PHYS 430 - Second Midterm
4 November 2006

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary.
The exam contains a total of 121 points, with 21 bonus points.
YOU HAVE 4 HOURS WITH A POSSIBILITY OF EXTENSION

1. Discuss the three distributions: the microcanonical, canonical and grand canonical. In particular, mention the quantities that are allowed to change or kept constant in one but not the others. Under what conditions would you use one or the other? (20 points)

2. Starting from the definitions of the free energy $F = -T \ln Z$ where $Z = \sum_{states} e^{-\beta T}$ and of the entropy $S = -\langle \ln \rho \rangle$ where ρ is the distribution function, show that (20 points)

$$dF = -SdT - PdV \tag{1}$$

   (You will not get any points if you use any of the relations at the end of this exam without explicitly demonstrating it first.)

3. Consider a chain of N arrows, one attached after the other. Each arrow can point either towards the left or towards the right. Let ℓ be the length of each arrow. If the total length of the chain is L, calculate its entropy $S(L)$. (20 points)

4. Consider a classical ideal gas consisting of N non-interacting particles having the energy momentum relation $\epsilon = cp$ where p is the magnitude of their momenta and c is some positive constant. Using canonical distribution: (25 points)

   (a) Show that (5 points)

$$Z(T, V, N) = \frac{Z(T, V, 1)^N}{N!} \tag{2}$$

where $Z(T, V, N)$ is the partition function of N particles enclosed in the volume V and having temperature T.

(b) Calculate $Z(T, V, 1)$ from the classical expression

$$Z(T, V, 1) = \int \frac{d^3x d^3p}{(2\pi\hbar)^3} e^{-\beta\epsilon} \tag{3}$$

and obtain the free energy F for the N particle system. (10 points)

(c) Calculate the energy E and the pressure P of the system in terms of T, V and N. (10 points)

5. Consider a system consisting of two particles that can only be in three states having energies 0 or $\pm\epsilon$. (36 points)

(a) Write down the possible states of the system, and calculate their energies. (You should have a total of 9 states) (9 points)

(b) For each possible energy that the system can have, calculate the entropy of the system if the system has that energy. and fill in the table below (Table 1). (10 points)

E	$\Delta\Gamma$	S

Table 1:

(c) Assume that to this system, we add a third particle that has the same properties, i.e. it can also be found in three different states that have energies 0 or $\pm\epsilon$. Repeat the step (a) and (b) for this system of 3 particles and fill in the Table 2. (7 points)

(d) Suppose that when the system had two particles, the system had a total energy $E = -2\epsilon$. When we add the third particle, how

E	$\Delta\Gamma$	S

Table 2:

should we change the total energy so that the entropy after the addition of the particle is the same as the entropy before the addition of the particle? This change in the energy is called the chemical potential of the system. (10 points)

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For anything else, you need to derive it.