1^{st} **Midterm - November 8, 2009**

Name and Surname:
Student ID:
Department:
Signature:

You should show your work. You will lose points if you do not show your work.

Discussion

Answer the following question with words only. You do not need to give a quantitative answer, a qualitative answer is enough. You will lose points if you use equations.

1. During the lectures, the force acting on a dielectric is derived using the following method:
   Consider a (assume rectangular with length given by l) capacitor, whose capacitance is C in the absence of a dielectric. A dielectric that has the same width as the capacitor is inserted such that the length of the dielectric inside is $l - x$. The new capacitance is given by $C_{eq} = C\frac{x}{l} + \epsilon_r \frac{L-x}{L}$. The energy stored in the capacitor is then given by $W = \frac{Q^2}{2C_{eq}}$. Then the force can be obtained as $F = -\frac{dW}{dx} = \frac{Q^2}{2C_{eq}^2}\frac{dC_{eq}}{dx} = \frac{1}{2}V^2\frac{dC_{eq}}{dx}$ which is constant.

   - In this derivation, it was assumed that the total charge of the capacitor is constant. If one assumes that the potential is constant (i.e. if the capacitor is attached to a battery), then $F = -\frac{dC_{eq}}{dx} = -\frac{d}{dx}\frac{1}{2}\frac{C_{eq}}{V}^2 = -\frac{V^2}{2}\frac{dC_{eq}}{dx}$ which differs from the previous result by an obvious minus sign. What is the reason of the discrepancy? What is missed in which derivation? (10 points)
   - Consider the case $x = 0$. In this case, the dielectric is completely inside the capacitor and assume that the dielectric fits perfectly. Due to the symmetry, the force acting on the dielectric should be zero. But it was shown that the force is independent of the value of x. Why does not our result work when $x = 0$? (10 points).

Short Questions

You can give short answers in the following questions. You do not need to show your work in detail.

2. Using Gauss' Law, calculate the electric field of a conduction sphere of radius R that has a total charge Q. Calculate the electric field both for $r > R$ and $r < R$. (10 points)

3. In the previous problem, assume that the sphere is covered by a dielectric shell of thickness R. What is the displacement field all over space? (10 points)

4. Show that a given polarization $\vec{P}$ is equivalent to a volume charge density $\rho_b = -\vec{\nabla}\cdot\vec{P}$ and a surface charge density $\rho_b = \vec{P}\cdot\hat{n}$, where $\hat{n}$ is the normal vector to the surface. (10 points) (The electrostatic potential due to the polarization of a material is given by

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int d^3r'\,\frac{\vec{P}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

Explicit Calculations:

In answering following problems, show your steps in detail. You should also explain why you do a specific step.

5. In cylindrical coordinates, write down the most general solution of the Laplace equation inside a cylindrical region using separation of variables. (Find the solution that is finite everywhere) (20 points)

6. Consider an infinite hollow cylinder made of a conducting material. Assume that the cylinder is grounded. Inside the cylinder, there is an infinite straight wire parallel to the main axis of the cylinder and a distance d away from it. The charge density of the wire is given as λ.

   (a) Find the electrostatic potential inside the tube using method of images (15 points)

   (b) Do the same as in previous part using separation of variables. (15 points)

   (c) What is the charge density on the surface? (10 points)

   (Note that the problem has translational symmetry along the axis of the cylinder. The solution should not contain any Bessel function or Legendre polynomial)

Some useful formulas:

Associated Legendre Polynomials $P_l^m(x)$ are solutions of the differential equation:

$$\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] + \left[l(l+1) - \frac{m^2}{1-x^2}\right]y = 0 \qquad (1)$$

that are finite for $-1 \le x \le 1$. Bessel functions $J_n(x)$ are solutions of the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0 \qquad (2)$$

that are finite at $x = 0$.