PHYS-430 RECITATIN 2

1.) In a certain quantum-mechanical system, it is known that the y component of the angular momentum, Ly is quantized and can take only three discrete values: -ħ, 0 or ħ. Also it is known that $< L_y >= \hbar /3$ and $< L_y^2 >= 2\hbar^2/3$, where  is a constant with units of angular momentum.
Find the probability density p(Ly) for Ly. Sketch the result.

2.) Consider a system in which the particles are 30 students confined to room with 50 chairs. Suppose the particles have no kinetic energy so they occupy chairs in the room and there is no interaction energy so there is at most one particle per chair. What is the entropy of this system?

3.) In a simple model of gas, the total volume accessible to N particles is divided into V cells, with $N \le V$ , so that each cell can be either empty or occupied by a single particle.
a) Obtain an expression for g(V,N), i.e., the multiplicity function or number of microstates of this system.
b) Obtain an expression for the entropy per particle s(v) = S/N in the thermodynamic limit (i.e. $N \to \infty$ ),where v=V/N is the specific volume.

4.) The Poisson Distribution. We obtain the binomial distribution. Recall, the probability

$$p_n = \binom{N}{n} p^n (1-p)^{N-n} ,$$

Consider the binomial distribution in the limit that $N \to \infty$, $p \to 0$, but $Np \to r$ .
a) Show that the probability of obtaining n is

$$p_n = \frac{r^n}{n!} e^{-r} .$$

This is called the Poisson distribution. This distribution occurs in counting, for example, counting nuclear decays. Can you see why this is applicable? (Consider a time interval in which, on the average, there will be r decays and divide the interval into a large number, N, of equal subintervals. Then the probability of a decay in any interval is r/N, and if N is large enough you don't need to worry about two or more decays in an interval...)
(b) Show that the mean and variance of the Poisson distribution are both r. What is the fractional uncertainty?

5.) Traffic jams have some interesting physics associated with them. In a simple model, suppose the probability a car has speed in the range $(v, v+dv)$ is

$$f(v)dv = Av\exp\left(-\frac{v}{v_0}\right)dv$$

Here $v_0$ is a characteristic speed and A is a normalization constant. Find A.  What is the average speed?  Is the average speed the same as the most probable speed?  Roughly plot the distribution function.