PHYS 430 - FINAL EXAM
27 MAY 2006

Name and Surname:
Student ID:
Department:
Signature:

Section I

1. Compare the three distributions that we have studied: microcanonical, canonical and grandcanonical. How do they differ? Under what conditions would you prefer one over the other? and why? (the last part is a personal question and there is no right or wrong. You have to justify yourself by answering to the "why" part)(20 points)

Section II

In this section you will be asked to solve slightly modified versions of the sample questions that I have distributed. Since these questions are not identically the same, and since you should not be credited for your ability to memorize solutions, you will not gain any points (not even partial) if you solve the unmodified version in the sample questions.

2. Consider two particles which can be only in 3 states that have energies ϵ, 2ϵ and 3ϵ. If the system is at a temperature T, write the partition function of the system and calculate the energy $\langle E \rangle$ if the particles are:

   (a) Distinguishable particles(5 points)

   (b) Identical particles satisfying the Boltzmann distribution (5 points)

   (c) Identical particles which are fermions (5 points)

   (d) Identical particles which are bosons.(5 points)

3. Consider a subsystem consisting of two single particles states in a metal which have energies ϵ and 2ϵ. If the metal is at temperature T and has a chemical potential μ, what is the average number of electrons in the subsystem?(8 points) What is the relative fluctuation in the number of particles in the subsystem? (12 points)

4. Suppose that you have a gas of N identical particles entrapped in a cylinder of radius R and height h and are subject to a potential $V = \alpha\rho$ where α is some constant and ρ is the distance from the center of the cylinder. Assume that the kinetic energy of the particles is related to the momentum through $\epsilon = cp$ where c is some constant and p is the magnitude of the momenta of the particle.

(a) Calculate the the density of particles as a function of the radius from the center. Properly normalize the density so that the integral of the density over the whole cylinder gives N, the total number of particles. (10 points)

(b) Calculate the partition function of the system. (10 points)

(c) Calculate the pressure the gas exerts on the cylinder surface. (10 points)

Section III-Presentation Questions

5. Answer **FOUR** of the following questions. Each question worths 5 points. If you have made a presentation, you will not gain any points if you answer the question related with your presentation. Write in the boxes below which questions you have answered. I will only consider the questions that are mentioned in the boxes, any other answers will be ignored.

(a) What is the Fermi energy? Calculate the Fermi energy of a free electron system.

(b) Calculate the pressure of a degenerate electron gas.

(c) Show that if your have an interacting gas such that the total energy of the system is:

$$E = \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m} + U(\vec{x}_1, \vec{x}_2, ..., \vec{x}_N) \quad (1)$$

where U is the interaction potential, show that the free energy of the system can be written as:

$$F = F_{id} - kT \ln \left[1 + \frac{1}{V^N} \int \cdots \int dV_1 dV_2 \cdots dV_N \left(e^{-\beta U} - 1 \right) \right] \quad (2)$$

(d) Consider a cavity which contains a photon gas in equilibrium. What is the average number of photons that have energy between ϵ and $\epsilon + d\epsilon$

(e) Show that if you have two phases in equilibrium, the two phases should have the same chemical potential. (*Hint:* Consider the two phases as two subsystems that can exchange particles).

(f) What is the distinction between Einstein theory and Debye's theory of solids?

(g) Consider the chemical reaction $2O_3 \leftrightarrow 3O_2$ at constant temperature. What is the relation between the chemical potentials of O_2 and O_3 in equilibrium.

(h) Is information physical? Explain.

Section IV - BONUS QUESTION

6. For most solids, if you increase the pressure, the melting point of the solid phase becomes higher;i.e. the substance becomes more solid. This is not the case for water. If you have ice at the melting point, by an increase of pressure the ice melts. Explain this using La Chateliers principle. (*Hint:* Contrary to other substances, ice has a larger volume per particle, then water at 1 atm and $0°C$)

You can use the following formulas/definitions without deriving them:

$$\begin{aligned}
dE &= TdS - PdV + \mu dN \\
dF &= -SdT - PdV + \mu dN \\
dW &= TdS + VdP + \mu dN \\
d\Phi &= -SdT + VdP + \mu dN \\
F &= E - ST \ ; \quad W = E + PV \ ; \quad \Phi = E - ST + PV \\
S &= \ln \Delta\Gamma(E) \ ; \quad \Delta\Gamma(E) = \Delta E \frac{\partial}{\partial E}\Gamma(E) \\
\ln N! &\simeq N \ln N - N \\
\int_0^\infty dx x^n e^{-\gamma x} &= \frac{n!}{\gamma^{n+1}}, \qquad \int_0^R d\rho \rho e^{-\alpha\rho} = \frac{1}{\alpha^2}\left[1 - (1+\alpha R)e^{-\alpha R}\right] \\
\beta &= \frac{1}{T}, \qquad k = 1
\end{aligned}$$

For anything else, you need to derive it.