PHYS 430 - Second Midterm
9 December 2006

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary. You will lose points if you make unnecessary assumptions.
The exam contains a total of 110 points, with 10 bonus points.
YOU HAVE 4 HOURS WITH A POSSIBILITY OF EXTENSION

1. During the class, we had shown that, if we take into account the bosonic or fermionic properties of an ideal gas, the equation of state can be written as:

$$PV = NT\left[1 \pm \frac{\pi^{3/2}}{2g}\frac{N\hbar^3}{V(mT)^{\frac{3}{2}}}\right] \quad (1)$$

up to higher order corrections. In the equation of state, the $+$ sign is for fermions and $-$ sign is for bosons, hence if one takes equal amounts of a bose gas, a classical gas and a fermi gas, and puts them into equal volumes, the pressure of the fermion gas will be larger than the pressure of the classical gas which will be larger then the pressure of the boson gas. Give a physical explanation why this is reasonable. (Do not use equations!). (20 points)

2. Consider a system consisting of two particles which can only be in three different states that have energies 0, ϵ and 2ϵ. If this system has a temperature T, Write down the partition function if the particles are: (20 points)

   (a) Distinguishable particles (5 points)
   (b) Identical particles obeying Boltzmann statistics (5 points)
   (c) Identical particles obeying Fermi statistics (5 points)
   (d) Identical particles obeying Bose statistics (5 points)

3. Consider a degenerate system of relativistic electrons ($\epsilon = cp$) confined to move on a two dimensional surface that has surface area A. Calculate the Fermi energy and the pressure of this system. (20 points)

4. In Fig. 1, the PV diagram of a heat engine is given. The axis of the diagram are P/P_0 and V/V_0, where P_0 and V_0 are some reference pressure and volume values. Plotted using these axis, the cycle is a semicircle. Calculate the efficiency of this engine. (20 points)

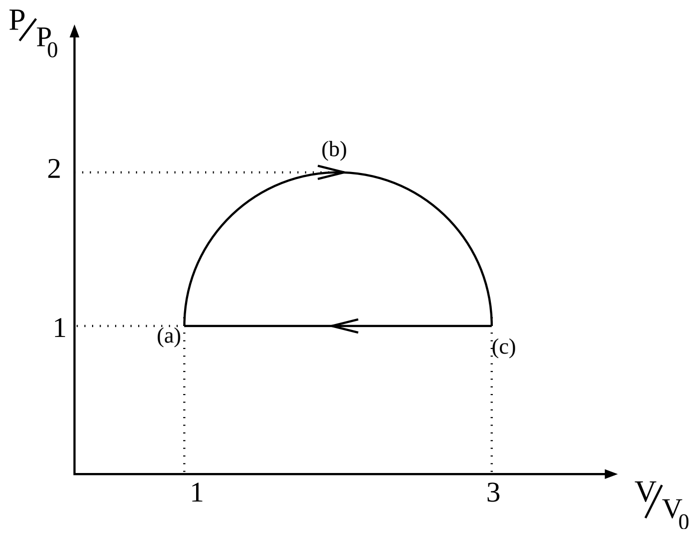

Figure 1: PV diagram for Question 4

5. Consider a gas of fermions or bosons whose equation of state is given by

$$PV = NT\left[1 \pm K\frac{N}{VT^{\frac{3}{2}}}\right] \tag{2}$$

where $+(-)$ sign is for fermions(bosons) and K is some positive constant. Using only the equation of state, show that the internal energy of this system can be written as:

$$E = \frac{3}{2}NT\left[1 \pm K\frac{N}{VT^{3/2}}\right] + E_0 \tag{3}$$

where E_0 is some constant. (30 points)

In order to show this:

(a) At a given fixed temperature, express the derivative

$$\left(\frac{\partial C_V}{\partial V}\right)_T$$

in terms of the equation of state. In order to solve this differential equation, you also need a boundary condition. The boundary condition can be obtained by considering the limit $V \to \infty$. In this limit, your result should reproduce the classical ideal gas result. (10 points)

(b) At a fixed volume, evaluate the derivative

$$\left(\frac{\partial E}{\partial T}\right)_V$$

in terms of the equation of state. (10 points)

(c) From the previous two parts, you have two differential equation for the energy:

$$\left(\frac{\partial E}{\partial V}\right)_T \tag{4}$$

which you have evaluated in the previous part and

$$\left(\frac{\partial E}{\partial T}\right)_V = C_V \tag{5}$$

Integrate both equation to obtain E. (You have to explain how you solve these equations. The result is already given in the question.)(10 points)

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST\ ;\quad W = E + PV\ ;\quad \Phi = E - ST + PV\ ;\quad \Omega = F - \mu N \\
\ln N! &\simeq N\ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}$$

For anything else, you need to derive it.