday?
Compare the results for both Stirling's approximation and direct multiplication. Assume for simplicity that the year always consists of 365 days.

4-) Suppose an event characterized by a probability p occurs n times in $N \geq n$ trials. Show that the probability of n such occurrences is

$$w(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \quad .$$

5-) Experts on college examinations often state that 10% of a class should get grades of "A". That is the practically the same as saying that the probability of one student getting a grade of "A" is one in ten, or 1/10, or 0.1. Assume that probability is correct, and that grades on examinations are completely random (like throwing dice). Then, in a class of 24 students (such as this one) what is the probability that 12 students will get grades of "A" on an examination?