Phys 430 - 2^{nd} HOMEWORK

1. How do you interpret negative temperatures? Are you convinced that negative temperatures are hotter than positive temperatures? If yes, why? If no, why? Show that heat always flows from negative temperatures to positive temperatures. (10 points)

2. Consider an ideal gas consisting of point particles whose energy momentum relation is given by $\varepsilon = cp$ whare c is some constant and p is the magnitude of the momentum of the particles. For this gas, calculate:

   The number of states $\Gamma(E)$ that have energy less then or equal to E (10 points)

   The entropy, S, of the system (5 points)

   The temperature, T, of the system (5 points)

   Calculate the thermodynamic potentials $E(S,V)$, $F(T,V)$, $W(S,P)$ and $\Phi(T,P)$ (10 points)

   Calculate C_P and C_V (5 points)

3. Consider again the system of N non-interacting dipoles that we have studied in the class. We had shown that if one takes only one of the dipoles, the probabilities that it will have energy $\pm\varepsilon$ are given by

$$P(\pm\varepsilon) = Z^{-1} e^{\mp\beta\varepsilon} \tag{1}$$

   where $\beta = T^{-1}$. Now, take two of the dipoles.

   What are the possible states that the dipoles can be found in? (5 points)

   Show explicitely (showing all your work) that the probability that these two dipoles are in the state n is given by

$$P_n = Z^{-1} e^{-\beta\varepsilon_n} \tag{2}$$

   where ε_n is the energy of the n^{th} state and $Z = e^{-2\beta\varepsilon} + 2 + e^{2\beta\varepsilon}$ (15 points)

4. Express the following derivative in terms of derivatives of the equation of state and C_p or C_v. Find processes for which each derivative is relevant. (3 points each)

$$\left(\frac{\partial E}{\partial P}\right)_T, \left(\frac{\partial W}{\partial V}\right)_T, \left(\frac{\partial E}{\partial T}\right)_P, \left(\frac{\partial W}{\partial P}\right)_T, \left(\frac{\partial W}{\partial T}\right)_V$$

5. Consider a system whose equation of state is given by

$$\left(P + a\frac{N^2}{V^2}\right)(V - Nb) = NT$$

for some positive constants a and b (this is called the Van der Waals' equation of state). For this system

Calculate the derivatives

$$\left(\frac{\partial E}{\partial V}\right)_T$$

and

$$\left(\frac{\partial E}{\partial T}\right)_V$$

. (10 points)

By integrating you results, find $E(V, T)$ for this system upto a constant. (10 points)

In the limit $V \to \infty$, the van der Waals' equation of state reduces to the ideal gas equation of state. The energy of the Van der Waals' gas in this limit also reduces to the energy of the ideal gas. By considering this limit, and comparing the energy with the energy of the ideal gas, fix the constant in the expression for the energy of the Van der Waals' gas.(bonus: 10 points)