PHYS 430 - First Midterm
2 April 2006

Name and Surname:
Student ID:
Department:
Signature:

INSTRUCTIONS

Write all your steps with explanations of why you do those steps. The questions might contain extra information of too few information. If the question does not contain sufficient information, make necessary assumptions, stating why those assumptions are necessary.

1. Comment on the following concepts (Just writing equations will not gain you any points): (3 points each, 21 points total)

   Negative Temperatures

   Micro canonical Distribution

   P-V Diagram

   Thermodynamical Potential

   Reversible process

   Ergodic Hypothesis

   Entropy

2. Consider a gas of identical particles confined to move only on the surface of a rectangular area of side length L. The energy momentum relation for the particles is given as $\epsilon = \alpha(p_x^2 + p_y^2)$ where p_x and p_y are the components of the momenta along two perpendicular directions on the surface and α is some constant. Calculate the entropy of this system if the system has energy E. (10 points). Calculate the temperature, T(5 points), of the system and the surface tension σ(5 points) of the system. (Hint: For a two dimensional system, $dE = TdS - \sigma dA$ where dA is a differential change of the area of the system and σ is called the surface tension) (20 points)

3. Consider a Carnot' cycle that uses the Van Der Waals gas as the working gas. Show explicitly that the efficiency of this Carnot' cycle is given by $\eta = 1 - \frac{T_{cold}}{T_{hot}}$. In order to show this you need to go through several steps: (61 points)

(a) First you need to calculate the adiabatic curve for the Van der Waals gas. To calculate the adiabatic curve, consider the derivative

$$\left(\frac{\partial V}{\partial T}\right)_S$$

First change your variables to (V, T) from (S, T) and obtain the relation between the volume and temperature in an adiabatic process. Then substituting T from the Van der Waals equation of state, show that in an adiabatic process:

$$(V - Na)^{5/3}\left(P + a\frac{N^2}{V^2}\right) = const$$

(15 points)

Draw the P-V diagram for the Carnot cycle. (5 points)

Calculate the work done on the gas, the heat absorbed by the gas, and the change in the internal energy of the gas for each of the 4 stages of the Carnot cycle. (You need t calculate $3 \times 4 = 12$ results. 3 points each of the results (36 points)

Calculate the efficiency (5 points)

(Hint: The Van der Walls equation of state is

$$\left(P + a\frac{N^2}{V^2}\right)(V - Nb) = NT$$

and the energy of a Van der Waals gas can be written as:

$$E = \frac{3}{2}NT - a\frac{N^2}{V}$$

)

4. Calculate the rate of change of the temperature of a Van der Waals' gas undergoing a Joule Thompson process. (Hint: a Joule Thompson process is a process that changes the pressure of the gas keeping the heat function constant). The relevant information about the Van der Waals gas is given in the previous problem.

You can use the following formulas/definitions without deriving them:

$$
\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty x^n e^{-x} &= n! \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}
$$

For multi dimensional spherically symmetric integrals, you can use the formula:

$$\int d^D x f(r) = \frac{2\pi^{D/2}}{\left(\frac{D}{2}-1\right)!} \int_0^\infty dr r^{D-1} f(r) \tag{1}$$

where $r^2 = \sum_{i=1}^D x_i^2$

For anything else, you need to derive it.