PHYS-430 RECITATIN 3

1.) A quantum harmonic oscillator with frequency ω can exist in states with energies (relative to zero point energy) of $\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, ... If there are N identical oscillators, the number of ways to obtain an energy $n\hbar\omega$ is

$$g(N,n) = \frac{(N+n-1)!}{n!(N-1)!}$$

(a) Find the entropy of this system of oscillators when N is large (so that you can use the Stirling approximation and you can ignore 1 compared to N).
(b) Find an expression for the energy in terms of the temperature.

2.) Consider a system composed of N non-interacting, distinguishable, spin-1/2 particles. In an applied external magnetic field B the "up" and "down" states of each spin have respectively energies $-\mu B$ and $+\mu B$ where the magnetic moment μ is a constant.
(a) What is the total number of states of this system?
(b) How many states are available to the system if there is an applied B and if the system's total energy is E where $-N\mu B < E < N\mu B$ and E is an integer multiple of μB.
(c) Write the entropy of the system as a function of E, B, and N. Make use of Stirling's formula to express any factorials that appear in your expressions.
(d) Calculate the temperature T of the system as a function of E, B, and N.
(e) Solve for the energy E as a function of the temperature T, B, and N.

3.) Consider a lattice with N spin-1 atoms. Each atom can be in one of three spin states, S_z = -1; 0; +1. Let n_{-1}, n_0, and n_1 denote the respective number of atoms in each of those spin states. Assume that no magnetic field is present, so all atoms have the same energy.
(a) Find the total entropy as a function of n_{-1}, n_0, and n_1
(b) Which configuration (n_{-1}; n_0 ;n_1) maximizes this entropy?
(c) What is the entropy in this maximized configuration?