PHYS 430 - Makeup Exam
22 January 2007

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information or too few information. If the question does not contain sufficient information, make the necessary assumptions, stating why those assumptions are necessary. You will lose points if you make unnecessary assumptions. You will also lose points if you write information which is irrelevant to the question.
The exam contains a total of 120 points, with 20 bonus points.
YOU HAVE 4 HOURS WITH A POSSIBILITY OF EXTENSION
Justification for taking the Make-up Exam:

1. Explain the following concepts with your own words. Do not use equations, for each equation or irrelevant information you write, your will lose 1 point. (4 points each, 20 points total)
   (a) Maximum Efficiency
   (b) Debye temperature
   (c) Ensemble average
   (d) Phase space
   (e) Equipartition theorem

2. **Short questions** To the questions below, you can give short answers, giving short explanations of how you obtain that answer.(5 points each, no partial credit, 20 points)
   (a) Consider two system, system A which is at a temperature $5K$ and system B, which is at $10K$. If the two systems are brought into contact, what will be the direction of heat flow?
   (b) In question (a), if the body A is instead at a temperature $-5K$, what will be the answer?
   (c) Assume that you have two fermions in a box at a temperature T. Let the energy of the n^{th} state be $n\epsilon$. What is the partition function if the system has total energy 2ϵ?

(d) In question (c), what will be the partition function if the particles are bosons?

3. Consider a system of N spin-$\frac{1}{2}$ particles. Assume that the Hamiltonian of the system is given by $H = -\alpha B \sum_i \sigma_3$ where σ_3 is the third Pauli matrix. If the system has total energy E, what is the temperature of this system? (15 points) Sketch the dependence of T on E (5 points). *Hint:*

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

4. The free energy F of an ideal gas can be written as

$$F = -NT \ln \frac{eV}{N} + Nf(T) \tag{2}$$

where $f(T)$ is an in general unknown function of T only and it parameterizes the internal structure of the particles making the gas. If it is known that the specific heat c_V is independent of temperature, show that the most general form of $f(T)$ is (10 points)

$$f(T) = -c_V T \ln T - \zeta T + \epsilon_0 \tag{3}$$

Using the explicit for of $f(T)$, calculate the thermodynamical potentials S, E, and W.(10 points)

5. Consider a cylinder filled with a gas made of monatomic particles. Assume that the cylinder has height h and radius R. If there is an external potential acting on the particles which has the form $V(r) = \alpha r$ where α is a constant and r is the distance of the particle from the central axis of the cylinder, calculate the pressure exerted on the lateral surface of the cylinder. (15 points) Show that for $\alpha = 0$, one recovers the ideal gas equation of state. (5 points)

6. Consider a Carnot Engine that works between two reservoirs at temperature T_1 and $T_2 < T_1$ and which utilizes a monatomic Van der Walls gas as the working medium. For each of the four stages of the Carnot cycle, calculate the change in the energy, the work done and the heat absorbed. (12 points) Show explicitly that the efficiency of this Carnot engine is (8 points)

$$\eta = 1 - \frac{T_2}{T_1} \tag{4}$$

The Equation of state of the Van der Waals gas is

$$\left(P + a\frac{N^2}{V^2}\right)(V - bN) = NT \tag{5}$$

The adiabatic equation for the Van der Waals gas is

$$\left(P + a\frac{N^2}{V^2}\right)(V - bN)^\gamma = NT \tag{6}$$

where $\gamma = 5/3$. The energy of the Van der Waals gas can be written as:

$$E = \frac{3}{2}NT - a\frac{N^2}{V} \tag{7}$$

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
d\Omega &= -SdT - PdV - Nd\mu \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \ ; \quad \Omega = F - \mu N \\
\ln N! &\simeq N\ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1 \\
\int d^3Z &= \int_0^\infty z^2 dz \int_{-1}^1 d\cos\theta_Z \int_0^{2\pi} d\phi_Z = \int_0^\infty \rho d\rho \int_0^{2\pi} d\varphi_Z \int_{-\infty}^\infty dZ_3 \\
\int_0^\infty dx \frac{x}{e^x - 1} &= \frac{\pi^2}{6} \ , \quad \int_0^\infty dx \frac{x^2}{e^x - 1} = 2.404 \ , \quad \int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \qquad (8) \\
& \qquad (9)
\end{aligned}$$

For anything else, you need to derive it.