1^{st} Homework, Part I
Due October 9, 2009

1. Show that the Levi-Civita tensor is a scalar, although it has 3 indices, i.e. show that under a rotation,

$$\epsilon_{ijk} \to (det\Lambda)\epsilon_{ijk} = \epsilon_{ijk} \quad (1)$$

where Λ is the rotation matrix. *Hint:* First, show that Levi-Civita tensor remains completely anti-symmetric even after rotation. Then, calculate what ϵ_{123} transforms into after the rotation.

2. Simplify the following expressions such that the result contains at most on vector product.
   (a) $(\vec{A} \times \vec{B}) \times (\vec{C} \times \vec{D})$
   (b) $\vec{A} \times (\vec{B} \times (\vec{C} \times \vec{D}))$
   (c) $(\vec{A} \times \vec{B}) \cdot (\vec{C} \times \vec{D})$

3. Show the following relations:
   (a) $\vec{\nabla}(fg) = f\vec{\nabla}g + (\vec{\nabla}f)g$
   (b) $\vec{\nabla}(\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times B) + \vec{B} \times (\nabla \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$
   (*Hint:* Express both sides in terms of components and simplify)

4. (a) Calculate the line integral of $\vec{V}(x, y, z) = y\hat{z}$ along a semi circle of unit radius that starts from the point $(x, y, z) = (1, 0, 0)$, passes through the point $(x, y, z) = (0, 1, 0)$ and ends at the point $(x, y, z) = (-1, 0, 0)$
   (b) Consider the $x > 0$ portion of the sphere of unit radius. Calculate the surface integral of the function in the previous section on this surface.
   (c) Calculate the volume of a sphere using Cartesian coordinates.